CONSEPTION OF MATHEMATICS TEACHING AS A NECESSARY PART OF ANY CONCEPTION (OF DEVELOPMENT) OF MATHEMATICS EDUCATION

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Abstract
We claim that any conception of mathematics education must contain a part devoted to the definition of concepts. Namely, when we are speaking about any conception of mathematics education we must have clear understanding of words "mathematics" and "education". Moreover, and this is quite important, we must know precisely what is meant under "mathematics teaching". This last knowledge we call "conception of mathematics teaching".

After that we propose and examine our own conception of mathematics teaching which we call "the conception of humanitarianly-oriented mathematics teaching".

Key words: *mathematics, education, teaching of mathematics, conception of mathematics education, conception of mathematics teaching*

1. INTRODUCTION

In the last few decades we have in Russia (and USSR) sufficiently large amount of conceptions (of development) of mathematical education. All these conceptions were written in so far as there exists unique understanding of the words "mathematics", "education", "teaching", etc. Our first goal here is to show that it is not right. So, in order to somewhat "conception" of mathematics education may be named conception one must define concepts (the mentioned words).

In this section we only show that the words mentioned above may have different meaning and thereby show the necessity of what we call "conception of mathematics teaching".

1.1. Let us begin with the question: "What is education?"

Not so long ago in the USSR and Russia we have approximately the following answer to this question:

"Education is socially organized and regulated process of continuous transference of socially significant experience from previous to following generations. The main way to receive education is to take a course of training in the system of educational institutions." (Леднёв 1991)

According to this definition education was the process of transferring and receiving. And what one can see in the most classrooms were in excellent accordance with this formulation. "Who is not ready to the lesson? Who did not prepare their homework?" Visible on the faces of the students is a mixture of astonishment and sympathy to stupid teacher. What is he saying? What homework? We came here to receive a transfer. Have you transfer — give it us. No — leave us alone.

It is very doubtful that we would like to continue this story. So, one must find another definition of education. Fortunately, there exists one, and we remember it in the sequel.

1.2. Now, turn to another question: "What is mathematics?" The following passage from the book on preparation of pupils to the Final State Examination in Russia (Ященко & Семёнов & Захаров 2009) will help us to see the need of precise answer to this question. The authors wrote (can you believe?):

"Uncertainty and lack of knowledge of the subject can be compensated by the knowledge of special tricks and techniques <...>

The first part [of exam] is obligatory for all. ... Namely this part is most difficult for students because the form and content of this part of the exam is very different from what was taught in the school for 9 years."

After reading this text can we not ask: WHAT IS MATHEMATICS? Is it "what was taught in the school for 9 years"? Or it is very different from that collection of puzzles of the so called "Final State Examination"?

Now we see: we must have generally acknowledged extremely clear and precise answers to the questions:

WHAT IS EDUCATION?

WHAT IS MATHEMATICS?

WHAT DOES IT MEAN TO TEACH MATHEMATICS?

WHAT FOR WE NEED TO TEACH MATHEMATICS IN GENERAL SCHOOL?

After formulation of these questions let us turn to the main task of this article: formulation and examination of our conception of mathematics teaching. We begin with formulation of the conception.

THE CONCEPTION OF HUMANITARIANLY ORIENTED TEACHING OF MATHEMATICS

1. *Mathematics is proof.*
2. *To teach mathematics means systematically encourage students to discovery their own proofs.*
3. *Teaching mathematics is indispensible mean of forming of homo cultural: thinking, moral, free.*

Then we proceed to examine this conception. We begin with

2. HEGEL'S CONCEPTION OF EDUCATION

Hegel examines the notion of education in his work "Philosophical Propaedeutic". It is quite interesting that this examination he includes in the part of his work named "Duties of the Individual to Himself". Just so! Not a transfer to be received but a duty, and not merely a duty but the duty of the individual to himself. Let us cite.

"Duties of the Individual to Himself

Man, as an individual, stands in relation to himself. He has two aspects: his individuality and his universal essence. His Duty to Himself consists partly in his duty to care for his physical preservation, partly in his duty to educate himself, to elevate his being as an individual into conformity with his universal nature."

