

BINOMIAL HYPERFILTERS. THE APPLICATION OF THE SPECTRAL METHOD IN THE NEURAL STRUCTURES

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Abstract

The given paper describes the design and the method of hyperlinear filter's synthesis. On its basis the scheme of the binomial filter is offered that will provide the increase in the efficiency of the neural structures. Methods of calculations using the spectral representations of Walsh are given. The estimates of the synthesized structures efficiency are indicated.

Keywords: hyperlinear and binomial filters, neural structure, tangent bundle

1. INTRODUCTION

The task that is being solved by a neural network (NN) may be represented by a logical function $f(x)$, that is defined on the n -dimensional space X of the input vectors of the x network, $x \in X$. In the works [1,2] it is shown that the placement of the input block of linear transformations in the network which performs the operation $\sigma \otimes x$, (mod2), provides the simplification of the NN structure. The paper [3] presents the idea of using a tangent space bundle X to increase the approximating capabilities of artificial neurons. It was noted that the tangent bundle X over the base of some subspace $T \triangleleft X$ provides the increase in the class of linearized threshold functions and this allows us to simplify NN, that performs the logical function (LF) $f(x)$. At the same time, the search for an effective linearization σ and optimal stratification T is implemented easily by applying the Walsh spectral presentations of the logical functions. The corresponding mathematical apparatus is described in the book [4].

On this basis, the given paper describes the search for linearization method σ and bundle T as in the case of the synthesis of a number of neurons operating in parallel with a threshold activation in order to get the exact realization of the Boolean functions of the arbitrary class.

2. THE OPTIMAL BUNDLE OF THE SPACE OF A LOGICAL FUNCTION

The tangent space bundle X on the base T is technically realized by the application of the decoder $m < n$ with the inputs to which simple linear filters ...are connected, determined by linearization vectors $\omega^j = (\omega_1^j, \omega_2^j, \dots, \omega_n^j)$, which form vector basis of the subspace T .

The initial LF $f(x)$ contains k_1 constituent unit and k_0 constituent of zero: $k_1 = |X^1|$, $X^1 = \{x^1 : f(x^1) = 1\}$, $k_0 = |X^0|$, $X^0 = \{x^0 : f(x^0) = 0\}$, $X = X^1 \cup X^0$. In order to obtain the optimal result of the neurostructures synthesis it is advisable that the tangent layers X_1, X_2, \dots, X_t of the space $X = X_1 \cup X_2 \cup \dots \cup X_t$ contained approximately equal in power subsets of single $X_i^1 = \{x^1 : x^1 \in X_i\}$ and null $X_i^0 = \{x^0 : x^0 \in X_i\}$, $i = 1 \div t$ constituents, that is $|X_i^1| \approx |X_j^1|$, $|X_i^0| \approx |X_j^0|$, $i \neq j$. Consequently, the first vector of the base bundle T , is to be chosen among the arguments ω minimal in absolute value of coefficients $s(\omega)$, $\omega \neq 0$ of the spectrum $S \Pi \Phi f(x)$. This is explained by the fact that every coefficient $s(\omega)$ characterizes space bundle X in accordance with the definition of the Walsh function $w_\omega(x)$: $X = {}^+X_\omega \cup {}^-X_\omega$, ${}^+X_\omega \cap {}^-X_\omega = \emptyset$,

where ${}^+X_\omega = \{x : w_\omega(x) = +1\}$, ${}^-X_\omega = \{x : w_\omega(x) = -1\}$, wherein
 $s(\omega) = \sum_X f(x) \cdot w_\omega(x) = \sum_{{}^+X_\omega} f(x) \cdot w_\omega(x) - \sum_{{}^-X_\omega} f(x) \cdot w_\omega(x) = |{}^+X_\omega^1| - |{}^-X_\omega^1|$, where
 ${}^+X_\omega^1 = \{x : (w_\omega(x) = +1) \& (f(x) = 1)\}$, ${}^-X_\omega^1 = \{x : (w_\omega(x) = -1) \& (f(x) = 1)\}$.

