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CALCULATION OF REINFORCED CONCRETE TANKS FOR VARIOUS APPLICATIONS

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Abstract

This article discusses the calculation of multipurpose reinforced concrete tanks which is made by the method of complete discretization. A complete picture of evolution of deformation stress state of the structure considering creep and construction history is considered. The comparative results of research with and without the technology of construction of buildings are given.

Key words: *Method of Full Discretization, creep, displacements, deformations, stresses, concrete construction, deformation stress, tank, history of construction.*

1. INTRODUCTION

Prospective transition of CIS to the European design standards in the construction industry requires development of new and modification of known engineering methods for calculation and design of construction. Creation and development of such methods should be based on fundamental research that can become the basis for the development of principally new, innovative technologies.

One of such directions is the further development and perfection of methods of analysis of structures and grounds, more precise estimate of evolution of the deformation stress state (DSS) and the yield of the building foundation in time. It is important to consider inelastic and nonlinear properties of building materials and structures, spatial and temporal in homogeneity, i.e. construction technology and other factors influencing their deformability, strength, durability and stability.

The basis for calculating any projected building object is its closed mathematical model, usually represented by systems of differential or integro-differential equations with boundary and initial conditions.

However, the analytical solution of such systems of equations can be obtained only in special cases for objects with a simple geometric shape with simple boundary and initial conditions. The way out was found by introducing various types of discretization of a continuous problem in which an infinite set of numbers representing the unknown function is replaced by a finite number of unknown parameters. As a result, the system of differential equations for real-world problems tends to turn into large systems of algebraic equations that can be solved only with the help of modern computer technology. One method of solving the assigned problems is the method of full discretization (MFD) which is a modified version of the finite element method (FEM) for solving various problems of elastic creep (Ter-Emmanuilyan & Polumordvinov, 2013).

MFD is a versatile, relatively simple, algorithmic and visual engineering method, which is non-stepping by time. It enables to determine discrete values of displacements, strains and stresses in the estimated time interval. The method was developed in two variants: as a method of displacements and as a method of forces. It can be combined with other engineering numerical methods such as the boundary elements method (BEM), finite difference method (FDM), and others.

MFD takes into account: a heterogeneous creep and aging of materials of construction of any structures and foundation soils, physical and geometric nonlinearity, plasticity, anisotropy; multimodulus elastic creep, the influence of temperature, the presence of prestressed or non-prestressed reinforcement in concrete, discrete graphics of erection of structures (increase or decrease in volumes, change of operational loads, etc.).

Let us consider basic operations required for the solution of the linear theory of creep by the method of complete discretization in the general case of structurally inhomogeneous in elastic properties of creep and age making body (Ter-Emmanuilyan, TN, 2006)

1. The most appropriate kernels of integral equations and corresponding functions $\delta(t,\tau)$ are selected considering the materials and conditions of the creep theory.

2. Introduction to the studied time interval of uniform or non-uniform grid of order p.

3. Calculation of inverse matrices $\left(\underline{E}^r\right)^{-1}$ of matrix «modules» $\underline{E}^r$ using selected theories of creep or only from the experimental data.

4. Adoption of matrix «coefficient» μ with time-constant diagonal elements, or its calculation for some materials (eg, concrete).

5. Selection of type and shape of finite elements and mesh nodes and their optimal numbering.

6. Calculation of stiffness matrices $\underline{k}^r$ of final elastic elements.

7. Deployment of each stiffness elements matrix into the generalized stiffness elements matrices $\underline{k}_t^r$ in accordance with elastic properties of creep and age of each element.

8. Construction of blocks of the generalized stiffness matrix of system by summing of elements of the generalized stiffness matrices of finite elements over the elements comprising respective nodes i and j by the formula $\underline{K}_{tij} = \sum_{r \in i,j} \underline{k}_{tij}^r$ and then construction of the matrix $\underline{K}_t$.

9. Building the vector $\underline{R}_t$ of time-varying external load values in all free nodes of the system in accordance with the specified conditions of the problem.

10. Solution of linear algebraic equations system $\underline{K}_t \overline{q}_t = \overline{R}_t$ with respect to components of vector $\overline{q}_t$ of evolution of motion of free nodes of the system.

11. Definition of vectors of evolution of displacements of nodes of all finite elements $\overline{q}_t^r$ from the general vector $\overline{q}_t$.

12. Calculation of deformation and stress by the formulas $\overline{\varepsilon}_t^r = \underline{B}_1 \overline{q}_t^r$ and $\overline{\sigma}_t = \underline{D}_t \overline{\varepsilon}_t$ respectively in all finite elements and at all time points of a discrete grid.

Implementation of item 10, i.e. direct solution of system of linear algebraic equations, encounters significant technical difficulties, as in general the system has a high order of *3Ns* for practical problems and an order of *2Np* for planar problems.

Therefore, due to peculiarities of the structure of the matrix $\underline{K}_t$, in the case of time-constant coefficient of transverse deformation two solutions for these systems were developed which require much less computer time than the direct solution of the system.

Two ways of solving these systems are developed taking into account peculiarities of structure of the matrix for a homogeneous elastic creep body, in the case of time-constant coefficient of transverse deformation. These methods of solution require far less computer time than the direct solution of the system.

Analysis of development and peculiarities of structure of the generalized stiffness matrix $\underline{K}_t$ for an ensemble of heterogeneous finite elements allowed us to construct the method of solution of the resolving system of equations - an algorithm that reduces the solution of the system, such as planar or axisymmetric problems of *2Np*-th order, to the sequence of *p* solutions of systems of *2Np*-th order.

