THE STATISTICAL PROCESSING OF RESULTS OF IMITATION MODELING OF BRIGHTNESS DISTRIBUTION

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Abstract

In this paper, we present some results of statistical processing of the numerical simulation data of the characteristics of vision systems through the atmosphere obtained with the help of a special software package created by us (Gendrina I.Yu., Kvach A.S.).

Keywords: *Vision system, angular brightness distribution, Monte Carlo method, regression*

1. INTRODUCTION

Various methods of statistical research such as correlation-regression analysis, dynamic series, variance analysis, etc. in the study of vision systems are using for studying and subsequent prediction of the patterns of radiation transfer. We have conducted Monte Carlo experiments to calculate the Point Spread Function (PSF) for linear system “underlying surface – atmosphere”. The one is defined as the system response to the input signal, representing a point mass, located at a certain point, and can be determined as the angular brightness distribution of surface-based point source measured with receiving device at the top of the atmosphere. Then we have attempted to apply elements of correlation-regression analysis for the study of the influence of various optical and geometrical parameters on the Point Spread Function.

Vision system (VS) is understood as an observation scheme including the underlying surface, a "cloudy environment" (atmosphere), and an optical device that captures incoming radiation. To study radiation transfer in such systems, the theory of systems and the theory of radiation transfer are traditionally used (Zuev V.E., Belov V.V., Veretennikov V.V.).

The main system characteristic for VS is the point spread function (PSF); it is defined as the response L of linear system to the input signal, representing a point mass $\delta(x-x_1)\delta(y-y_1)$, located at a certain point (x_1, y_1): $L[\delta(x-x_1)\delta(y-y_1)] = h(x, y; x_1, y_1)$.

An arbitrary object (function) $f(x, y)$ can be considered as a set of point masses. For instance:

$$f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x_1,y_1)\delta(x-x_1)\delta(y-y_1)dx_1dy_1$$

Then, a result of the system impact (image) can be represented in the form:

$$g(x,y) = L[f(x,y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x_1,y_1)h(x,y;x_1,y_1)dx_1dy_1$$

Obviously, regularities of the image distortion due to impact of any system can be studied by analyzing the effect of this system on the point spread function.

The model of the atmosphere includes the following characteristics: the total extinction coefficient $\sigma(\lambda,\vec{r}) = \sigma_{sc}(\lambda,\vec{r}) + \sigma_a(\lambda,\vec{r})$, , where $\sigma_{sc}(\lambda,\vec{r})$ – the scattering coefficient, $\sigma_a(\lambda,\vec{r})$– the absorption coefficient; $g(\lambda,\mu,\vec{r})$ – aerosol phase function. Here $\vec{r} = (x,y,z)$ – radius-vector of the current point in space, $\mu(\vec{\omega}',\vec{\omega})$ – cosine of the scattering angle of radiation coming from direction $\vec{\omega}'$, in the direction ω, λ – is the wavelength of incident radiation.

The paper considers two models of the atmosphere:

1. vertically bounded plane-parallel layer-homogeneous aerosol-molecular;
2. vertically bounded plane-parallel aerosol-molecular, including overcast layer. For the cloud layer, different characteristics from those of the first model is assumed: coefficients of extinction, absorption, scattering, and the aerosol phase function.

2. METHODS AND INITIAL DATA

2.1. Methods

The geometric scheme of calculations is as follows: at the lower boundary of the atmosphere $z = z_0$ there is a point source of unit capacity. At the upper boundary of the atmosphere ($z = H$) there is an optical receiver that can receive scattered radiation coming from different directions (observation angles). The brightness of the scattered radiation is solution of the integro-differential transport equation (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.), which can be practically solved only by approximate or numerical methods.

