

TESTS FOR THE DISCIPLINE “MATHEMATICAL METHODS AND MODELS IN ECONOMICS”

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Abstract

This paper presents the tests developed and used by the author for assessing the extent to which the material of training courses “Mathematical Models and Models in Economics”, “Operations Research in Economics”, “Methods of Optimal Solution” is assimilated by students of the National Research Tomsk State University and the Tomsk State University of Control Systems and Radioelectronics.

Key words: *educational test, test reliability, test validity, achievement test, mathematical models, linear programming*

1. INTRODUCTION

Mathematical models of economic processes, including linear programming models, are actively used by management of large companies in various socioeconomic activities and industries. In particular, a survey of vice-presidents for production from Fortune 500 companies shows that models for optimal allocation of scarce resources, inventory management models, etc. are the most popular tools for solving production problems [<http://www.konfliktov.net/>]. It should be noted that Fortune Global 500 is an annual ranking of the top 500 corporations worldwide as measured by revenue. The list is compiled and published annually by Fortune magazine. Thus, it is difficult to overestimate the importance of acquiring mathematical methods and models both by future economists and specialists in other disciplines who study economics as additional education.

Control of acquired knowledge is the important issue in study of any discipline. The basic conventional tools for such a control are writing papers or essays for checking practical abilities and skills and answering quizzes for checking of how the theoretic material is assimilated. These tools are used in examinations, passes, etc. The transformation of the social, economic and demographical environment in Russia has entailed the continuous reformation of the educational system and required qualitatively new approaches to training of students, including new approaches to assessment of their achievements. The priority has been given to qualitative methods for assessing knowledge and skills of students. Nowadays, one of the key methods for educational measurement is testing. An example would be the State Final Attestation System or the Unified State Exam (USE) which are known to be the primary method for final assessment of high school students. Moreover, the USE results serve as selection criteria for admittance to Russian universities. The training of pupils is oriented at passing tests. When pupils leave a high school and go to universities, the conventional assessment methods become an additional stress for them in the quite different training environment. That is why, universities are increasingly introducing tests to their training process, and teachers are facing the problem to develop new and adapt existing (part of which is uploaded in Internet) tests for various courses.

2. CONTENT OF THE COURSE “MATHEMATICAL METHODS AND MODELS IN ECONOMICS”

The course “Mathematical Methods and Models in Economics” is based on linear programming models and methods [Haas, Kremer]. The focus is made on such linear programming problems (LPP) as optimal production planning, optimal mixing, optimal cutout, and transportation problem. The simplex method in the general setting and tableau format are considered as the basic method. The

method of potentials is described for solving the transportation problem. The graphical method of solution to linear programming problems is studied in detail for better understanding of the simplex method. One of the most important concepts in the linear programming theory is the concept of duality [Kantorovich]. We study in detail mutually dual linear programming problems (DLPP), the meaning of dual variables and their relationship with variables of an original problem.

3. SPECIFICS OF EDUCATIONAL TESTS AS A KNOWLEDGE ASSESMENT TOOL

“The educational test is the system of specific tasks presented in order of ascending difficulty which allows one to qualitatively assess the knowledge structure and level of test subjects (2). At the same time, Avanesov considers the test [Avanesov, 1999] as “a standardized task the accomplishment of which allows one to measure psychophysiologic and personality characteristics, as well as knowledge, abilities and skills of test subjects”. The first studies devoted to the theory of tests were published in the early XX century at the interface of psychology, sociology, pedagogy, and other so called behavioral sciences. Foreign psychologists call this science the psychometrika, while educationists – the educational measurement.

In our opinion, the advantage of tests is the possibility for detailed differentiation of students' knowledge. Diverse questions in a test allow one to assess actual level of knowledge. However, it cannot be denied that subject's intuition plays significant role in passing tests. To a certain degree, tests can be considered as a tool for developing intuition.

Nowadays, test-based assessment of knowledge is used for different courses, including ones in the higher education system. We offer you the set of tests used by the author for assessing knowledge of students in the courses “Mathematical Models and Models in Economics”, “Methods of Optimal Solution”, etc.