The second and subsequent vectors of the basis T shall be defined among the minimal in absolute value spectral coefficients $s_i(\omega)$, $\omega \neq 0$, $i = 1 \div t$ of the fragments $f_i(x)$ LF $f(x)$, which correspond to the previously allocated tangent subspaces X_i . So, for example, let r vectors of the base T be previously determined and, correspondingly, $R = 2^r$ of the tangent subspaces X_i , $i = 1 \div R$. The fragments of the functions $f_i(x) = \begin{cases} f(x) : x \in X_i \\ 0 : x \notin X_i \end{cases}$ and their spectra S_i have also been determined. Then the next basic vector ω_{r+1} is selected among arguments ω , for which $\sum_{i=1}^R |s_i(\omega)| = \min, s_i(\omega) \in S_i, \omega \neq 0$.

For a partially defined LF $f(x)$ while determining the base of the bundle T it is necessary to follow the same conditions that are performed simultaneously for the fully determined functions $f^1(x)$ and $f^0(x)$, which correlate with the initial function in the following way:

$$f^1(x) = \begin{cases} 1, & \text{if } f(x) = 1, \\ 0, & \text{if } f(x) \neq 1. \end{cases} \quad f^0(x) = \begin{cases} 1, & \text{if } f(x) = 0, \\ 0, & \text{if } f(x) \neq 0. \end{cases}$$

3. THE DETERMINATION OF THE BINOMIAL HYPERFILTER

The level of approximation LF $f(x)$ through the function of the neuron $\nu(x)$ may be increased by the input linearization σ of the neuron ν , that generates function $\nu^\sigma(x) = \nu(\sigma \otimes x)$, where σ - is a binary matrix $m \times n$. The lines $\sigma_i, i = 1 \div m$, of the matrix σ are determined successively in accordance with the arguments $\omega^i \neq 0$ of the dominant coefficients $s(\omega^i)$ of the spectrum S of the function f , $|s(\omega^i)| = \max|S|$. The same arguments determine the linear filters

$$\lambda_1(x), \lambda_2(x), \dots, \lambda_m(x) \text{ at the inputs of the neuron } \nu: \lambda_i = \omega_1^i x_1 \oplus \omega_2^i x_2 \oplus \dots \oplus \omega_n^i x_n, i = 1 \div m.$$

With the increase of dimension and complexity $f(x)$ the effectiveness of the simple filters application $\lambda_i(x)$ decreases. It is possible to facilitate the solution of the problem of approximation of such functions in three ways.

First, by applying the tangent space bundle X . To do this, the bundle base T , tangent layers X_1, X_2, \dots, X_t and the corresponding to them fragments $f_j(x), j = 1 \div t$, of the function $f(x)$. Then the spectral densities of the fragment functions $D_j = |S_j|$ are determined, their sum $D = \sum_{j=1 \div t} D_j = \sum_{j=1 \div t} |S_j|$, where S_j - is the spectrum of the fragment $f_j(x)$ and coefficients $d(\omega^i) \in D: d(\omega^i) = \sum_{j=1 \div t} |s_j(\omega^i)|$ are analyzed. The linearization arguments ω^i are selected from

the condition $d(\omega^i) = \max\{D\} = \max\left\{\sum_{j=1 \div t} |s_j(\omega^i)|\right\}$. The quality improvement of the corresponding filter $\lambda_i(x)$ is explained by the fact that $\sum_{j=1 \div t} |s_j(\omega^i)| \geq |s(\omega^i)|$. When implementing the filter calculated in this way it is necessary to take into account the signs of the coefficients $s_j(\omega^i)$ in each of the layers $j = 1 \div t$. Taking into account the specifics of the Walsh representations, if the input signal x belongs to the layer X_j , i.e. $x \in X_j$, and at the same time $s_j(\omega^i) \geq 0$, then the signal of the filter $\lambda_i(x)$ shall be inverted. Having this aim in view, in case of a hardware implementation, $mod2$ is modally added to the signal of j -output of the decoder. At the input of the line for forming a simple filter $\lambda_i(x)$, representing a chain of two-input adders $mod 2$, it is necessary to establish a connection with j -th output of the decoder.