Let us consider the above mentioned algorithm which is tested by solving many test and industrial planar and axisymmetric problems. This algorithm is built as an addition to the algorithm and program of solution FEM elastic problems. This algorithm consists of consecutive implementation of the following operations (Ter-Emmanuilyan, NYa & Ter-Emmanuilyan, TN, 1981).

1. All the terms of each element of the stiffness matrix of the elastic system, (i.e. all the elements k_{ijmn}^r of each of the stiffness matrix of finite elements) are stored in RAM or computed every time when they are called.

2. For the obtained stiffness matrix of the system, we firstly solve the simple elastic problem by finding N unknown components of nodal displacements, which are designated as $x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}$ for convenience, i.e.

vector: $\overline{q}_{t1} = \left[x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, ..., x_n^{(1)} \right]^T, (t_1 = 0)$.

In this case the elastic system can be written as:

$$\underset{[N\times N]}{\underline{A}} \cdot \overline{q}_{t1} = \underset{[N]}{\overline{b}^{(1)}}$$

or in expanded form:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & ... & & 0 & 0 & ... & 0 \\ a_{21} & a_{22} & a_{23} & ... & & & 0 & ... & 0 \\ a_{31} & a_{32} & a_{33} & ... & & & & ... & 0 \\ ... & ... & & & & & & & \\ ... & ... & & & \textit{лента} & & & & \\ ... & ... & & & & & & & ... \\ 0 & 0 & ... & & & & & ... & a_{NN} \end{pmatrix} \cdot \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \\ ... \\ ... \\ ... \\ x_N^{(1)} \end{pmatrix} = \begin{pmatrix} b_1^{(1)} \\ b_2^{(1)} \\ b_3^{(1)} \\ ... \\ ... \\ ... \\ b_N^{(1)} \end{pmatrix}.$$

$[\mathrm{N\times N}]$

Here *N* is now a full order of the elastic system.

3. Calculation of inverse matrices $\left(\underline{E}_p^r\right)^{-1}$ for different materials comprising the ensemble of finite elements with a subsequent reversion of these matrices, whereby a set of matrix "modules" $\underline{E}_p^r$ is obtained.

4. Calculation of the coefficients and right-hand sides of the equations for the first time step, using the elements of the second row of the matrix "modules" $\underline{E}_p^r$ and the solution of the obtained system, for what the following operations are performed:

a) Multiplication of $\frac{\underline{E}_{21}^r}{\underline{E}_0^r}$ by the appropriate members k_{ijmn}^r of the finite element of stiffness matrices and then summing over the elements as it is done in the formation of the ordinary system stiffness matrix. As a result $\sum_{r\ni ij} \left(\frac{\underline{E}_{21}^r}{\underline{E}_0^r} \right) k_{ijmn}^r$ will be obtained, which gives a symmetric band matrix of order N coefficients of the system, standing in the second rows and first columns of each its block of size p × p. The number of nonzero terms of the matrix equals the number of nonzero terms of the stiffness matrix of the elastic system.

$$\underset{[N\times N]}{\underline{C}_{21}} = \begin{pmatrix} C_{21} & C_{2,p+1} & C_{2,2p+1} & ... \\ C_{p+2,1} & C_{p+2,p=1} & C_{p+2,2p+1} & ... \\ C_{2p+2,1} & C_{2p+2,p+1} & C_{2p+2,2p+1} & ... \\ ... & ... & ... & ... \end{pmatrix}.$$

where

$$\begin{array}{lll} C_{11}=a_{11}, & C_{1,p+1}=a_{12}, & C_{1,2p+1}=a_{13}, \\ C_{p+1,1}=a_{21}, & C_{p+1,p+1}=a_{22}, & C_{p+1,2p+1}=a_{23}, \\ C_{2p+1,1}=a_{31}, & C_{2p+1,p+1}=a_{32}, & C_{2p+1,2p+1}=a_{33}, \\ C_{3p+1,1}=a_{41}, & C_{3p+1,p+1}=a_{41}, & \ldots \end{array}$$

b) Multiplying $\frac{\underline{\underline{E}}_{22}^{r}}{\underline{E}_{0}^{r}}$ by the corresponding k_{ijmn}^{r} and then summing, allows us to get $\sum_{r\ni ij}\left(\frac{\underline{\underline{E}}_{22}^{r}}{\underline{E}_{0}^{r}}\right)k_{ijmn}^{r}$ what gives the coefficients of the system standing in the second rows and second columns of each block, i.e. $\underline{C}_{22}$ matrix:

$$\underline{C}_{22}=\begin{pmatrix} C_{22} & C_{2,p+2} & \ldots \\ C_{p+2,2} & C_{p+2,p+2} & \ldots \\ C_{2p+2,2} & C_{2p+2.p+2} & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}.$$

в) Multiplication of matrix $\underline{C}_{21}$ by vector $\overline{q}_{t1}$ gives:

$$\underline{C}_{21}\overline{q}_{t1}=\begin{pmatrix} C_{21} & C_{2.p+1} & C_{2,2p+1} & \ldots \\ C_{p+2,1} & C_{p+2,p+1} & C_{p+2,2p+1} & \ldots \\ C_{2p+2,1} & C_{2p+2.p+1} & C_{2p+2,2p+1} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}\cdot\begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \\ . \\ x_N^{(1)} \end{pmatrix}=\begin{pmatrix} f_{1(21)} \\ f_{2(21)} \\ . \\ f_{N(21)} \end{pmatrix}=\overline{f}_{(21)}.$$