One of the most universal methods for solving this problem is the simulation method, or the Monte Carlo method. The basis of the Monte Carlo method is the integral transport equation of the second kind with a generalized kernel for the density of particles' collisions (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.):

$$f(\vec{x}) = \int_X k(\vec{x}',\vec{x}) f(\vec{x}')d\vec{x}' + \psi(\vec{x}) \text{ or } f = Kf + \psi$$

Here $\vec{x} = (\vec{r},\vec{\omega})$ – is the point of the phase space of coordinates and directions, $\psi(\vec{x})$– source function, K – integral operator with kernel $k(\vec{x}',\vec{x})$:

$$k(\vec{x}',\vec{x}) = \frac{\sigma_{sc}(\vec{r}\,)\cdot g(\mu)exp(-\tau(\vec{r}\,',\vec{r}\,))\sigma(\vec{r}\,)}{\sigma(\vec{r}\,')2\pi\left|\vec{r}-\vec{r}\,'\right|^2}\cdot\delta\left(\vec{\omega}-\frac{\vec{r}-\vec{r}\,'}{\left|\vec{r}-\vec{r}\,'\right|}\right).$$

In this paper, one of the algorithms of the Monte Carlo method was used - the method of local estimation (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.).

Algorithm for local estimation consists in calculating following functional:

$$J(\Omega_i) = \int\limits_{\Omega_i} \Phi(\vec{r}^{\,*},\vec{\omega}^{\,*})d\vec{\omega}^{\,*} = \int\limits_X l_i(\vec{x}\,',\vec{x}^{\,*}) f(\vec{x}\,')d\vec{x}\,' =$$

$$= M\sum_{n=0}^{N} Q_n \cdot l_i(\vec{x}_n,\vec{x}^{\,*})$$

$$l_i(\vec{x},\vec{x}^{\,*}) = \frac{exp(-\tau(\vec{r},\vec{r}^{\,*}))\cdot g(\mu^*)}{2\pi\left|\vec{r}-\vec{r}^{\,*}\right|^2}\Delta_i(\vec{s}^{\,*})$$

Here $s^* = \frac{\vec{r}^*-\vec{r}}{|\vec{r}^*-\vec{r}|}$, $\mu^* = (\vec{\omega}, s^*)$, $\Delta_i(s^*)$ – is the indicator of region Ω_i. Φ – flux of particles at given point $\vec{x}^*$. Q_n– weight of the particle, $f(\vec{x})$ – density of collisions.

2.2. Ininial data

We will consider the process of radiative transfer through aerosol-molecular atmosphere, which comprises a layer overcast, by neglecting the reflection from underlying surface. We used the following data (Gendrina I.Yu., Kvach A.S.):

Wavelength (μm) in transparent windows: 0.530; 0.694; 0.860; 1,060; 3,390; 10.60.

Lower boundary of atmosphere $z_0 = 0$ km above Earth's surface, upper boundary of the atmosphere $H = 30$ km above the Earth's surface.

Optical thickness for a cloudless atmosphere are presented in Table 1.

Wavelength, µm	Optical thickness
0,53	0,158
0,694	0,124
0,86	0,098
1,06	0,092
3,39	0,067
10,6	0,041

Table 1. Optical thickness of the cloudless atmosphere

Lower boundary of the cloud layer - 1 km above the Earth's surface, upper boundary - 2 km above the Earth's surface. The optical models of the cloud layer – “haze H” and “cloud C1” (Deirmendjian D.)

In this work, we considered isotropic and Lambertian models of sources of radiation. In the first case the density of the initial areas looks like: $p(\vec{\omega}) = \frac{1}{2\pi}$. In the second case this density is: $p(\vec{\omega}) = \frac{\mu}{\pi}$, where $\mu = arccos\ \theta$, θ - zenith angle of initial direction.

3. SIMULATION RESULTS

Quantitative values brightness of scattered radiation for the cloudless atmosphere are presented in our previous publication (Gendrina I.Yu., Alekseenko M. A.). Similar values for various models of the cloud atmosphere are given in Tables 2 - 5.