Let the decoder $DC(\delta)$ with the input linear transformation σ_{DC} have p inputs, then one of its outputs $\delta_j, j = 0 \div (2^p - 1)$ will be activated, $\delta_j(x) = 1$, if $j = \sigma_{DC} \otimes x$. If there is a connection of the j -th output of the decoder with the input of λ_i filter, then the value of the signal for hyperfilter γ^j for $x \in X_i$, constructed in this way is determined by the expression $\gamma^j(x) = \lambda_i(x) \oplus \delta_j(x)$. If $\delta_j(x) = 1$, then $\gamma^j(x) = \lambda_i(x) \oplus 1 = \bar{\lambda}_i(x)$, otherwise $\gamma^j(x) = \lambda_i(x)$. The example of hyperlinearization implementation in the form of a logical scheme is shown in Fig.1.

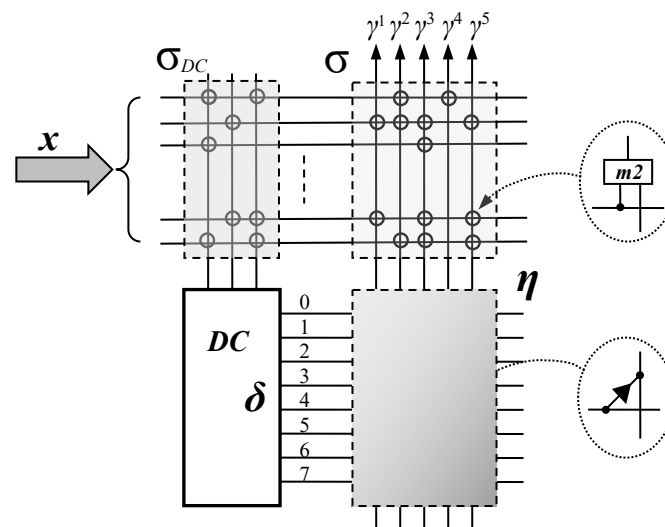


Fig. 1. Logical scheme of hyperfilters in case of tangent stratification of space X on the basis T of $p = 3$ dimension.

In the software implementation a lot of connections of decoders' outputs with linear filters $\lambda_1(x), \lambda_2(x), \dots, \lambda_m(x)$ inputs are represented in the form of binary matrix η of dimension $P \times m$, where the number of outputs of the decoder $P = 2^p$. The calculation of the values of many hyperfilters is determined by the expression $\gamma = \sigma \otimes x \oplus \eta(\sigma_{DC} \otimes x)$, where $\sigma_{DC} \otimes x$ represents the number of the line in matrix η .

Secondly, local linear hyperfilters $\mu^r(x)$, $r = 1 \div m$ of multiplicity k , where $\mu^r(x)$ shall be used. As a result, we shall get a linearized neural function $v^\mu(x) = v(\mu^1(x), \mu^2(x), \dots, \mu^m(x))$, which approximates the predefined function $f(x)$ more effectively. There appears an opportunity to realize more effectively $f(x)$ as a binary sum of a simple row of the similar linearized neurons $f(x) = v_1^{\mu^1}(x) \oplus v_2^{\mu^2}(x) \oplus \dots \oplus v_d^{\mu^d}(x)$. However, in this case, if we increase the size of the complex functions $f(x)$ further on, then the relevant to them rows will elongate considerably.

The matter is that each subsequent $(i + 1)$ -neural function of $v_{i+1}^{\mu^{i+1}}(x)$ row shall obligatory approximate the residual function $f_{i+1}(x) = f(x) \oplus v_1^{\mu^1}(x) \oplus v_2^{\mu^2}(x) \oplus \dots \oplus v_i^{\mu^i}(x)$ with more and more decreasing range as the serial number $(i + 1)$ increases. As a result, it becomes necessary to use more and more local filters μ^{i+1} of the increasing multiplicity and this leads to considerable growth of the total number of the hyperfilters used.

That is why, thirdly, there should be used binomial combination of hyper filters γ of the type

$$\beta(x) = \prod_{j=1}^k \beta^j(x) = \prod_{j=1}^k (\gamma^{j1}(x) \vee \gamma^{j2}(x)).$$

The logic diagram is shown in Fig.2. Such binomial hyper filter β of the multiplicity k contains $2k$ of simple hyperfilters γ , being equivalent at that to 2^k local hyper filters μ of the multiplicity k , but containing a sum of k simple hyper filters γ with multiplicity 1.