г) Then we perform substraction of second columns from right sides of rows *p+2, 2p+2*, i.e. vector $\overline{f}_{(21)}$ from the vector $\overline{d}_{(2)}$. In the result the right side of a new system of equations $\underset{[N\times N]}{\underline{C}_{22}}\cdot\overline{q}_{t2}=\overline{b}^{(2)}$ will be constructed.

$$\underline{\overline{d}}_{2}\,\overline{f}_{(21)}=\begin{pmatrix} d_2 \\ d_{p+2} \\ d_{2p+2} \\ \ldots \\ \ldots \\ \ldots \end{pmatrix}-\begin{pmatrix} f_{1(21)} \\ f_{2(21)} \\ f_{3(21)} \\ \ldots \\ \ldots \\ \ldots \end{pmatrix}=\overline{b}^{(2)}.$$

д) Then we solve the system of equations for the first time step, having exactly the same order, structure and number of non-zero coefficients, like the elastic system:

$$\underset{[N\times N]}{\underline{C}_{22}}\cdot\overline{q}_{t2}=\overline{b}^{(2)}.$$

And finally we find $\overline{q}_{t2}^{T} = \left[x_1^{(2)} x_2^{(2)} ... x_N^{(2)} \right]$, i.e. the vector of components of displacements of nodes of the system at time t_2.

5. Then we calculate coefficients and right-hand sides of the equations for the second step, using the elements of the third row of the matrix "modules" $\underline{E}_s^r$. The resulting system of equations is solved, that determines the vector of components of displacements of nodes, corresponding to time t_3:

$$\overline{q}_{t3}^{T} = \left[x_1^{(3)} x_2^{(3)} x_3^{(3)} ... x_N^{(3)} \right].$$

6. Let us consider the formation of the coefficients and right-hand sides of the system at an arbitrary *(ℓ -1)*-th time step using ℓ-th rows matrix "modules" $\underline{E}_p^r$, where $2 \le \ell \le p$;

a) First, after multiplication and summation, we find $\sum_{r \ni ij} \frac{\underline{E}_{l2}^r}{\underline{E}_0^r} k_{ijmn}^r$ that gives the coefficients standing in ℓ-th rows and ℓ-th columns of blocks, i.e. matrix $\underline{C}_{l1}$. Then using $\frac{\underline{E}_{l2}^r}{\underline{E}_0^r}$ we get $\underline{C}_{l2}$, and similarly the other matrices, i.e.

$$\underline{C}_{l1}, \underline{C}_{l2}, \underline{C}_{l3}, ..., \underline{C}_{l,l-1}, \underline{C}_{ll}; \quad 2 \le \ell \le p.$$

b) We multiply resulting matrices, except the last $\underline{C}_{\ell\ell}$, by the corresponding displacement vectors

$$\underline{C}_{l1} \overline{q}_{t1} = \overline{f}_{l1},$$
$$\underline{C}_{l2} \overline{q}_{t2} = \overline{f}_{l2},$$
$$\underline{C}_{l3} \overline{q}_{t3} = \overline{f}_{l3},$$
$$...$$
$$\underline{C}_{l,l-1} \overline{q}_{t,l-1} = \overline{f}_{\ell,\ell-1}.$$

c) Then the sum of the vectors is subtracted from the right sides of *ℓ-th, p + l, 2p + l, ...*, and the other rows, i.e. from vector $\overline{d}_l$ composed of $\overline{d}_l, \overline{d}_{p+l}, \overline{d}_{2p+l}, ...$:

$$\overline{d}_\ell - \sum_k^{\ell-1} \overline{f}_{\ell k} = \overline{b}^{(l)}$$

as a result the vector of the right side of the new system is obtained.

d) Then the system of the *N*-th order:

$$\underline{C}_{l,l} \overline{q}_{t,l} = \overline{b}^{(l)}$$

is solved and

$$\overline{q}_{tl}^{T} = \left[x_1^{(l)} x_2^{(l)} x_3^{(l)} ... x_N^{(l)} \right]$$

is found.

A similar cycle of operations is performed for all the other steps in time, until the last, *p-1*, step.

As a result, we obtain the desired law of changes in the components of displacements of all nodes of the system in this time interval.

Thus one-time solution of a large system of resolving equations of order *Np* is replaced with the sequence of *p*-solving of systems of equations of order N.

7. Hereinafter vectors of evolution of the deformation and stress in all finite elements are determined.

The algorithm provided in this section is convenient for programming and saves computing time and computer memory.

Most of the software for solving problems of creep theory by the method of full discretization is written using this algorithm.

2. CALCULATION OF RECTANGULAR CONCRETE TANK

Let us consider joint in time calculation of two-tier reinforced concrete building and its foundation in the spatial formulation in the example of modular station tanks of biological sewage treatment with capacity of 6400 m^3/day designed for filtration fields in the village Tasboget, Akmola Oblast, Kazakhstan (Figure 1).

The tanks of the modular station are made of concrete B25 with the following mechanical properties: initial modulus is 30000 MPa, the calculated resistance to axial compression is 14.5 MPa, the calculated resistance to axial tension is 1.0 MPa. There was used the reinforcement armature of Class A-III with mechanical properties: modulus of elasticity is 200 000 MPa, the design resistance is 367.7 MPa.