Angles of reception, deg	0,53 µm	0,694 µm	0,86 µm	1,06 µm	3,39 µm	10,6 µm
4,5	5,01E-07	5,29E-07	5,46E-07	5,18E-07	4,91E-07	3,14E-07
13,5	1,36E-07	1,01E-07	9,47E-08	8,07E-08	3,58E-08	3,65E-08
22,5	4,30E-08	4,23E-08	3,50E-08	2,85E-08	8,37E-09	7,65E-09
31,5	1,84E-08	1,20E-08	1,19E-08	7,86E-09	3,73E-09	2,17E-09
40,5	8,43E-09	6,32E-09	5,25E-09	3,75E-09	6,66E-10	9,15E-10
49,5	2,46E-09	3,19E-09	2,69E-09	1,97E-09	5,41E-10	4,77E-10
58,5	1,71E-09	1,36E-09	1,37E-09	1,17E-09	3,49E-10	2,72E-10
67,5	1,10E-09	7,91E-10	7,98E-10	6,56E-10	2,15E-10	1,25E-10
76,5	1,07E-09	7,76E-10	5,14E-10	5,12E-10	9,37E-11	4,19E-11
85,5	5,61E-10	9,50E-11	1,58E-10	1,05E-10	1,06E-11	5,66E-12

Table 2. Brightness of scattered radiation for the atmosphere with a cloud layer of the "Haze H" type, isotropic source, W/µm·m^2

Angles of reception, deg	**0,53 μm**	**0,694 μm**	**0,86 μm**	**1,06 μm**	**3,39 μm**	**10,6 μm**
4,5	9,65E-07	1,03E-06	1,05E-06	9,78E-07	9,28E-07	5,78E-07
13,5	2,41E-07	1,75E-07	1,62E-07	1,34E-07	5,35E-08	4,92E-08
22,5	7,21E-08	6,84E-08	5,63E-08	4,52E-08	1,15E-08	8,23E-09
31,5	2,97E-08	1,85E-08	1,77E-08	1,11E-08	4,57E-09	1,74E-09
40,5	1,25E-08	8,56E-09	6,67E-09	4,44E-09	4,25E-10	5,99E-10
49,5	3,16E-09	3,89E-09	2,99E-09	2,19E-09	3,32E-10	2,04E-10
58,5	1,98E-09	1,48E-09	1,42E-09	1,12E-09	1,97E-10	1,01E-10
67,5	1,06E-09	6,65E-10	6,47E-10	4,48E-10	1,49E-10	3,24E-11
76,5	9,77E-10	5,90E-10	2,88E-10	2,86E-10	4,65E-11	1,94E-11
85,5	5,10E-10	4,38E-11	1,20E-10	6,60E-11	2,55E-12	2,01E-12

Table 3. Brightness of scattered radiation for the atmosphere with a cloud layer of the "Haze H" type, Lambertian source, $W/\mu m \cdot m^2$

Angles of reception, deg	**0,53 μm**	**0,694 μm**	**0,86 μm**	**1,06 μm**	**3,39 μm**	**10,6 μm**
4,5	5,01E-07	5,45E-07	5,69E-07	5,18E-07	4,91E-07	3,38E-07
13,5	1,36E-07	1,01E-07	9,47E-08	8,07E-08	3,59E-08	4,21E-08
22,5	4,30E-08	4,23E-08	3,50E-08	2,85E-08	8,39E-09	9,36E-09
31,5	1,84E-08	1,20E-08	1,19E-08	7,86E-09	3,73E-09	2,66E-09
40,5	8,43E-09	6,45E-09	5,16E-09	3,70E-09	6,67E-10	9,83E-10
49,5	2,44E-09	3,10E-09	2,69E-09	1,97E-09	5,41E-10	4,98E-10
58,5	1,59E-09	1,36E-09	1,39E-09	1,19E-09	3,49E-10	2,76E-10
67,5	1,13E-09	7,77E-10	8,25E-10	7,02E-10	2,17E-10	1,37E-10
76,5	9,73E-10	7,45E-10	5,32E-10	5,11E-10	9,01E-11	3,96E-11
85,5	5,77E-10	8,95E-11	1,78E-10	1,12E-10	1,08E-11	5,50E-12

Table 4. Brightness of scattered radiation for the atmosphere with a cloud layer of the "Cloud C1" type, isotropic source, $W/\mu m \cdot m^2$

Table 2-3 contains the results for the brightness of scattered radiation for the atmosphere with a cloud layer of the "Haze H" type.

