A STATISTICAL ANALYSIS OF THE IMPACT OF THE INFORMAL ECONOMY ON THE COMPORTMENT OF THE EXCHANGE RATES IN ALBANIA

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Abstract

The use of foreign currencies as partial substitute of national currency is qualified herein as contributors for a relatively good conduct of the Euro-ALL FX rates. Its effect is considerable due to the significant size of the informal sector where this economic event takes place, and by other cash inflows from Euro zone-economies. At the same time, those occurrences have contributed on the over-evaluation of the national currency but imposed high nonstationary of the rate of return distribution and large multifractality spectrum of the exchange prices.

Key words: exchange rates, informal economy, stationarity

A GENERAL VIEW ON EXCHANGE RATE FORMATION

The effects of informal or underground economy on socio-economic and financial indicators are well-known and studied thoroughly from the scientific and econometric perspectives. Its relationship and dependence with labour markets indicators, financial indexes, productivity, and domestic product are modelled and evaluated quantitatively, see (Dell’Anno, R. 2023), (Schneider & Enste 2000), (Dell’Anno & Schneider, 2009), (Alberola & Urrutia, 2019), (Elgin et al, 2021), for example. From a general point of view, direct relationship between IE and exchange rates may exists, and some scholars have considered them in specific modelling, as in (Edeme, 2022) for example. It is expected also that those relationships would be specific due to the specifics of the currency FX rates for a certain economy etc. In this regard, we have analysed herein the effects of the informal economy FX rates, for a specific economy where the informal sector is sizeable, there are considerable flows of foreign currencies injected continuously in this economy, a specific foreign currency is used in everyday transaction without being converted in the national currency, etc. Mutual interaction of the shadow economy with money aggregates or other quantities related with FX rates, as remittances and informal economy (Chatterjee & Turnovsky, 2018), monetary policies and informal economy, economy size effect on FX rates (Mohsin et al, 2007), (Betts & Devereux, 2000) etc., suggest to include them on standard models discussed in the literature, as for example in (Siregar, 2007), (Macdonald, 1995), (Hamilton, 2018). Note that general models described in FX (Macdonald, 1995), (Hamilton, 2018), (Kuzmin, 2022), (Gray, 1979), etc., do not include the shadow economy directly. In some less general address as (Edeme, 2022) the shadow economy and FX rates appear together in a simple auto regressive model in the form \( V(t) = \sum_{i=1}^{p} a_i V_{t-i} + u_t \). Also, we have attempted to adopt a MIMIC approach for this analysis using our system data, but the unknown amount of the foreign currency used in IE pose a critical setback for quantitative assessment. We are skipping those results for a while to consider an empiric description of the relationships based on the dynamics and statistical findings. Herein we will consider particularly the Euro-ALL FX behavior as typical case associated with specifics motioned above. In principle, the effects of informal economy on the FX rate can be recognised directly from the presence of the money aggregates in the respective equations of the standard theoretical approaches. So, according to the equilibrium approach, the real logarithmic exchange rate is given by a linear formula

\[
E_{\text{real}} = (E_{\text{nominal}} + p_T - p_T) - ((1 - \beta)(p_{NT}^T - p_T) - (1 - \beta')(p_{NT}^{NT} - p_T^{NT}) \quad (1)
\]

having \( E_{\text{nominal}} = (m_t - m_f) - a_1(y - y_f) - a_2(i_t - i_f) + \epsilon_t \), with \( p \) the prices level in both countries, \( i \) the interest rates, \( m \) is the money, \( y \) is the GDP measure and \( \epsilon_t \) is a stochastic term, usually taken as Weiner process, see (Betts & Devereux, 2000), (Siregar, 2007), (Kuzmin, 2022) (Gray, 1979) for more details. The superscript \( f \) indicates “foreign”, and \( T \) and \( NT \) assigns tradable and not tradable nature. Evidently from (1) the informal economy affects the FX rates the GDP measure (y), and money
aggregate (m). Potentially the price level (p) in (1) could appear in two measures, for informal and formal sector respectively. Adding to that, the price level itself is affected also by the money supply disturbance. So, from the dynamical model

\[
\frac{d [e]}{dt} [p] = \left[ \frac{\alpha \beta}{\gamma \delta} \right] [e] + \frac{[e]}{\theta} m
\]