Figure 1 - General view of the biological treatment of sewage

The model of concrete with "smeared" reinforcement were used in the calculation of the building to account for the rheological properties of the material of the tanks. In this case, the behaviour of elastic creeping material (concrete or reinforced concrete with "smeared" reinforcement) can be described by equations of the hereditary theory of aging with a measure of creep by S.V.Aleksandrovskiy (Alexandrovskiy, 1973)

$$\delta(t,\tau) = \frac{1}{E}\left(1 - e^{-\beta\tau}\right) + \psi(\tau) - \psi(\tau)\frac{e^{\gamma\tau} - A_2}{e^{\gamma\tau} - A_2} + \Delta(\tau)\left[1 - e^{-\alpha(1-\tau)}\right]$$

here

$$\Delta(\tau) = C_1 - C_3 + \frac{A_1 - A_3}{\tau}, \quad \psi = C_3 + \frac{A_3}{\tau},$$

the values for parameters of creep are taken in accordance with recommendations of N.H.Aratyunyan (Aratyunyan, 1952): $\alpha = d^{-1}$, $\beta = 0{,}206\ d^{-1}$, $\gamma = 0{,}03\ d^{-1}$, $A_1 = 4{,}62 \cdot 10^{-5}$ d/(kgf/cm^2), $A_2 = 1$, $A_3 = 3{,}416 \cdot 10^{-5}$ d/(kgf/cm^2), $C_1 = 0{,}975 \cdot 10^{-5}$ (kgf/cm^2)$^{-1}$, $C_3 = 0{,}756 \cdot 10^{-5}$ (kgf/cm^2)$^{-1}$ are the parameters of creep, E_o =30000 МПа.

Then we use the measure and kernel of Zh.S.Erzhanov (Erzhanov & Karimbayev, 1975) to describe the behaviour of elastic creeping ground:

$$\delta(t,\tau) = \frac{1}{E}\left[1 + \frac{\Delta}{\alpha}(t-\tau)^{\alpha}\right]$$

obtained from the difference kernel

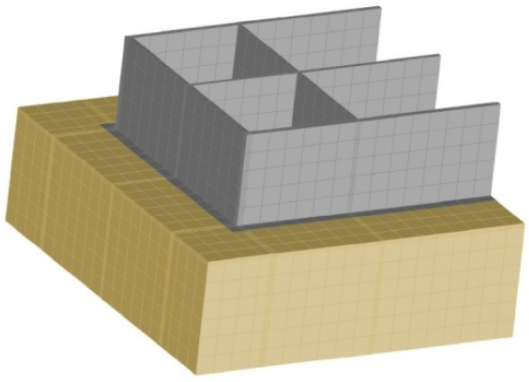
$$L(t-\tau) = \Delta(t-\tau)^{\alpha-1},$$

where Δ, α are the creep parameters.

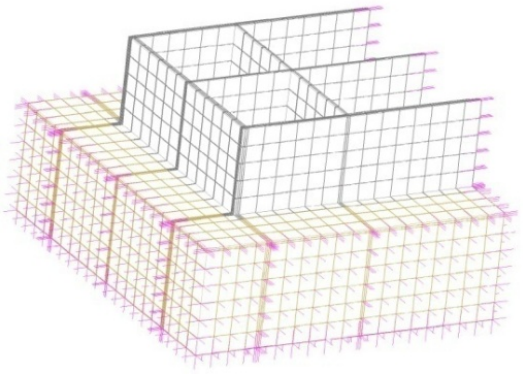
The influence of the weight of a building on the deformation stress state of the soil mass is considered with assumption of smallness of the deformation with respect to the size of the deformed region. The same assumption remains valid when the yield of a building is considered near walled pits, underground structures, natural slopes, etc. The hypothesis about the smallness of the deformation is not applicable only when deformations comparable to the size of the deformable region (i.e. the process of sliding slope, buckling, etc.) are considered (Shashkin & Shashkin, 2002).

The analytical model of building is shown in Figure 2-a. Due to the presence of symmetry plane, only half of the tank is considered in the calculation.

This design scheme is split into 6212 octanodal prismatic volume elements connected in 7714 nodes. Boundary conditions in the form of rod connections are shown in Figure 2-b:



a)



b)

Figure 2 - Design scheme of the tank

Figure 3 shows the projections of the tank with building axes.