OPTION No. 1

1. Specify the canonical form of a linear programming problem (LPP):

$$L(x_1, x_2, x_3) = 5x_1 - 4x_2 + 3x_3 \rightarrow \max$$

$$a) \begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 - 3x_2 + 2x_3 = -1 \\ 4x_1 - 2x_2 - x_3 = 1 \end{cases}$$

$$L(x_1, x_2, x_3) = 3x_1 + 2x_2 + 6x_3 \rightarrow \min$$

$$b) \begin{cases} x_1 + x_2 + x_3 \leq 2 \\ x_1 - 3x_2 + 2x_3 = -1 \\ 4x_1 - 2x_2 - x_3 = 1 \end{cases}$$

$$L(x_1, x_2, x_3) = x_1 - 6x_2 + 2x_3 \rightarrow \max$$

$$c) \begin{cases} x_1 - 4x_2 + 2x_3 = -2 \\ 3x_1 - x_2 - 4x_3 = -1 \\ x_1 - x_2 - x_3 = 1 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

2. A linear programming problem does not have a solution if:

a) system of constraints is inconsistent; b) a linear form does not have a lower limit; d) a domain is not limited; e) inequalities in constraints have different signs.

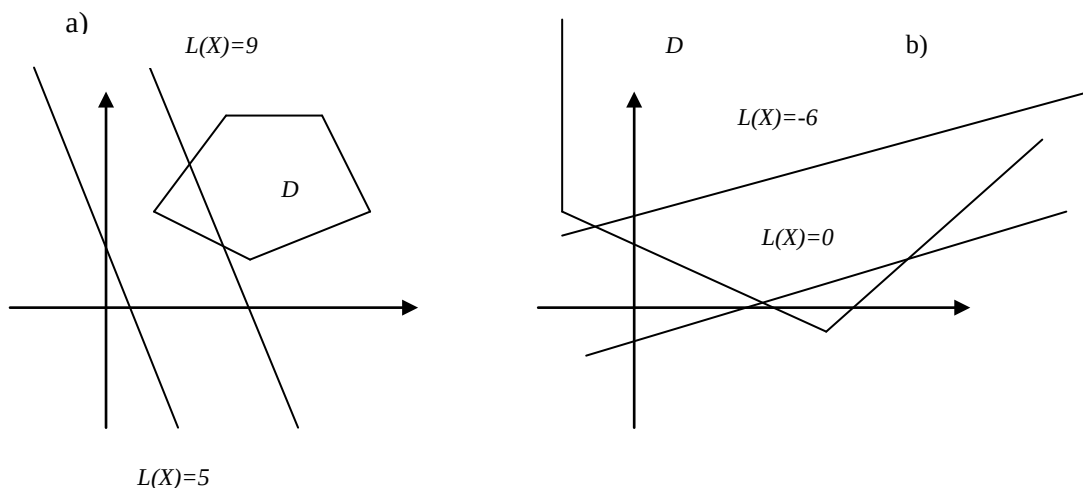
3. The domain of a linear programming problem is:

a) nonnegative polyhedron; b) convex polytope; c) closed polygon; d) closed polygon which limits a convex polytope.

4. The solution to a linear programming problem is achieved:

a) within a domain; b) at the boundary of a domain; c) at the vertex of a domain; d) at the closest point to the origin.

5. Choose the designs of a linear programming problem which affords a maximum and a minimum to the linear form for fig.a) and fig.b):



6. Find errors in the notation of the first simplex table:

B	β	x_1	x_2	x_3	x_4	x_5	x_6
x_4	4	2	-1	3	0	0	0

x_5	5	3	0	-6	1	5	0
x_6	-11	4	10	-0,5	0	0	1
$L(X)$	-5	1	3,4	-6,5	0	0	4

7. Write the optimal plan of a linear programming problem and optimal value of a linear form (including type of extremum) for the following last simplex table:

B	β	x_1	x_2	x_3	x_4	x_5
x_2	4	0	1	-3	0	2,5
x_4	5	0	0	4	1	-1,2
x_1	1	1	0	0,5	0	3
$L(X)$	-5	0	0	-6,5	0	-2,5