(2)

where the variables used here represent the distance x-x* from their equilibrium values, clearly m = M – M* ≠ 0 makes the equation inhomogeneous. Note that constant parameters in (3) are interrelated, implicating that the saddle point solution (e*, p*) discussed in (Gray, 1979), evolve themselves and the systems system jumps from one manifold to another in a very complicated way. Specifically, the solution \( \left[ \frac{\alpha \beta}{\gamma \delta} \frac{e^*}{p^*} \right] m_{real} \) depend on the \( m_{real} \) which is not necessarily zero as presumed on the original model. This behaviour can be favoured if the foreign currency, is used in the informal market. So, the corresponding system for this case is at least 3D with richer in dynamics compared to the 2D one expressed by (2). Also, in a formal market \( a \frac{de}{dt} + b \frac{dp}{dt} + c \frac{dm}{dt} \sim 0 \) so if the changes on money supply are not frequent, the system is predicted to jump from an attractive basin to another, producing a complicated but deterministic behaviour. In this case the authorities intervene appropriately to keep the values of the parameter (e) under control by imposing policies in the system, as to behave theoretically according to the fixed points of the system (2). A better picture for those effects can be understand by considering the Krugman target zone model elaborated in (Lera & Sornette, 2015).

\[
e = m + v + \frac{y E[de]}{dt}
\]

(3)

where specifically v is the exogenous velocity shock comprising all effects not controlled by national authority and is considered a Brownian motion. The expected change E[e] encompass all market influences usually based on the anticipate activities of actors when e \( \rightarrow e_{floor_bank} \) predefined by Central Bank. Letting details aside, again, the presence of the foreign currency in everyday transaction makes (3) more complicated. So the effective policy claiming \( \frac{de}{dv} = v_{floor} \) when the bank expects (v = \( v_{floor} \) = \( s_{floor} \), imposes \( \frac{dm}{dv} + 1 + y \frac{E[de]}{dt} = 0 \), but dm is not zero at all do to the foreign currency presence. It implies that the boundary limit of FX rate, the \( e_{floor} \), is not realised in the intended \( v_{floor} \) value, and therefore, an extra policy \( v_{extra} \) should be imposed. Those movements make time data series more heterogenous and the distribution of the relative changes more nonstationary. Also, the bid-ask spread is expected also with rich dynamics. Those effects can be analysed by measuring the stationarity of the RoR states, recognizing heterogeneity characteristics of the FX series and corresponding return, and so on. On the other side, we can use those statistical and dynamics characteristics to analyse and specific effects of the informality on FX.

**INFORMAL SECTOR IN ALBANIA**

All those arguments are useful in the analysis of the FX in Albania. The country’s economy belongs to the developing and transitive category, with low-medium per capita income, low GDP. The size of the informal economy as considerable, estimated at 30%-40% level for last years, see (Elgin et al, 2021), Fig.1. It has a specific distribution, and after 2018, it populated the left side of it. Interestingly, it resulted that the value obtained by MIMIC model differs from them of the DGE model by some percent of the GDP, Fig. 1. From 2009-2018 this gap goes to 2.5%-3.5% of the GDP or about 10% of the EI calculated. Notice also that calculation techniques reveal particularities that can be used for our analysis So, MIMIC is based on slow-moving variables whereas DGE is based on more volatile variables. The difference observed between to methods of calculation in (Elgin et al, 2021) is an indicator that more hidden effects and variables are present. The amount of foreign currency used as substitute of national currency is very likely among them. It is unknown by value but expected to be considerable. So, a part of the remittances, almost all in EURO, counting around 9.63% of the GDP those last year (World Economics, 2023), reach
the country out of bank system, contributing to the increase of the use of Euro directly as payment currency without being converted in ALL. Also, the tourism inputs at around 2 billion USD annually (Tsallis, 2017), mostly in EURO and USD, and considering the informality level, contribute proportionally to the amount of foreign currency used as national currency’s substitute. It is expected that relatively, the FX EURO-ALL and USD are permanently under pressure to lay down the ‘natural level” estimated by formal (1) and (2) above, even without considering the boundaries set by (3). Notwithstanding this, it resulted in the fact that the financial system has resisted last crisis, which merits a careful analysis like the one provided in (Kovaci and Prenga, 2022).

Finally, the distribution of the Euro-ALL FX values for periods [Jan. 2015- Feb 2023] and [Oct 2019-Feb.2023] reveals specifics. Firstly, the respective histograms consist in two separated parts and remarkably narrower than all other currencies FX histograms. For the period considered, this histogram for Euro-ALL has become narrower, and this is more highlighted for the low value state, as seen by comparing solid and doted lines in Fig. 2.