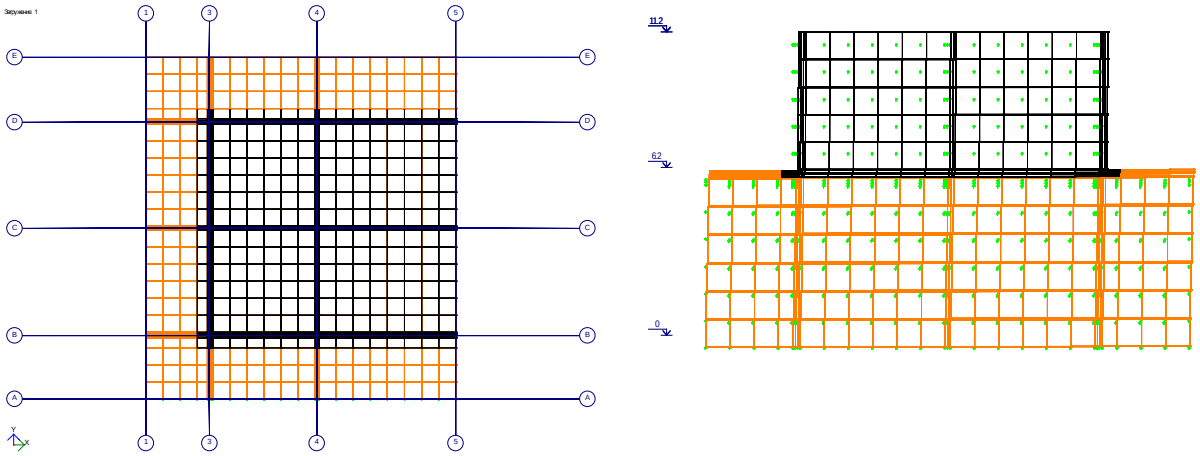
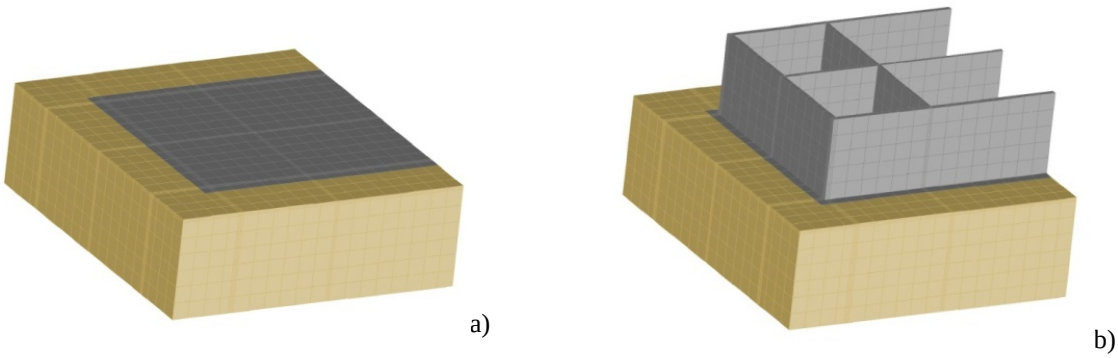


Figure 3 – Frontal and upper projections of the tank

Building of the tank is carried out in two stages. The first stage is the concreting of the base plate. The duration of this phase is 30 days (5 time points. Figure 4-a). The own weight of the base plate is taken into account at this stage. The second stage is the concreting of the walls of the tank. The own weight of the base plate and the tank walls is taken into account at this stage. The duration of this stage is 90 days (5 time points. Figure 4-b). In the third stage the tank is filled with water. The own weight of the base plate and the tank walls is taken into account at this stage. Duration of this stage is 480 days (10 time points) (Figure 4-b).



a) b)

Figure 4 - Stages of construction of building.

The timeline for calculations is taken uneven, with increasing intervals to the end of the scale. This makes it possible to estimate the evolution of the deformation stress state of the tank, both during periods of construction and initial operation.

Information about the components of nodal displacements in 7714 nodes for the 20 time points is obtained as a result of the calculations. The components of vectors of deformations and stresses for each time point, at all 6212 finite elements are calculated. Thus, the complete picture of evolution of the displacement vectors, deformations and stresses in space and time is obtained.

Let us consider some typical results of calculations of the change of the deformation stress state of the reservoir in time. Software module in AutoCAD was developed during the study. This module was used for graphical interpretation of the results obtained. The input data for this module are parameters of FEM of the construction model (nodes, elements, supports, loads, stiffness) and the results of solution of the problem MFD for given construction (vectors of displacements, deformations and stresses in each of the time points of the interval).

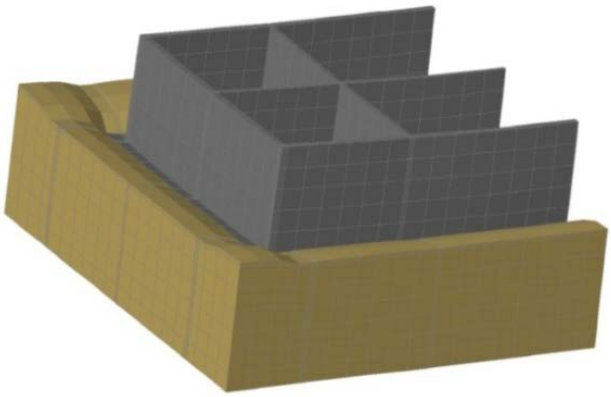


Figure 5 - Deformed diagram computational region in 600 days

The figure shows the deformed scheme of computational region (scale deformation increased 30-fold) for the last time point - in 600 days after the construction of buildings.

Figure 6 (a, b, c, d) shows the contours of vertical displacements for t_4 = 20, t_9 = 70, t_{14} = 200, t_{20} = 600 days, which illustrate the evolution of vertical displacements.