8. The following LPP are given: a)
$$\begin{cases} L(X) = 2x_1 + x_2 \rightarrow \max \\ 2x_1 + 3x_2 \leq 5 \\ 5x_1 + 2x_2 \leq 7 \\ -x_1 + 2x_2 \leq 3 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$
 b)
$$\begin{cases} L(x_1, x_2) = x_1 - x_2 \rightarrow \max \\ -2x_1 + x_2 \leq 2 \\ x_1 - 2x_2 \leq -8 \\ x_1 + x_2 \leq 5 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

$$L(Y) = 5y_1 + 7y_2 + 3y_3 \rightarrow \min$$

Assign symmetric DLPP to them: a)
$$\begin{cases} 2y_1 + 5y_2 - y_3 \leq 2 \\ 3y_1 + 2y_2 + 2y_3 \leq 1 \end{cases}$$

$$L(Y) = -2y_1 + 8y_2 - 5y_3 \rightarrow \min$$

$$L(Y) = 5y_1 + 7y_2 + 3y_3 \rightarrow \min$$

b)
$$\begin{cases} -2y_1 + y_2 + y_3 \geq 1 \\ y_1 - 2y_2 + y_3 \geq -1 \\ y_1 \geq 0, y_2 \geq 0 \end{cases}$$
 c)
$$\begin{cases} 2y_1 + 5y_2 - y_3 \geq 2 \\ 3y_1 + 2y_2 + 2y_3 \geq 1 \\ y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \end{cases}$$

$$L(Y) = 5y_1 + 7y_2 + 3y_3 \rightarrow \min$$

$$L(Y) = 2y_1 - 8y_2 + 5y_3 \rightarrow \min$$

d)
$$\begin{cases} 2y_1 + 5y_2 - y_3 \leq 2 \\ 3y_1 + 2y_2 + 2y_3 \leq 1 \\ y_1 \geq 0, y_2 \geq 0 \end{cases}$$
 e)
$$\begin{cases} -2y_1 + y_2 + y_3 \geq 1 \\ y_1 - 2y_2 + y_3 \geq -1 \\ y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \end{cases}$$

$$L(Y) = -2y_1 + 8y_2 - 5y_3 \rightarrow \min$$

$$L(Y) = 2y_1 - 8y_2 + 5y_3 \rightarrow \min$$

f)
$$\begin{cases} -2y_1 + y_2 + y_3 = 1 \\ y_1 - 2y_2 + y_3 = -1 \\ y_1 \geq 0, y_2 \geq 0 \end{cases}$$
 g)
$$\begin{cases} -2y_1 + y_2 + y_3 = -1 \\ y_1 - 2y_2 + y_3 = +1 \\ y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \end{cases}$$

9. Set up a correspondence between variables for each pair of symmetric dual problems from the previous clause. Find a graphical solution to the original problems and write a solution to both problems.

10. Determine the intervals and the stability region for the problem 1a).

OPTION No. 2

1. Specify the right notations of the transportation problem:

$$L(X) = \sum_{i=1}^m c_{ij} x_{ij} \rightarrow \min \quad L(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \rightarrow \min \quad L(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \rightarrow \min$$

a) $\begin{cases} \sum_{i=1}^m x_{ij} = b_j, j = \overline{1, n} \\ \sum_{j=1}^n x_{ij} \geq a_i, i = \overline{1, m} \\ x_{ij} \geq 0, i = \overline{1, m}, j = \overline{1, n} \end{cases}$ b) $\begin{cases} \sum_{i=1}^m x_{ij} = b_j, j = \overline{1, n} \\ \sum_{j=1}^n x_{ij} = a_i, i = \overline{1, m} \\ x_{ij} \geq 0, i = \overline{1, m}, j = \overline{1, n} \end{cases}$ c) $\begin{cases} \sum_{i=1}^m x_{ij} = b_j, j = \overline{1, n} \\ \sum_{j=1}^n x_{ij} \leq a_i, i = \overline{1, m} \\ x_{ij} \geq 0, i = \overline{1, m}, j = \overline{1, n} \end{cases}$

$$L(X) = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \rightarrow \min \quad L(X) = \sum_{i=1}^m \sum_{j=1}^n x_{ij} \rightarrow \min$$

d) $\begin{cases} \sum_{i=1}^m x_{ij} \leq b_j, j = \overline{1, n} \\ \sum_{j=1}^n x_{ij} = a_i, i = \overline{1, m} \\ x_{ij} \geq 0, i = \overline{1, m}, j = \overline{1, n} \end{cases}$ f) $\begin{cases} \sum_{i=1}^m c_{ij} x_{ij} = b_j, j = \overline{1, n} \\ \sum_{j=1}^n c_{ij} x_{ij} = a_i, i = \overline{1, m} \\ x_{ij} \geq 0, i = \overline{1, m}, j = \overline{1, n} \end{cases}.$