Fig. 1. The world rank as per informal economy size, 1993-2018. Right panel the difference between calculation by MIMIC and DGE methods. The graph is based on the data analysed in reference 5.
Those introductory observations suggest that FX rates Euro-ALL exhibit specifics and particularities. Also, by employing series on (1), (3) we observed significant inconveniences between estimated and real values, and constant parameter on those models varies when referring different time span. We also classify this finding as other indicators that a hidden variable related with particularities of our system, the sizable informal economy, and the use of Euro for direct transaction in this sector, impose specific behaviour if the corresponding FX series. We proposed to employ statistical tools and dynamics analysis to explore more about those specifics.

THE NONLINEARITY ANALYSIS.

For statistical analysis financial time series, it is practical to consider distributions of the relative changes of prices \( r(t) = \frac{p(t+d)-p(t)}{p(t)} \), the logarithmic return \( R(t) = \log[P_{t+1} - P_t] \) or the volatility \( \nu = \frac{\sigma}{\sqrt{T}} \) where \( T \) is a the period of interest and \( \sigma \) is the standard deviation of the return. For our purpose, we will consider stationarity and stability issues as indicators of the supposed particularities. It can be realised by using standard mathematical approaches, e.g., using Levy stability for example. Though, intriguing results can be obtained by contemplating the Tsallis’ distribution analysis based on the q-Gaussian features. The q-Gaussian is a specific case of the student distribution also called t-LocalizationScale and has the general form given in (Umarov et al, 2008), (Tsallis, 2017)

\[
G_q(x) = \frac{1}{\sqrt{2\sigma_q z_q}} \left( 1 - (q - 1) \left( \frac{x - \mu_q}{2\sigma_q} \right)^2 \right)^{\frac{1}{1-q}} + \left( \frac{2\sqrt{\pi}}{(3-q)\sqrt{1-q}} \right)^{\frac{1-q}{1-q}} \frac{1}{r^{1-q}} \quad ; q < 1
\]

\[
\sqrt{\pi}; q = 1
\]

\[
\sqrt{q-1} \frac{1}{r^{1-q}} ; 1 < q < 3
\]

where \( Z_q = \left\{ \begin{array}{ll}
\frac{2\sqrt{\pi}}{(3-q)\sqrt{1-q}} & ; q < 1 \\
\sqrt{\pi} & ; q = 1 \\
\sqrt{q-1} \frac{1}{r^{1-q}} & ; 1 < q < 3
\end{array} \right. \)
where the $\Gamma(.)$ are gamma functions. According to (Tsallis, 2011), (Tsallis, 2011), (Tsallis, 2017) the parameter $q$ in (4) known as $q_{\text{stationary}}$ or $q_{\text{entropy}}$, measures the distance from the stationarity. Note that the distribution itself is stationary and with finite variance given by $\sigma \sim \frac{1}{5-3q}$ so

$$1 < q_{\text{fit}} < \frac{5}{3} \rightarrow (\sigma > 0) \cap (\sigma > \infty): \text{stationary distribution} \tag{5}$$

$$\frac{5}{3} < q_{\text{fit}} < 2 \rightarrow (\sigma \sim \infty): \text{Nonstationary distribution}$$

$$2 < q_{\text{fit}} < 3 \rightarrow (\sigma \text{ undefined}): \text{Nonstationary distribution}$$

$$q_{\text{fit}} > 3: \text{No distribution object}$$

Q –Gaussian has a power law nature that makes it particularly useful to describe the fat tails observed in real-world data systems (Tsallis, 2011), (Tsallis, 2011), (Pavlos, 2014), (Antoniadesa, 2021). It is found very effective in describing statistical features for financial indexes (Borland, 2002), (Tsallis, 2011), (Tsallis, 2017), etc. Note that technically, when fitting (4) with the real data histogram, the optimisation of the histogram itself and specific fitting techniques are important, due to the high sensitivity of the $q$-function from the $q$-parameter $q$. For this purpose we have used a composite algorithm based on NLS and Particle Swarm Optimization techniques that we used successfully elsewhere (Prenga et al, 2021). In the first step of the analysis, we have identified the nonstationary grade and the mean for analysing it. So, we considered the distribution of the Rate of Return (RoR) for daily FX series, spanning a relatively long period, 2015, 2023. In this stage, data series are discretised, and the histogram is optimised using Freedman-Djакович based on the empirical evidence for the un-stationarity of the distributions. Next, we tried the fitting of standard distributions with our data. Formally we used all the set of the MATLAB, totalling 17, but a few of them are expected as candidates, because most of the distributions are related with specific physical processes, with no econometric or financial basis. The ranking criteria for the goodness of fit is based on negative likelihood, Akaike Information, Bayesian Information Criteria etc, but here we will present only the ranging as by Negative Likely Hood, because there is no significant change on the result. So, for the RoR of all FX rates and for the ONIR, the best fitted distribution from the standard set has been the tLocationScale distribution, Figure 3.