Tables 4-5 contains similar data for the atmosphere with a cloud layer of the "Cloud C1" type.

These types vary in value of the average cosine of scattering phase function:

$$\bar{\mu} = \frac{1}{2}\int_{-1}^{1} \mu g(\mu) d\mu.$$

It is known that this parameter characterizes the elongation of aerosol phase function. For example in case $\lambda = 0{,}694$ μm the average cosine amounts to 0,745 for type “Haze H” and 0,857 for type “Cloud C1”.

Angles of reception, deg	**0,53 µm**	**0,694 µm**	**0,86 µm**	**1,06 µm**	**3,39 µm**	**10,6 µm**
4,5	9,65E-07	1,04E-06	1,07E-06	9,78E-07	9,29E-07	6,16E-07
13,5	2,41E-07	1,75E-07	1,62E-07	1,34E-07	5,35E-08	5,33E-08
22,5	7,21E-08	6,84E-08	5,63E-08	4,52E-08	1,15E-08	8,99E-09
31,5	2,97E-08	1,86E-08	1,77E-08	1,11E-08	4,57E-09	1,89E-09
40,5	1,25E-08	8,72E-09	6,57E-09	4,40E-09	4,26E-10	6,13E-10
49,5	3,14E-09	3,82E-09	2,99E-09	2,19E-09	3,32E-10	2,07E-10
58,5	1,79E-09	1,49E-09	1,43E-09	1,13E-09	1,97E-10	1,01E-10
67,5	1,10E-09	6,29E-10	6,68E-10	5,02E-10	1,49E-10	3,56E-11
76,5	8,10E-10	5,40E-10	3,08E-10	2,74E-10	4,40E-11	1,87E-11
85,5	5,29E-10	4,13E-11	1,30E-10	6,85E-11	2,55E-12	1,98E-12

Table 5. Brightness of scattered radiation for the atmosphere with a cloud layer of the "Cloud C1" type, Lambertian source, W/µm · m^2

4. DISCUSSION

In our previous studies we found the regressional relationship between the brightness of the upwelling radiation and various geometrical parameters such as angle of observation, the height of the lower and upper boundary of the cloud layer. In this paper we estimated the effect of the type of radiation source located on the underlying surface on the brightness value for different wavelengths. Two statistical mechanisms were used to assess this effect: using the dummy variables and by identifying the trend.

Let us describe the first scheme. We introduce a dummy variable *D* corresponding to different types of source and assuming the following values: *D*=0 for an isotropic source and *D*=1 for a Lambertian source.

We will derive an equation for the dependence of the brightness y on angle x in the form of multiple regression:

$$y = b_0 + \frac{b_1}{x} + \gamma_1 D.$$

Using the testing of statistical hypothesis mechanism, we estimate the statistical significance of the coefficient γ_1. The significance of the coefficient shows the significant influence or its absence of the source type on the brightness value.

The second scheme involves building a trend on the available data combined into one sample regardless of the type of the source. A high value of the determination coefficient for the equation can obviously speak of the homogeneity of the data, i.e. of the absence of a significant influence of the source type on the brightness value.

Tables 6-7 contain the values of the coefficients b_0, b_1 and γ_1, as well as the confidence intervals for the significance level of 0,05. The data given shows that coefficients b_0 and b_1 are statistically significant, and the coefficient γ_1 is statistically insignificant. From this we can conclude that the influence of the source type is insignificant.

	Value	Lower 95%	Upper 95%
b_0	-9,70E-08	-1,56E-07	-3,82E-08
b_1	3,55E-06	2,95E-06	4,15E-06
γ_1	6,16E-08	-1,14E-08	1,35E-07

Table 6. The values of the coefficients b_0, b_1 and γ_1, as well as the confidence intervals for the significance level of 0,05. 0,53 µm. Haze H.