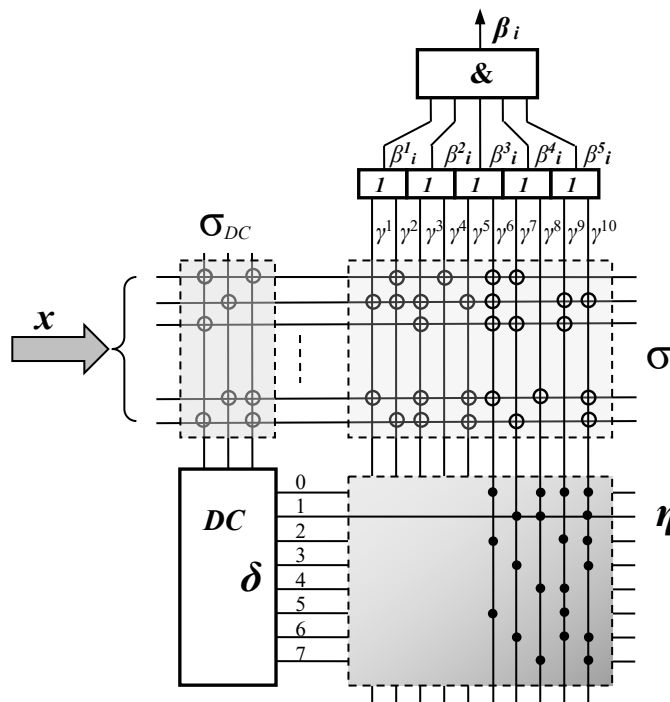


Fig. 2. The Logical Scheme of the Binomial Hyperfilter

The calculation of the elementary binomial $\beta^1(x) = \gamma^1(x) \vee \gamma^2(x)$ for a certain function $f'(x)$, that represents partially a definite fragment of the initial function f in the field $X' \subset X$, is performed in the following sequence:

- the calculation of the spectral density D' of the function f' based on T ;
- the choice $d'(\omega) = \max\{D'\}$ and the determination of the hyperfilter $\gamma^1(x)$, in accordance with the argument ω of the coefficient $d'(\omega)$;
- the determination of the area $X'' = \{x : (x \in X') \& (\gamma^1(x) = 0)\}$ and the corresponding to it fragment of the function $f'' \subset f'$;
- the calculation of the spectral density D'' of the function f'' based on T ;
- the selection of $d''(\omega) = \max\{D''\}$ and the determination of hyperfilter $\gamma^2(x)$, according to argument ω of the coefficient $d''(\omega)$.

If it turns out that $f'' = 0$, then area $X'' = \{x : (x \in X') \& (\gamma^1(x) = 1)\}$ is overridden and the corresponding to it fragment $f'' \subset f'$. In this case, $\beta^1(x) = \gamma^1(x)$ and further on the calculation of the binomial $\beta^2(x)$ for the fragment $f''(x)$ is performed.

The given sequence of calculations provides the increase in trim value $\theta = \log_2(k_1/k_0)$, where k_1, k_0 - of the number of single and zero constituents of the function f' on $X^{(1)} = \{x : (x \in X') \& (\gamma^1(x) \vee \gamma^2(x) = 1)\}$ - in the area of binomial $\beta^1(x)$ approximation.

The calculation of each consequent elementary binominals $\beta^j(x)$, $j = 2 \div k$, in the hyperfilter β is performed in the area/field $X^{(j-1)} = \bigcap_{r=1}^{j-1} X^{(r)}$ for the corresponding to it fragment of the function $f^{(j-1)} \subset f'$.

4. THE REPRESENTATION OF THE LOGICAL FUNCTION BY THE COMPOSITION OF THE FIBONACCI NEURON AND THE MODULES OF BINOMIAL HYPERFILTERS

It is quite acceptable to implement simple LF in the form of a binary sum of binomial hyperfilters: $f(x) = \beta_1(x) \oplus \beta_2(x) \oplus \dots \oplus \beta_q(x)$. However, with the increase in complexity the same difficulties arise that we see in the implementation of LF through local hyper filters $\mu(x)$. The reason is that for residual in a series of binomial hyperfilters $\beta_r(x)$, $r \geq 5$ the functions they approximate have a decreasing rank and decreasing in absolute value spectral coefficients. But the main difficulty is connected with the substantial decrease in the monotonicity of the residual approximating functions. As a result of this, the use of neurons with threshold activation becomes ineffective. Together with this, to put it simply, a great number of zero constituents k_0 for residual approximated functions it is a ballast, that must be disposed every time during synthesis of each of hyperfilters $\beta_r(x)$. Having this aim in view, a chain of cascade connections of the elements INHIBIT is introduced, in which direct non-inverted inputs receive signals of disjunctions of binomial hyperfilters $B(x) = \beta_1(x) \vee \beta_2(x) \vee \dots \vee \beta_m(x)$.