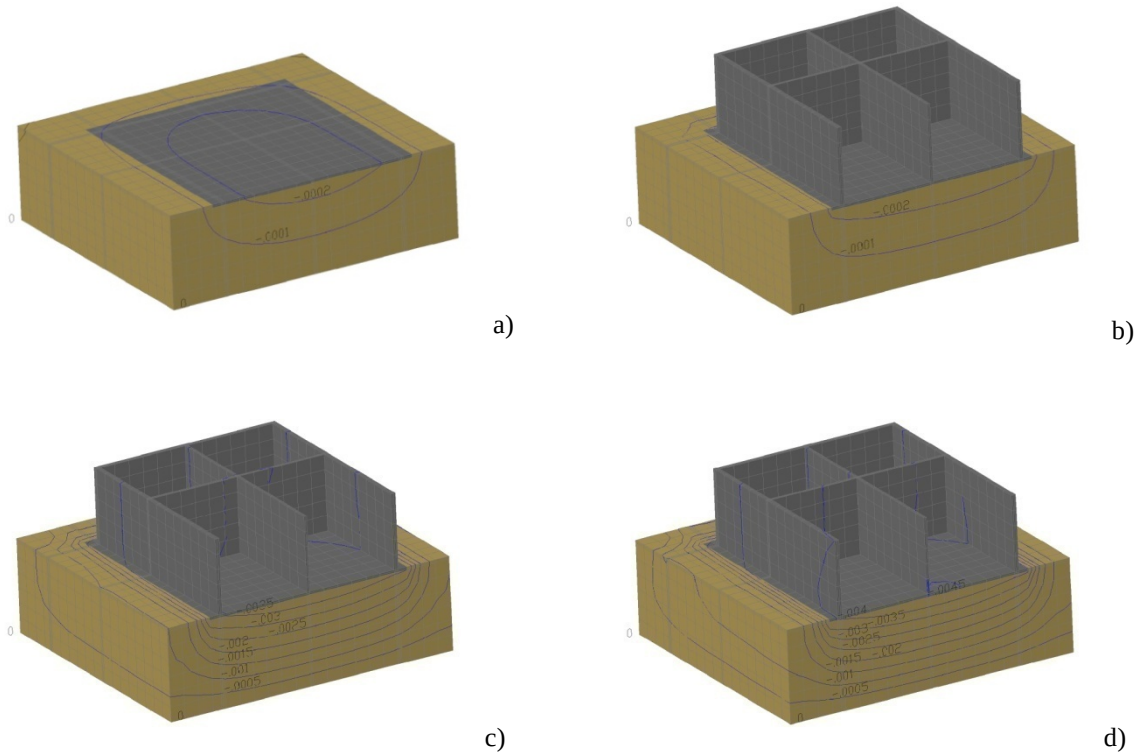


Figure 6 - Contours of vertical displacements for times t_4 (a), t_9 (b), t_{14} (c), t_{20} (d).

Isolines of the horizontal components of displacements for time points t_9 = 70 t_{20} = 600 days are shown in Figure 7 (a, b,c,d).

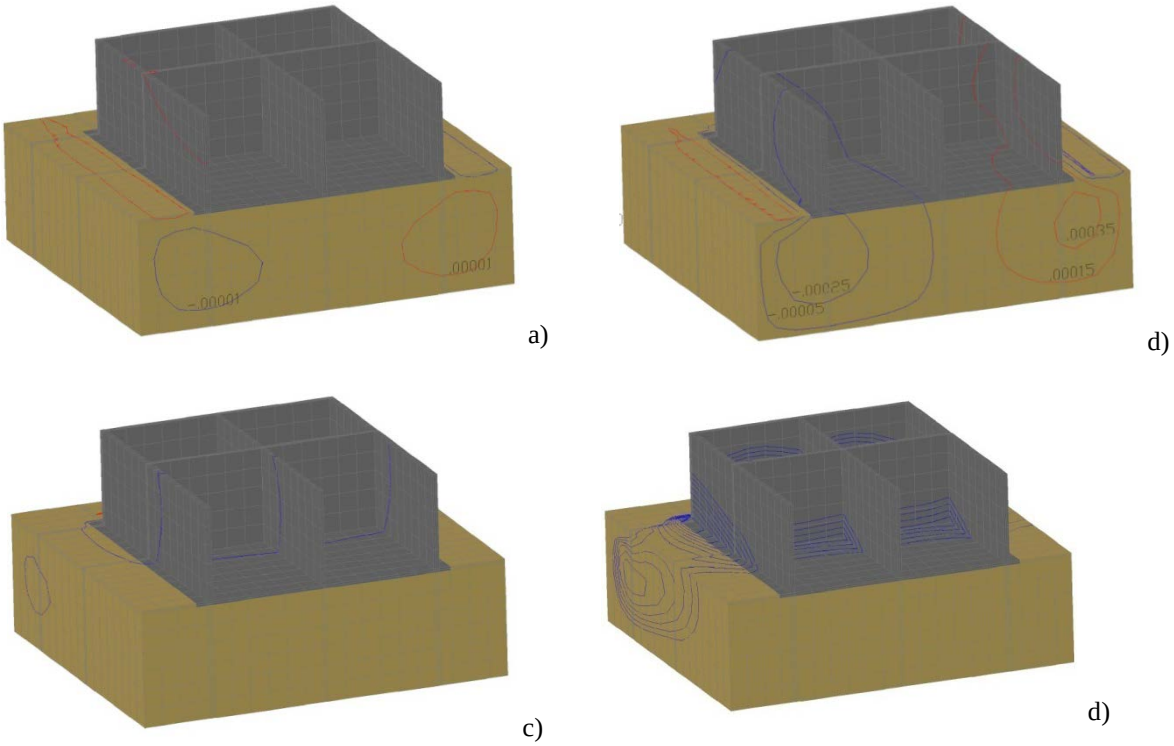


Figure 7 Contours of horizontal components of displacements for times t_9, t_{20}.