![RoR PDF Euro ALL](image-url)
Fig. 3. Best fitted Distribution are tLocationScale-a q-Gaussian family

Theoretically, this distribution belongs to the family of q-Gaussians presented in (4). This is a very important finding, because we can use the rules (5) objectively. Finally, we have repeated the fit using optimal histogram by alternating Scott, Freedman Diaconic, Struggle, and an Entropy Maximization technique for the bins-size, according to the best goodness. The q-Gaussian has resulted that the best goodness of fit among other standard distributions. In figure 4 is presented such a discussion.

Fig. 4. Estimating errors by CDF fit q-Gaussian is given by solid lines
According to those preliminary steps, we can use the q-Gaussians and their parameters for analyzing the features of the FX rates.

**COMPARATIVE EVIDENCE OF THE STATIONARITY LEVELS FOR FX RATES OF ALL TOWARD MAIN CURRENCIES**

For evidencing the effects of the informal economy on EURO-ALL FX due to the use of the Euro as domestic currency substitute, we propose to compare its statistical and dynamics features with those of other main currencies FX as USD-ALL, GBP-ALL and, CHF-ALL. By hypothesis, the informal sector is expected to act as a reservoir and a depositing room for the most present currency in the informal economy. It supposedly feeds the needs or absorbs the surpluses of Euro quantities whenever market forces arising from import-export disbalances, or price inequalities, create them. To realize this, methodologically we should use indicators, and next, by employing a known or ad hoc modelling like MIMIC for example, the claimed effect can be measured in the framework of a hidden variable analysis.

As said above, this work is focused on the first phase, the evidence and estimation of the informal economy on the FX behaviour, letting the deeper investigation for another address. So, for evidencing the specifics and particularities for FX and some financial indicators, we have considered series $y(t_0, t_{end})$, where $t_0$ is assigned empirically in January 2015 and $t_{end} = June \ 2020 + n \ast 24 \ days$. We chose the step by $n\ast24$ working days empirically, intending to mimic the periodic Central Banks and/or state agencies tentative to stabilize the FX value. We note that those bodies have declared the unchangeable policy to maintain stable FX rates for economic, historical, and financial reasons, (bankofalbania.org), (financa.gov.al). The spanning period cover a stabilized economic period 2015-2018 followed by a period of economic difficulties caused by a natural disaster (earthquake), covid closure period and world economic troubles imposed from Ukraine war global impact. The idea is that if there were signature of the features mentioned above, they would be highlighted in the periods of economic crisis or difficulties. We started the discussion by assessing the stationarity level for some FX and the Over Night Inters Rates (ONIR) by analysing their RoR distributions. Next, the fitting of the form (4) is performed by employing the nonlinear last squares fitting (NLS) and a particle swarm optimization (PSO) algorithm separately and if the parameters obtained did not coincide, the fit was rejected. Based on the remarks above we moved to the verification of the stationarity issues by evaluating the distance $d = q - 1$. We obtained that the distributions of the corresponding RoR for our FX series in the period [2015, March 2023], were non-stationary because $q_{\text{Series stationary}} \gg 1$. By spanning series [2015, January 2020+n*24 days], we observe that the stationarity measure of the Euro/ALL FX varies significantly above and below the 1.67 limit, in a specific way, differently from the others, Figure 4. The RoR for USD, GBP and CHF FX series remained in a relatively low level of the non-stationarity. We may explain this behaviour by contemplating the reservoir idea disused above, and the priority of FX rate on the monetary policy activities.
Fig. 5. The evolution of the RoR distribution stationarity parameter by time. The Euro-ALL series shows a specific and pattern compared to the others.