Let for the function f the areas $X^1 = \{x : f(x) = 1\}$, $X^0 = \{x : f(x) = 0\}$ is determined, and for disjunction $B(x)$ the area of single values $X_B^1 = \{x : \exists \beta_i(x) = 1\}$ is also determined. It is always possible to choose a certain number m of binomial hyperfilters $\beta_i(x) = \prod_{j=1}^k \beta_i^{(j)}(x)$, i.e. such a

disjunction $B(x)$, that $X^1 \subseteq X_B^1$. In this case, it may turn out, that $X^0 \cap X_B^1 \neq \emptyset$. Let

$$X_{B-f} = X^0 \cap X_B^1. \text{ Let's define the function of the correction } f_c(x) = \begin{cases} 1, & \text{if } x \in X_{B-f} \\ 0, & \text{if } x \in X^1 \\ \text{"-"}, & \text{in other cases} \end{cases},$$

where symbol "-" represents the undefined function value. Then, using a logical operation INHIBIT, we get $f(x) = B(x) \cdot \overline{f_c(x)}$. It is clear that this method can be applied both to the approximation of the partial function $f_c(x)$, that already contains fewer k_1, k_0 numbers of single and zero constituents in relation to function $f(x)$: Then $f_c(x) = B'(x) \cdot \overline{f'_c(x)}$, where disjunction B' of binomial hyperfilters, approximating the function $f_c(x)$, is accompanied by the corresponding function of the correction $f'_c(x)$. In this case, there it is formed a two-cascade inhibition scheme - $f(x) = B(x) \cdot \overline{B'(x) \cdot \overline{f'_c(x)}}$. This procedure can be continued even further, achieving on (r) repetition the formation of the elementary correction function $f_c^{(r)}$, -

$$f(x) = \overline{\overline{\overline{\overline{\overline{B(x) \cdot B'(x) \cdot B''(x) \cdot \dots \cdot B^{(r)}(x) \cdot f_c^{(r)}(x)}}}}}}$$

The basic blocks of this construction are modules which implement disjunctions $B^{(t)}$, $t = 0 \div (r+1)$ of the binomial hyperfilters where $B^{(0)} = B(x)$, and module $B^{(r+1)}$ represents the function of correction $f_c^{(r)}$. The output block implements Fibonacci neuron $[\Phi]$ and serves as a cascade connection of the elements INHIBIT, shown in Fig.3.

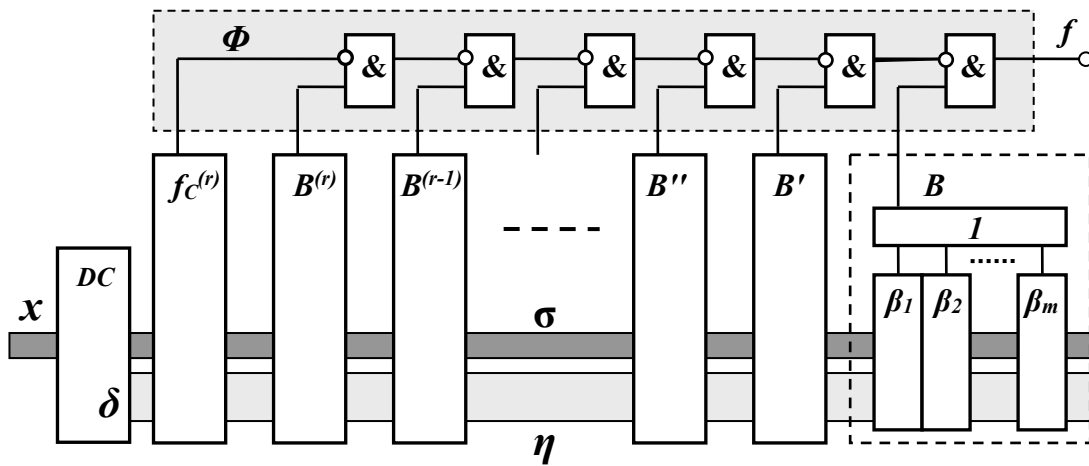


Fig. 3. The Scheme of the Logical Function with the Modules of Binomial Hyperfilters