2. The transport problem is solvable when, and only

when.....: (write the formula)

3. Open transport problems: with excess, if.....; with shortage, if.....

4. Specify type of the transportation problems from clause 1): closed -; with excess -; with shortage -

5. For solving the problem with shortage, the fictious.....should be introduced; for solving the problem with excess, the fictious.....should be introduced.

6. The auxiliary problem of the following type must be introduced for solving the transportation problem:

a) $\overline{c_{ij}} = \begin{cases} c_{ij}, (i, j) \in E \\ M, (i, j) \notin E \end{cases}$ b) $\overline{c_{ij}} = \begin{cases} c_{ij}, (i, j) \in E \\ 0, (i, j) \notin E \end{cases}$ c) $\overline{a_i} = \begin{cases} a_i, (i, j) \in E \\ M, (i, j) \notin E \end{cases}$ d) $\overline{b_j} = \begin{cases} b_j, (i, j) \in E \\ M, (i, j) \notin E \end{cases}$

e) $\overline{c_{ij}} = \begin{cases} 0, (i, j) \in E \\ M, (i, j) \notin E \end{cases}$ f) $\overline{c_{ij}} = \begin{cases} 0, (i, j) \in E \\ c_{ij}, (i, j) \notin E \end{cases}.$

7. In order to solve the problem with the additional constraint $x_{ij} \geq p$:

a) the additional consumer should be introduced; b) inventories and needs should be changed; c) the additional supplier should be introduced; d) the supplier A_i and the consumer B_j should be excluded from consideration.

8. In order to solve the problem with the additional constraint $x_{ij} \leq q$:

a) the additional consumer should be introduced; b) inventories and needs should be changed; c) the additional supplier should be introduced; d) the supplier A_i and the consumer B_j should be excluded from consideration.

9. The following problem will lead to the problem with inhibitions: a) with the additional constraint $x_{ij} \geq p$; b) with the additional constraint $x_{ij} \leq q$.

10. The theorem of solubility of the problem with inhibitions is expressed by the following conditions: a) $M > c \cdot \sum_{i=1}^m a_i$; b) $M \geq c \cdot \sum_{i=1}^m a_i$; c) $M < c \cdot \sum_{i=1}^m a_i$.

11. In the formula from clause 10: $c = \dots\dots\dots$; $\sum_{i=1}^m a_i$ - for
.....transportation problem.

12. The problem with inhibition is deemed solved if the optimal plan of the auxiliary problem contains.....

13. The optimal plan of the auxiliary problem for the problem with exclusions is generated in the following way:
.....

14. Fill in the following cell:

	B_1	B_2	B_3	B_4	Resources
A_1	47 5	8 6	20	65	11 6 X
A_2	33	0	7	28	22
A_3	24	70	3	46	8
A_4	0	0	0	0	86
Requirements	5 X	54 48	65	3	127

15. Fill in the first cell:

	B_1	B_2	B_3	B_4	Resources	between two.....costs
A_1	47	8	20	65	11	
A_2	33	0	7	28	22	
A_3	24	70	3	46	8	
A_4	0	0	0	0	86	
Requirements	5	54	65	3	127	
.....between two. costs						

16. Write the first plan for the following transportation problem by the method of minimal cost, check for optimality, make one improvement of the plan and compare values of the linear form:

	B_1	B_2	B_3	B_4	3
A_1	47	5	41	38	59
A_2	34	41	70	17	73
A_3	21	41	38	50	53
Π	5	70	38	8	

17. Write the auxiliary problem for the problem from clause 16 provided that transportation is prohibited: $A_1 \rightarrow B_3$, $A_2 \rightarrow B_4$, $A_3 \rightarrow B_1$.

18. Write the auxiliary problem for the problem from clause 16 if $x_{22} \geq 40$, $x_{33} \leq 20$:

19. Write the optimal design and the optimal value of the linear form for the original problem taking into account the optimal design of the auxiliary problem:

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