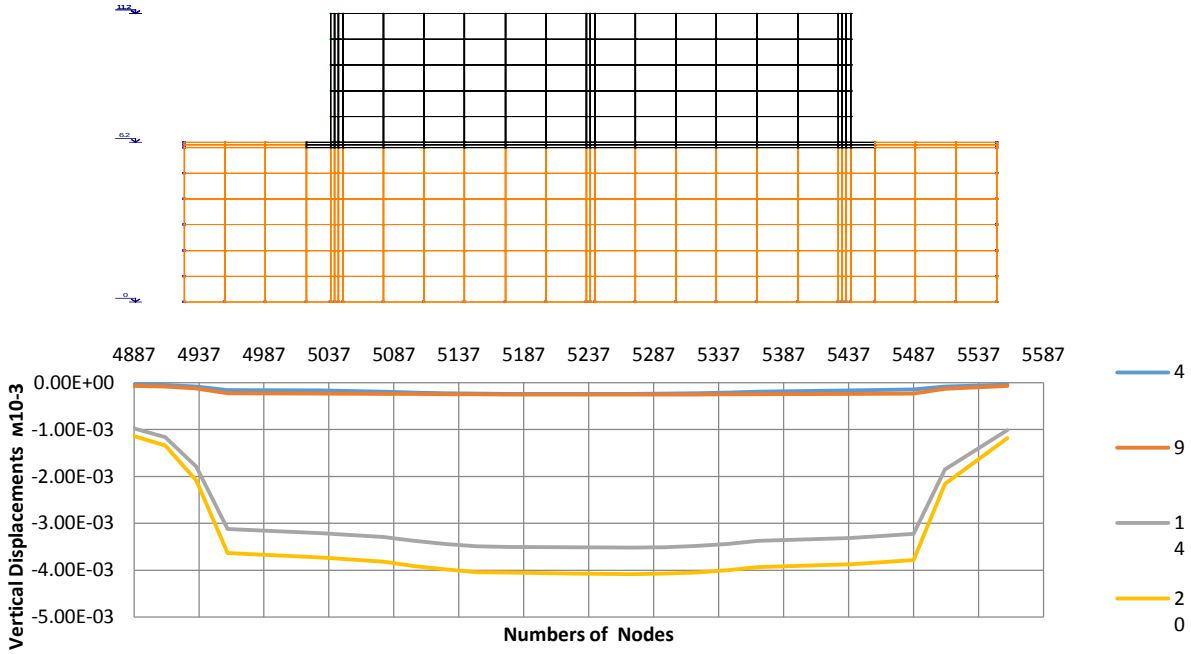


Figure 8. Diagrams of yield of nodes (m) along the axis 4. (Curves correspond to the times: 1) t_4; 2) t_6; 3) t_{14}; 4) t_{20}.

Figure 8 shows diagrams of yield of tank elements located along the axis 4, and Figure 9 shows diagrams of yield of tank elements along C-axis for four time points corresponding to the four stages of erection: t_4, t_9, t_{14} and t_{20} (respectively curves 1,2,3,4).

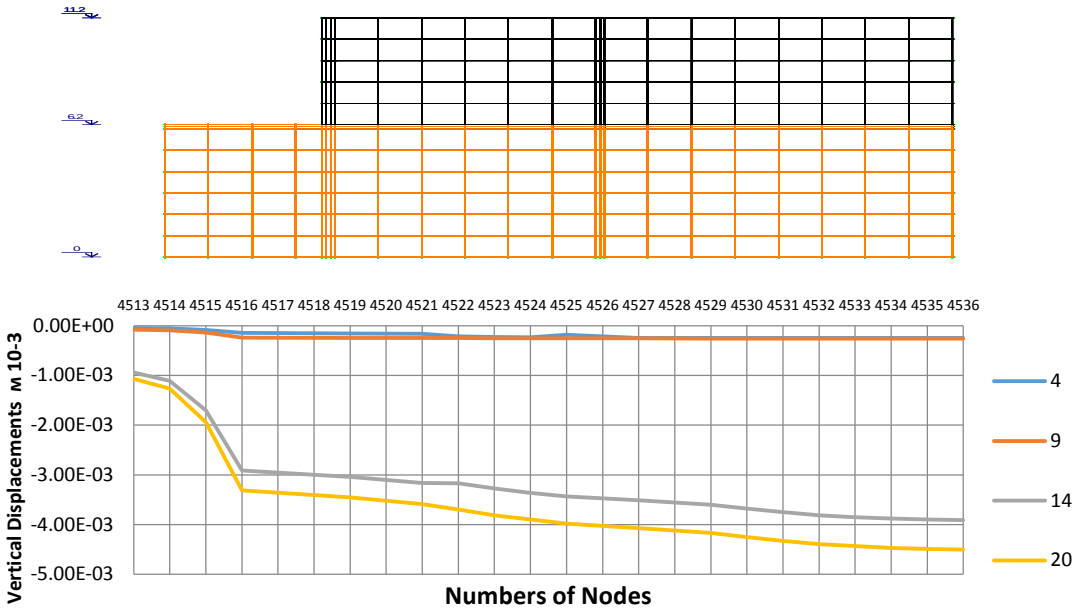


Figure 9. Diagrams of yield (m) of nodes along the axis C.

(Curves correspond to the times: 1) t_4; 2) t_6; 3) t_{14}; 4) t_{20}.