Let us peak out:

1. not **to receive** education**,** but **to educate himself;**
2. to educate himself **is its duty;**
3. this is duty of individual **to himself;**
4. **to educate** himself means **to elevate,** not to receive**;**
5. to elevate **into conformity with his universal nature.**

Compare this goal, to elevate human's being as an individual into conformity with his universal nature, with another goal, all-rounded development of personality, which was considered as the main goal of general education in the USSR and Russia. One could think that these two goals are identical. But those who are acquainted with set theory easily will see the difference. From the set theoretical point of view, all-roundedness corresponds to the operation of union while universal human nature (all-humanness) — to that of intersection. And Hegel is talking about intersection, not about union.

General education must care about qualities common to all people. Isn't it clear? We must live in community, so the main our care must be about what is common to all member of community. Our individual differences are our own business rather than that of general education.

In the literature were already mentioned defects of all-rounded development as a goal of general education (Лазарев 2000): it is non-historic, non-real (it is not real to develop **all** sides of personality particularly because not all of them are known) and non-verifiable (it is impossible to verify whether **all** sides of personality were developed or not). But we can add to this list of defects one more defect which, in our opinion, is more significant than all mentioned:

This goal is dangerous, both to individual and to community.

Remember: a couple of years ago almost every week we learned about new discoveries on the human gene map. We learned then that there exists the gen of alcoholism, the gen of cruelty, the gen of suicide … And this means that there exist peoples having genetic predisposition to alcoholism, to cruelty, to suicide … Can we, after learning all of that, proclaim as before all-rounded development of personality the main goal of general education? No! Quite the reverse, we must hardly think which sides of individuality is worth to develop and which is not.

Thus we see that Hegel was much more penetrating and more far-seeing than we are. Abilities, he said, one must elevate rather than develop, and elevate not in the direction of our please but in the direction of all-human nature.

Now the question arises naturally: what is this all-human nature? There exist different answers to this question ("Several have defined man as "an animal which laughs" (Bergson 2009)) but most of thinkers believe that this is reasonableness (the presence of reason, of mind, of intellect, ability to think, rationality). According to this point of view (and to Hegel)

education is elevation to the reasonableness,

or, as Hegel himself wrote,

making man's rational side become his guiding principle.

We cite (Hegel, ibid):

"*Explanatory*: Man is, on the one hand, a natural being. As such he behaves according to caprice and accident as an inconstant, subjective being. He does not distinguish the essential from the unessential. Secondly, he is a spiritual, rational being and as such he is not by nature what he ought to be. The animal stands in no need of education, for it is by nature what it ought to be. It is only a natural being. But man has the task of bringing into harmony his two sides, of making his individuality conform to his rational side or of making the latter become his guiding principle."

Yet another thought must be underlined in this extremely thoughtful text. Namely, that peculiarities of the individual are measured while comparing them with the Whole. And standard for this measurement is Vocation.

"The Vocation is something universal and necessary, and constitutes a side of the social life of humanity. It is, therefore, one of the divisions of human labor. When a man has a Vocation, he enters into cooperation and participation with the Whole. Through this he becomes objective. The Vocation is a particular, limited sphere, yet it constitutes a necessary part of the whole, and, besides this, is in-itself a whole. If a man is to become something he must know how to limit himself, that is, make some specialty his Vocation. Then his work ceases to be an irksome restraint to him. He then comes to be at unity with himself, with his externality, with his sphere. He is a universal, a whole."

Let us underline one more thought of Hegel:

If a man is to become something he must know how to limit himself.

Turn attention: not to emancipate, but **to limit**!

Because this is written while considering the notion of education, it shows that the notion of education is in close connection with the two other sides of human community: that of culture and that of human's freedom.

Now we turn to consideration of these two phenomena (in their connection with that of education, naturally).

3. SCHWEITZER'S CONCEPTION OF CULTURE

The first and, in our opinion, most deep investigation of the notion of culture was made by Albert Schweitzer in his remarkable work "*Kulturphilosophie*" (in two volumes, Bern, Paul Haupt, 1923). (Schweitzer 1923) is English translation of the 1-st volume.

In the following citations one must take into account that Schweitzer himself in his 1923 German edition speaks about 'Kultur' while his English translator uses words "civilization" and "progress".