	Value	Lower 95%	Upper 95%
b_0	-9,70E-08	-1,56E-07	-3,82E-08
b_1	3,55E-06	2,95E-06	4,15E-06
γ_1	6,16E-08	-1,14E-08	1,34E-07

Table 7. The values of the coefficients b_0, b_1 and γ_1, as well as the confidence intervals for the significance level of 0,05. 0,53 µm. Cloud C1.

The same conclusion can be made based on figures 1-4, which present modeling data for different wavelengths, as well as trend lines. For clarity the figures also include form of the corresponding equation and the value of the determination coefficient.

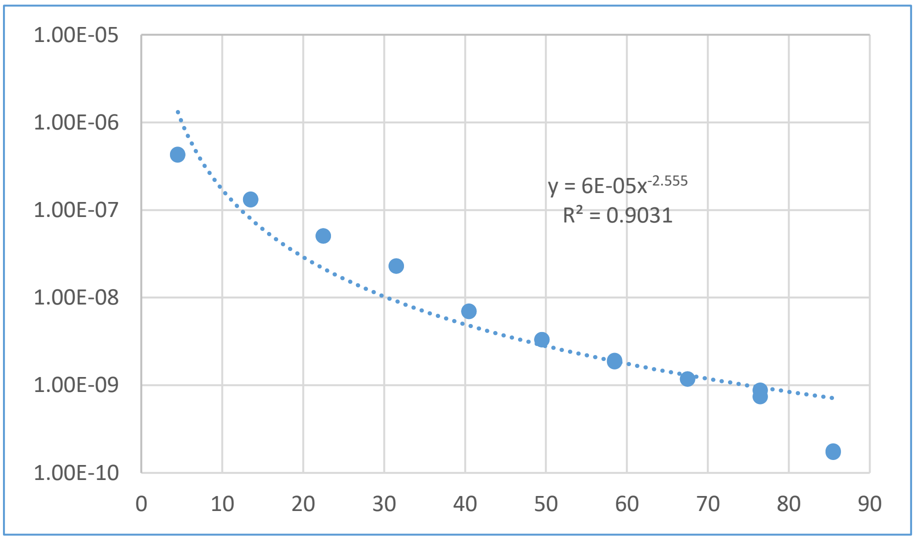

Fig. 1. Simulation results, 0,53 mkm, haze H.

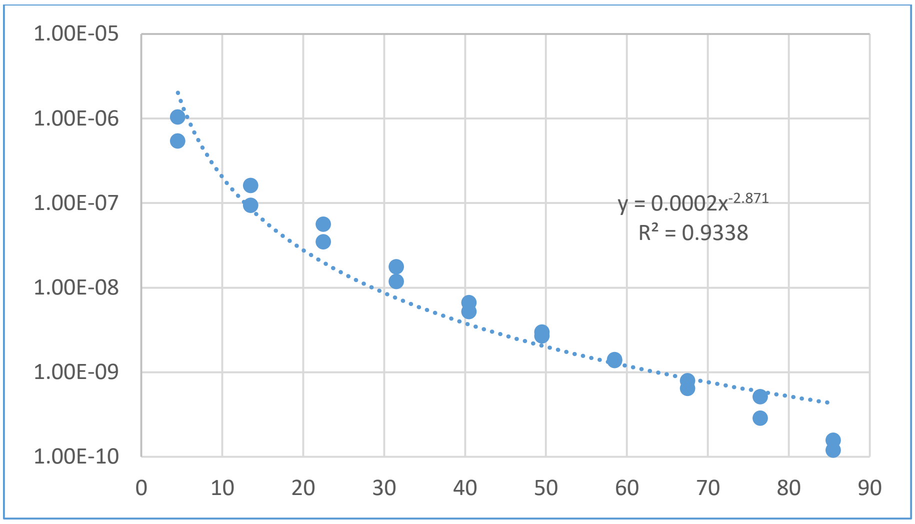

Fig. 2. Simulation results, 0,86 mkm, haze H.