Figure 9 shows the displacements (yield) of more typical node that is located at the intersection of the axes C and 5 depending on time for the entire range of the study.

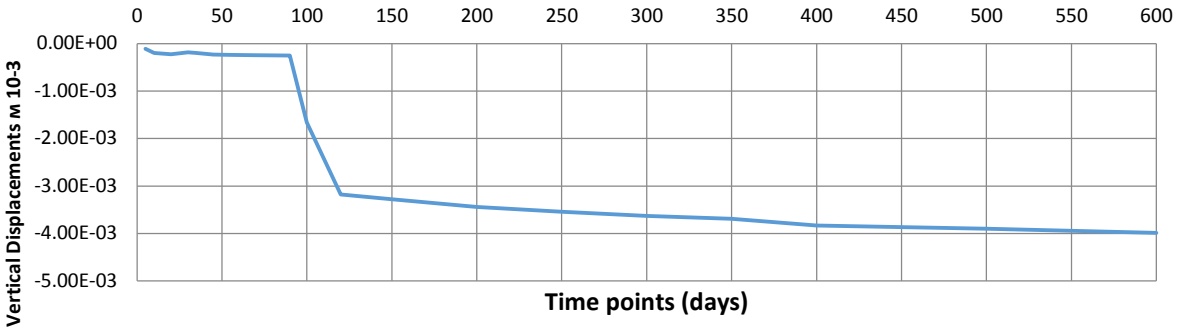


Figure 10. Diagrams of yield (m) of element at the intersection of the axes of C-5.

(Curves correspond to times: 1) t_4; 2) t_6; 3) t_{14}; 4) t_{20}.

Analysis of the solution reveals the significant incremental growth of displacement in time (Figure 10) and deformation (on the example of the vector components of ground deformations in the element number 2451 - Figure 11). Evolution of stress tensor components (on the example of the components of the stress vector in the ground element number 2451 - Figure 12) within each time step virtually are unchanged.

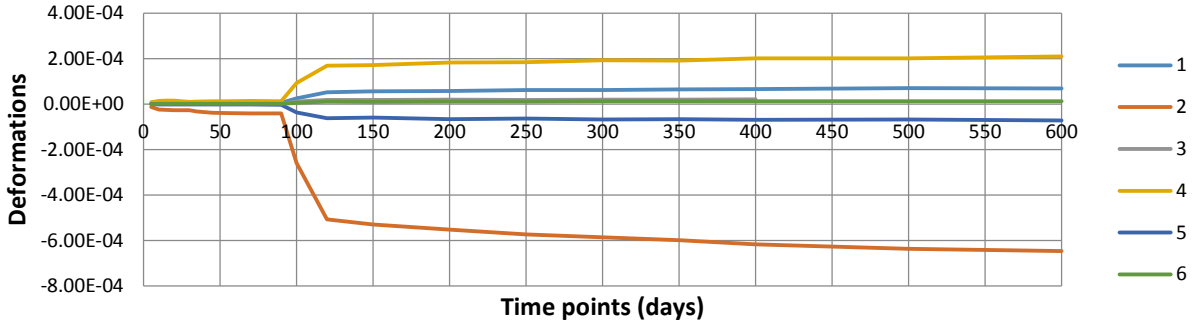


Figure 11. Dependence of the components of vector in the element of deformation of the soil mass number 2451 on time

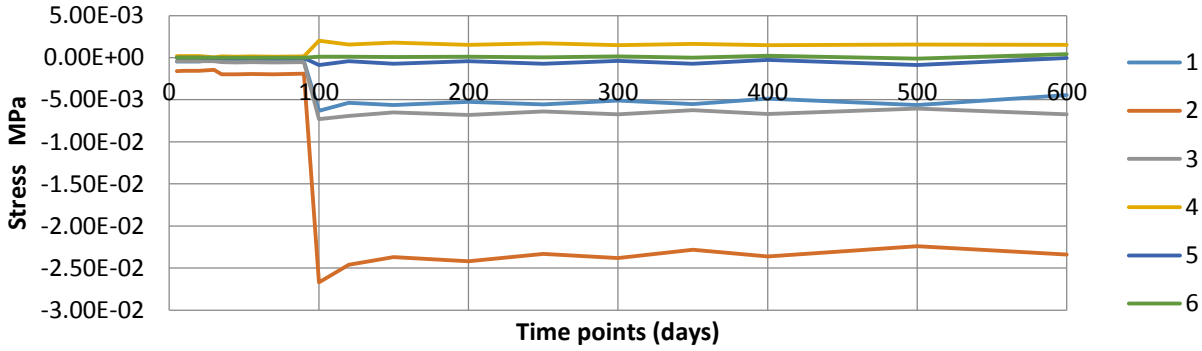


Figure 12. Dependence of the components of the stress vector in the element of the soil mass number 2451 on time.

4. CONCLUSIONS:

1. The developed method can be used in calculations and design of large-scale structures in the form of buildings, dams, and other objects taking into account their joint perfomance with the soil and bedrock, the temporal and spatial inhomogeneity due to creep materials. In addition, the developed software packages are convenient because of their user-friendly interface, advanced automation data input and processing of the results.
2. Methodology developed in practice and the results obtained allow to predict the change of DSS in time considering construction projects with high accuracy over a long period of operation, even with the possibility of reconstruction. Such consideration can lead to significant changes of DSS (by an order of magnitude and higher) for the entire period of operation.

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