Now, according to Schweitzer:

"Civilization (read: culture) is … twofold in its nature: it realizes itself in the supremacy of reason, first, over the forces of nature, and, secondly, over the dispositions of men."

Which of these two addendums of civilization (culture) is more significant? The latter, though it is the least open to observation. Why? Because the supremacy of reason over the forces of nature can lead mankind to the situation in which men and whole nations will have abilities to destroy each other, and then only supremacy of reason over dispositions of men will give to mankind a chance to prevent the catastrophe.

This latter and most significant addendum of culture is naturally to call spiritual culture (while the former — material culture). So, after Schweitzer,

spiritual culture is the supremacy of the reason over the disposition of men.

Noting that

the supremacy of reason over the disposition of men

is identical with Hegel's

making man's rational side become his guiding principle

(what Hegel called education) we can say:

education is the formation of the spiritual culture of personality,
that is, the supremacy of reason over the disposition of men.

4. SOCRATES' CONCEPTION OF HUMAN FREEDOM

Now consider the role of education in the solution one more problem of importance the problem of human's freedom.

Many people do not see here any problem. They merely think that freedom consists in the absence of limitations. But already Socrates calls the man fulfilling all his desires not free but slave — slave of his wishes. Actually the absence of limitations is by no means freedom but its ugly opposite. Genuine freedom is inconceivable without limitations. Every task requires applying of concentrated efforts in the necessary direction and therefore cutting off all that prevents to applying of these efforts. About this excellently says Austrian writer Ilse Aichinger in his remarkable story "Bound man" (in the essence, repeating in art form cited earlier words of Hegel about Vocation):

"… flying is possible only for those who keep himself in a check. … bonds rescue from the fatal advantage of free limbs which causes men to be worsted." (Aichinger 2014)

Serious reflections about freedom frequently are substituted by emotions. “Why God did not give us the freedom to fly as birds?” But is it real limitation of human’s freedom? No! Man is not bird, he needs human’s freedom, not that of bird.

It follows one must distinguish the freedom of bird, the freedom of cattle, the freedom of man. So, what does it mean to be the *free **man***?

To find answer to this question let us first think about bird. It is clear that bird’s freedom consists in the living without limitations according to bird’s nature. Analogously, cattle’s freedom consists in living without limitations accordingly cattle’s nature (manifestations of this cattle’s freedom are every day demonstrated on tv screen). And precisely in that manner human’s freedom consists in living without limitations according to human’s nature. This nature, as we already seen, consists in spiritual culture, that is, in the supremacy of reason over the disposition of men. So,

human freedom consists in ability to be guided, without limitations, by reason in his beliefs and intensions.

Remembering what is said earlier about education, we see that main goal of education may also be expressed as follows:

*to educate means to teach human being to be free **man***.

Take into account: human beings **must learn** to human’s freedom and they can learn it in only one way — by developing in themselves the ability to subordinate their beliefs and intensions to the voice of reason.

5. HUMANITARIAN EDUCATION = EDUCATION

English “humanitarian” has its origin in Latinas “humanitas” that means “human nature”. And because education always means *human’s* education, it must be built in accordance with human nature, that is, to be humanitarian. Of course, there may be schools and classes of biological, chemical, mathematical and other profiles, but teaching in all of them must be performed in accordance with human nature, that is, to be humanitarian. If one wants:

Humanitarian education is merely education,
that is,
elevation to the spiritual culture,
that is,
elevation to the supremacy of the reason over human’s convictions, intensions and believes,
that is,
making man’s rational side become his guiding principle.

6. WHAT IS MATHEMATICS?

6.1. *Mathematics from the point of view of matter*

From the point of view of matter

“mathematics is the science of quantitative relations and spatial forms in the real world.

Quantitative relations (in general philosophical sense of this term) are characterized, contrary to qualitative, only their indifference to the specific nature of those objects, they relate.

… spatial forms one may consider as a particular case of quantitative relations.

… mathematics learns only those relations indifferent to concrete nature of the objects they relates

… in the indicated general sense all relations learned by mathematics always are quantitative.” (Kolmogorov 1954; translation in English is mine. — A. N.)