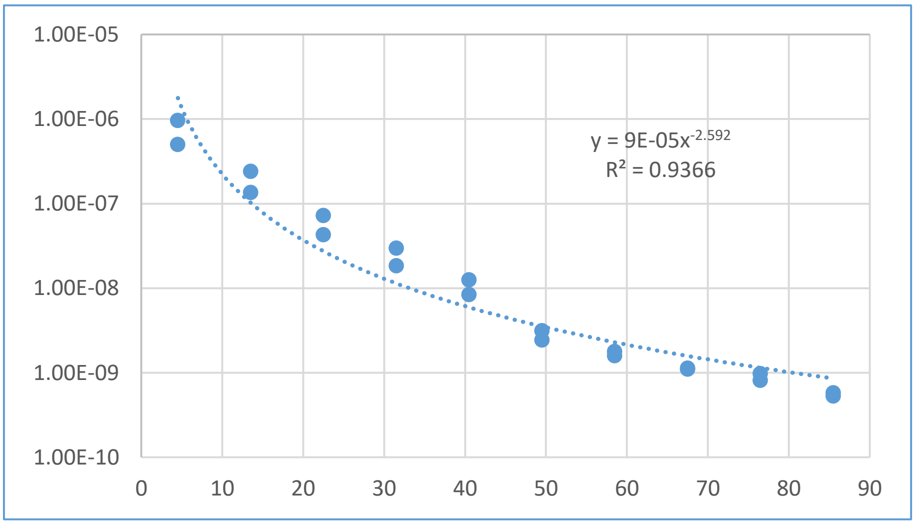

Fig 3. Simulation results, 0,53 mkm, cloud C1.

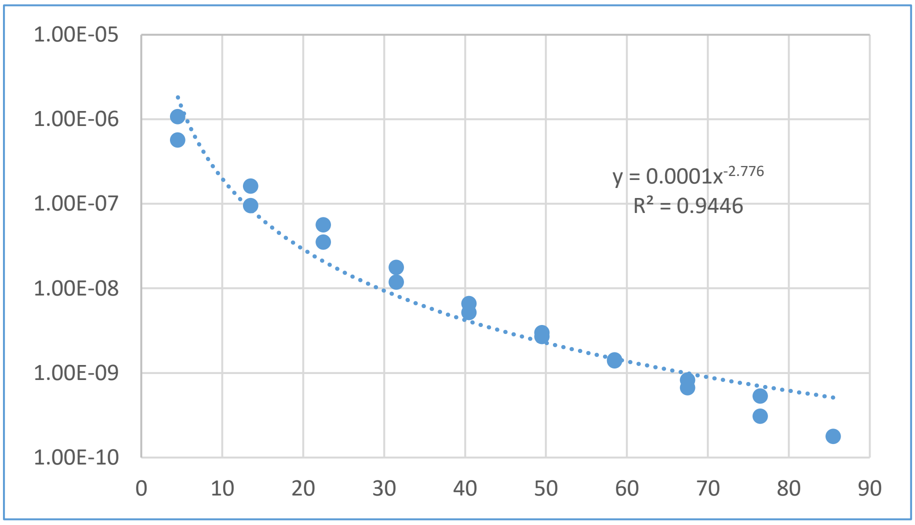

Fig 4. Simulation results, 0,86 mkm, cloud C1.

5. CONCLUSIONS

Statistical estimation of regression equations for significance was carried out on the basis of the F-test

The statistical significance of determination coefficient can be confirmed with the help of Fisher statistics: $F = \frac{R^2}{1-R^2}(n-2)$, where n is the number of observations.

With 90% confidence, it can be argued that the considered dependence is statistically significant.

Based on the above, it can be concluded that the brightness of the scattered radiation under the considered conditions is a monotonically decreasing function of the observation angle for each wavelength. Brightness values are almost independent of the source type and optical properties of the cloud layer.

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