The total length of the cascade, number $(r+2)$, depends on the approximation accuracy of the intermediate functions $f_c^{(i-1)}$, which is provided by modules $B^{(i)}$. The higher the accuracy of the approximation, the fewer is the number of modules.

High accuracy is ensured by the increase of the total number k_γ of the hyperfilters $\gamma^j(x)$ and the increase of the corresponding matrix costs $Z_M = k_\gamma(n+R)$. It is possible to optimize the costs by

adjusting the multiplicity k of the binomial hyperfilters $\beta_j(x)$ as part of disjunction $B^{(i)}, i = 0 \div (r + 1)$.

5. THE DETERMINING OF BINOMIAL HYPERFILTERS IN THE MODULE COMPOSITION

In the calculation process of disjunction $B^{(i)} = \beta_{(i)_1} \vee \beta_{(i)_2} \vee \dots \vee \beta_{(i)_p}$, each subsequent binomial hyperfilter $\beta_{(i)_{p+1}}$ is determined/defined for the approximation of the fragment $f'(x)$ of the function $f_C^{(i)}(x)$, bounded on the domain $X_{p+1} = \{x : B^{(i)}(x) = 0\}$. The domain X_{p+1} has the characteristics χ , expressed by the ratio of single and zero constituents $f'(x) : \chi = 2k_1 / (k_1 + k_0)$. In the process of synthesis the value $\chi \leq 1$. We shall determine here the accuracy of the approximation of the fragment $f'(x)$ for filter $\beta_{(i)_{p+1}}(x)$ as a ratio $\rho = k_1 / k_0$ single and zero constituents of the fragment $f'(x)$ in the area/domain $X^\beta = \{x : \beta_{(i)_{p+1}}(x) = 1\}$. It is evident that with the with the increase in multiplicity k of the filter $\beta_{(i)_{p+1}}(x) = \prod_{r=1}^{j-1} \beta_{(i)_{p+1}}^r$ by one, that is, with the addition of the next binomial $\beta_{(i)_{p+1}}^{(j)}(x) = \gamma_{(i)_{p+1}}^{(j1)}(x) \vee \gamma_{(i)_{p+1}}^{(j2)}(x)$, the value of ρ increases and the accuracy of approximation rises.

In this research paper the formation and the increase in multiplicity k of the filter $\beta_{(i)_{p+1}} = \prod_{r=1}^k \beta_{(i)_{p+1}}^{(r)}(x)$ for the fragment $f'(x)$ stopped at that value of k , if the accuracy of approximation ρ exceeded some previously calculated and correlated with fragment f' , value $\tilde{\rho}_f$, i.e. in case when $\rho > \tilde{\rho}_f$. The value $\tilde{\rho}_f$ varied within the interval $[0.5, 1.02]$, $(0.5 \leq \tilde{\rho}_f \leq 1.02)$ and was determined for the fragment $f'(x)$ with the involvement of χ characteristic by the expression $\tilde{\rho}_f = 0.5 + 0.52 \cdot (1 - (1 - \chi)^4)$.

6. EVALUATION OF THE EFFECTIVENESS OF THE BINOMIAL HYPERFILTERS APPLICATION

The described method of synthesis was tested on the individual logical functions, having a random with a uniform probability density distribution of single and null/zero values. Table 1. shows the results of calculations for fully defined complex LF, possessing a trim $\theta \rightarrow 0$, the rank of which is approaching the value 2^{n-1} .