So, when a harsh change as value of FX open (mechanically set of the Bid price) or other policy shock occurs on the Euro-ALL FX, the aftershock waving push the distribution in another stability level, but once happened, the cumulative reservoir effects contribute to keep this level unchanged. Note that the role of the Euro-ALL RoR in monetary policies is quite evident, so more activities are related to this variable. So, majority of the imports –exports trading is made with EU countries. So, in the observation in (OEC - the Observatory of Economic Complexity), for the year 2021, the country is ranked at 115 in the world in terms of GDP (current US$), 131 in total exports, 123 in total imports, 89 in terms of GDP per capita (current US$) and 72 according to the Economic Complexity Index (ECI). All those elements and others are relevant in understanding the complexity of the main currencies FX series behaviour. The different behaviour observed herein for the FX Euro-ALL RoR, can be considered indirect proof of a particular position of this currency. In this regard, among others, we highlight its use as a substitute for the national currency in the informal market. Interestingly, the distribution of the RoR for ONIR series shows a higher distance from the stationarity, and the changes are smothered than those found for EURO-ALL RoR distributions, Figure 5 (the green line). Also, that stationary q-index for ONIR and EURO FX series looks alternated in a complementary fashion. Acknowledging that the non-stationarity in a distribution is related with fluctuation features, the facts that the level of stationarity of two series jumps at the same time moment is a complex response of the interrelationship seen in the equation (1) and others not included therein, but again, the Euro-ALL FX RoR are apparently distinguishable from the others. It is another indicator that something different affects the Euro-ALL FX rates. For us, this is the combination of the specific position of the European Currency for the country’s economy with its use as domestic currency substitute in some parts of the market activities.

The perturbation implied from ONIR are amortized like the forced Bid values above, and the dephasing of stationarity observed needs a thorough analysis which we are skipping herein. Once the stationery level changes, a skewness is settled (the local -q-Gaussian is centered in another point), but the abundance in foreign currency help volatility to decrease. Apparently, this would be evidenced by low
q-standard deviation calculated by the fitting of the histogram to the model (4). Finally, by the time, a relaxation trend is observed for all series and corresponding distributions become more stationary as seen after coordinate 25 in the graph of Figure 5 (hence after February 2022). It reflects the increase of the length of series but also, it indicates the effect of the stabilization phenomena. Additional and valuable information is gathered from the bid-ask behaviors, which we will consider in another address because some of our bid-ask series were incomplete. Considering the period 2018-2023 when several economic difficulties have taken place, we observed that q-parameter for EURO-ALL RoR is the highest among others, at the edge of the stationary limit at $\frac{5}{3}$. However, the standard deviance of the RoR for EURO-ALL FX series for this period is 0.0012 smaller than others, respectively 0.0051 (GPB-ALL) 0.0048 (USD-ALL) and 0.0051(CHF-ALL). The same properties hold for the q-Standard deviation. We obtained the corresponding values at [0.0007 0.0048 0.0039 0.0036]. Note that as $q < \frac{5}{3}$ those parameters are defined and indicate the q-width of the histogram. Next, for ONIR RoR we obtained $q \sim 2.02$, q-standard deviance at 0.0127 and arithmetic standard deviation at 0.044, both much larger than the corresponding values for all FX currencies above.

As seen from those findings, arithmetic variances, and q-variance of the Euro-ALL RoR are smaller than for other FX currencies. Interestingly, both those parameters describe globally the statistics, therefore we concluded the averaged increments of the RoR EURO-ALL are the smallest compared with corresponding of other main currencies RoR, whereas the stationary indicator for the distribution is the highest. It suggests that a specific process correlated with the ongoing crisis caused the distribution to be highly nonstationary by injecting harsh and unexpected increments, and another pay back for the disbalances to keep the average amplitudes of the fluctuation, small. High amplitude are rare events as they happened locally and seems to be set aside of the characteristic distribution given by solid blue line in Figure 6, hence, probability not produced by the dominant FX process. So far, the specific nature of the distribution, highly un-stationary but with low variance, can be explained by the effectiveness of relaxation process with origin the reservoir behaviour which we adopted for the IE and using of Euro in its transactions. To know more about those behaviors, we need more information about local behaviour and fluctuations dynamics. The next observation reveals the fact that for other main currencies traded officially in the country, those behaviors are weakened.
CONCLUSIONS

By exploring differences between dynamical behaviour and measures for FX EURO-ALL and ALL-other main currencies traded in Albania, we obtained indicators that the process of the FX price formation for Euro currency is affected strongly by the size of the informal economy and the non-neglectable amount of euro currency used in the everyday transaction in this sector without being exchanged before. Considering the specific role of the Euro in the Import Export balances for the country, the behaviour of the Euro-ALL indicates a good adaptability of its volatility with frequent shock imposed by the monitory policies. The dynamics remain nearly stable in its un-stationary state, and all dynamical metric indicators as multiracial width, multiracial asymmetry, q-entropy are high, but stable. Also, the FX price showed resistance toward risky self-organization processes. Based on the specifics of the exchange rates Euro-ALL compared with others FX series, we concluded that those features are result of the structure of the informal economies of the country that permit the Euro to behave as national currency substitute. It also explains the abnormal strength of the national currency and its good comportments during last economic crisis on the country.

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