This leads us to the following remarkable definition of mathematics known as earlier as VI century AD (Kolmogorov did not mention it):

"Mathematics is the science of abstract quantity. Abstract quantity is what we learn purely speculatively distracting it in mind from the substance and random manifestations". (Cassiodorus, VI century AD; translation in English is mine. — A. N.)

6.2. *Mathematics from the point of view of method*

Abstract character of mathematics predetermines the special role of proof in it. Because of all objects of mathematics are abstract, all truths about them can be established only mentally, with the help of special mental constructions named reasoning. Reasoning which convinces in truthfulness of proposition is called proof of proposition. Mathematics is the only science in which truth may be established only by proof. This leads us to the definition of mathematics from the point of view of method.

MATHEMATICS IS PROOF.

"Since the time of the Greeks to say "mathematics" means to say "proof".

(Bourbaki)

"…mathematics is coextensive with demonstrative reasoning, which pervades the sciences just as far as their concepts are raised to a sufficiently abstract and definite, mathematico-logical level".

(Polya 1981)

6.3. *Why proofs inside mathematics?*

In fact, why proofs? Why not allow to pupils to believe the teacher? — The point is that the detachment of proposition is not the only intention of proof. It has at least two other intentions. Namely, proof helps us to discover the truth and to understand it. Look at the following example.

Problem: Represent the set of all points of coordinate plain *Oxy*, for which

$$\frac{1}{x} < \frac{1}{y}.$$

Solution: For all x and y in **R** we have:

$$\frac{1}{x} < \frac{1}{y} \leftrightarrow \frac{1}{x} - \frac{1}{y} < 0 \leftrightarrow \frac{y-x}{xy} < 0 \leftrightarrow \begin{cases} y-x<0, \\ xy>0; \end{cases} or \begin{cases} y-x>0, \\ xy<0; \end{cases} \leftrightarrow \begin{cases} y<x, \\ xy>0; \end{cases} or \begin{cases} y>x, \\ xy<0. \end{cases}$$

Now we see: in order to represent the given set, we need to take all points (x, y) with $y < x$ from the domain where $xy > 0$ and add to them points (x, y) with $y > x$ from the domain where $xy < 0$. So, we understand what is given set. How it has happened? We learn it from the text written above. What is this text? It is the proof. Proof of what it is? It is the proof of the assertion that the first inequality and the last disjunction are equivalent. Is it true? Of course, this text is proof!

Let us summarize: with the help of the proof we discover that the first inequality in it is equivalent to the last disjunction, we confirm our discovery and understand what is given set.

Let us peak out.

Three intensions of proof in mathematics:

- *to help to discover*,
- *to help to understand the discovery* and
- *to confirm the discovery*.

7. WHAT DOES IT MEAN TO TEACH MATHEMATCS?

Remember Herbert Spencer (Polya 1981):

"What does it mean to teach? — This means to impel systematically the students to their own discoveries."

And about mathematics we said that it is the proof. Combining these two formulations we get second proposition in our conception:

to teach mathematics means to impel systematically the students to discovery their own proofs.

8. WHAT FOR IS MATHEMATICS IN GENERAL EDUCATION?

8.1. Remember what tell us Hegel and Schweitzer:

education consists in making man's rational side the guiding principle,
or in elevation to the supremacy of the reason over human's convictions, intensions and believes.

Now ask: what mathematics can give to education? To understand this, the following example will be helpful. John Aubrey (1626-1697) wrote about Thomas Hobbes:

"He was 40 years old before he looked on geometry; which happened accidentally. Being in a gentleman's library, Euclid's Elements lay open, and "twas the 47 El. libri I" [Pythagoras' Theorem]. He read the proposition. "By God", said he, "this is impossible". So he reads the demonstration of it, which referred him back to such a proposition; which proposition he read. That referred him back to another, which he also read. Et sic deinceps, that at last he was demonstratively convinced of that truth. This made him in love with geometry" (Dick 1960).

What has happened with Hobbes in this episode? At the beginning he has one conviction but under influence of the proof his conviction became another. The proof forced him to subordinate his conviction to the voice of reason. Such influence has every proof that we discover or understand. This shows that mathematics, as an educational discipline, is, grinded by centuries, system of excellently selected exercises accustoming the men to supremacy of his reason over his convictions, beliefs and intensions.