Table 1

$n \backslash m$	8	10	12	14	16	18
2 k_γ	53	146	464	1549	5315	18947
ε	2.484	1.996	1.813	1.702	1.622	1.590
3 k_γ	40	126	386	1310	4544	16422
ε	2.500	2.215	1.885	1.759	1.664	1.629
4 k_γ	25	91	297	988	3525	12723
ε	2.344	2.311	2.030	1.809	1.721	1.650
5 k_γ	23	61	209	687	2478	8997
ε	3.594	2.502	2.245	1.929	1.815	1.716

In this table columns are denoted by the n dimension value of the function being analyzed, lines are denoted by value m of base T dimension of the applied tangent bundle. The numbers in the line, corresponding to symbol k_γ represent the number of synthesized hyperfilters. The numbers in a line, corresponding to symbol ε , represent relative matrix costs, $\varepsilon = k_\gamma(n + 2^m) / 2^n$. The value ε for the synthesized structure characterizes the ratio of the total number of the matrix elements σ and η in hyperfilters in relation to the volume of the storage capacity, corresponding to the function being analyzed.

While analyzing the value ε , one can notice that as it was claimed in the foundational work [4], the efficiency of linearization application rises with the increase in function dimension.

The effect of applying binomial hyperfilters is quite noticeable in case of neural networks synthesis, when neural networks solve the task of pattern recognition. These tasks are comparable to the problems of synthesis of partially defined LF. For example, calculations for partially defined functions from $n = 16$ variables of tangent bundle on base T of the dimension $m = 3$ are represented in Table 2.

Table 2.

δ	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05
k_γ	4536	4094	3655	3189	2733	2296	1826	1417	927	484	243
k_β	469	435	410	365	306	288	238	220	138	92	53
k_B	66	66	67	61	49	56	49	52	34	28	21
ε	1.661	1.499	1.338	1.168	1.001	0.841	0.669	0.519	0.339	0.177	0.089

Here, the value of δ indicates the relative power of the domain X of the function $\delta = |X| / 2^n$, k_β - the number of binomial hyperfilters $\beta(x)$, where k_B is the number of modules $B^{(t)}$ in the structure.

It is easy to notice that value ε of the matrix costs decreases in proportion to the decrease of the degree δ of partial LF certainty.

In conclusion, it is necessary to pay attention that right up to hyperlinearization, binomial hyperfilter $\beta(x)$ represents the inverse of the disjunctive analogue $q^\vee(x) = x_1x_2 \vee x_3x_4 \vee \dots \vee x_{n-1}x_n$ of the quadric form $q(x) = x_1x_2 \oplus x_3x_4 \oplus \dots \oplus x_{n-1}x_n$.

Indeed, $\beta(x) = \prod_{j=1}^k \beta^j = \prod_{j=1}^k (\gamma^{j1} \vee \gamma^{j2}) = \overline{\gamma^{11} \gamma^{12} \vee \gamma^{21} \gamma^{22} \vee \dots \vee \gamma^{k1} \gamma^{k2}} = \overline{q^\vee(\gamma)}$. Quadric form $q(x)$ according to its spectral characteristics is a filter of the white noise [5], for example, $D_q(\omega) = |S_q(\omega)| = const, \omega \neq 0$. Wherein, the spectral density $D_q^\vee(\omega) = |S_q^\vee(\omega)|$ of the disjunctive analogue $q^\vee(x)$ possesses several fixed values, that allows one to consider that binomial hyperfilters to be a disjunctive analogue of the white noise.

7. CONCLUSION

The procedures for synthesizing binomial hyperfilters and neural structures based on them is based on matrix and vector operations. Software implementation of synthesized neural structures also uses only matrix and vector operations in the field of residues module 2.

The concept of the error surface has not been used in this work, and the search of the global minimum of the error has not been aimed at. If the training sequence is not inconsistent and is adequately represented by a logical function, then the structure that realizes it with binomial hyperfilters provides absolute minimum of an error 0%.

The synthesized structure contains a simple series of independent functioning modules of the same type and in this way providing parallelism of calculations. The operation of hyperlinear transformation $\sigma \otimes x \oplus \eta$ of the input signal x can be performed for all modules simultaneously.

If a synthesized partial LF possesses, in contrast to a chaotically defined LF, a higher degree of monotonicity, then both the elements OR and the neurons with threshold activation and structural vector $A = (p; 1, 1, \dots, 1)$, $p > 1$ can be output elements of modules $B^{(t)}$. In this case, a reduction of matrix costs ε will be achieved.

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