8.2. "All thinking and all actions are influenced by conclusions that have been consciously or unconsciously drawn. This fact is fundamental; if it is not habitually well done, whatever edifice of culture may be built is warped and insecure. The conclusions which must be made by the score every day are of complicated nature; the facts are so many, so elaborate, so imperfectly known, that it is often very difficult to draw any conclusion, much less to be sure that we have the right one.

It is justly demanded that the school familiarize children with a mode of thought so ubiquitous, so important and so difficult. A subject suitable for this purpose should have three characteristics:

1. That its conclusions are certain. At first, at least, it is essential that the learner should know whether or not he has drawn the correct conclusion.
2. That it permit the learner to begin with simple and very easy conclusions, and to pass in well graded sequence to very difficult ones, as the earlier ones are mastered.
3. That the type of conclusions exemplified in the introductory subject be found in other subjects also, and in human intercourse in general.

These desiderata are possessed by mathematics in a far larger measure than by any other available subject."

And further:

"Are you to be a lawyer? How can you learn to analyze a complicated legal case, if you cannot learn to analyze a simple proposition of geometry?

Are you a student of history? How can you determine the influence of Napoleon on the world's development, if you are incapable of determining the influence of a coefficient in a simple relation of algebra?

Are you a linguist? How will you translate a masterpiece, with its myriad shades of meaning, from one language into another, if you cannot learn to translate a trifling "reading problem" into the corresponding mathematical symbols?

Are you to be a physician? How will you diagnose and eliminate a disease, with its complicated, ambiguous, and obscure symptoms, if you lack the faculties needed to diagnose and eliminate the unknown quantity out of an elementary equation?" (Young 1920)

Conclusion: in mathematics, in pure form and in the simplest situations, are worked out such thinking skills that will be necessary to every man on every field of activity.

At last let us display

9. WHY WE CALL OUR CONCEPTION HUMANITARIANLY ORIENTED

Return to the example with John Aubrey and Tomas Hobbes. The influence we see in that example is truly humanitarian in character. Moreover, there are no other educational disciplines which can be compared with mathematics in this relation. So,

mathematics is the only truly humanitarian educational discipline.

It was clear to all until the end of nineteenth century when has happened what Charles Percy Snow called tragic split of culture. But this is another story.

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APPENDIX

Let me add the following remarkable words of Albert Schweitzer:

"Our age has a kind of artistic prejudice against a reflective theory of the universe. We are still children of the Romantic movement to a greater extent than we realize. What that movement produced in opposition to the Aufklärung and to rationalism seems to us valid for all ages against any theory that would found itself solely on thought. In such a theory of the universe we can see beforehand the world dominated by a barren intellectualism, convictions governed by mere utility, and a shallow optimism, which together throw a wet blanket over all human genius and enthusiasm.

In a great deal of the opposition which it offered to rationalism the reaction of the early nineteenth century was right. Nevertheless it remains true that it despised and distorted what was, in spite of all its imperfections, the greatest and most valuable manifestation of the spiritual life of man that the world has yet seen. Down through all circles of cultured and uncultured alike there prevailed at that time a belief in thought and a reverence for truth. For that reason alone that age stands higher than any which preceded it, and much higher than our own.

At no price must the feelings and phrases of Romanticism be allowed to prevent our generation from forming a clear conception of what reason really is. It is no dry intellectualism which would suppress all the manifold movements of our inner life, but the totality of all the functions of our spirit in their living action and interaction. In it our intellect and our will hold that mysterious intercourse which determines the character of our spiritual being. These fundamental ideas which it produces contain all that we can feel or imagine about our destiny and that of mankind, and give our whole being its direction and its value. <…>

*"**Rationalism** is more than a movement of thought which realized itself at the end of the eighteenth and the beginning of the nineteenth centuries. It **is a necessary phenomenon in all normal spiritual life.***

***All real progress in the world is in the last analysis produced by rationalism.**"* (Schweitzer 1923)

And in another place (Schweitzer 1998):

"WITH ITS DISDAIN FOR THINKING, OUR GENERATION HAS LOST ITS FEELING FOR SINCERITY. IT CAN THEREFORE BE HELPED ONLY BY REVIVING THE VOICE OF THOUGHT".

This was written in 1923 (1998 is the year of reprint); it seems that it is written today!