

NAVIER'S APPROACH FOR BENDING ANALYSIS OF FUNCTIONALLY GRADED SQUARE PLATES

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Abstract

In last decades functionally graded materials become very popular for lots of industries such as automotive, naval, railroad, aerospace, etc. There are many papers in literature for bending, stability and vibration analysis of functionally graded plates. In this paper, bending of simply supported functionally graded square plates are studied. For functionally graded plates many shear deformation theory is offered. Various shape functions for defining displacement fields are used in these theories. In this study, several shape functions are discussed for deflection and stress distribution of functionally graded plate with sinusoidal loading. The exponential gradient form is assumed for change of material properties through thickness direction. Refined plate theory with different shape functions is used. Governing equations are derived from the principle of virtual displacements. The solution is obtained by Navier's double trigonometric series approach. Numerical results of deflection, normal stress and shear stress are presented for thin and thick square plates.

Key words: functionally graded plates, naviers solution, bending, stress, shear strain

1. INTRODUCTION

In the last 50-60 years, using composite structures in engineering applications has increased due to this fact many studies have been related to composite structures such as: shells, plates and beams (Aydogdu et al. 2011, Demirhan et al. 2011, Demirhan & Taskin 2015). The Classical Plate Theory (CPT) is simplest and useful model for thin plates, but it's not accurate for the thick plates because of the neglecting shear and normal deformation effects. To overcome the deficiencies of the CPT, many shear deformation theories accounting for transverse shear effects have been developed (Mechab et al. 2010, Kim & Reddy 2013, Li, Wang & Han 2010, Nguyen et al. 2014, Nie, Zhong & Chen 2013, Reddy & Wang 2000). The First-order Shear Deformation Theory (FSDT) accounts transverse shear strains as constant through the plate thickness but it requires shear correction coefficients (Reddy 2000). The Third-order Shear Deformation Theory (TSDT) developed by Reddy (2000) accounts for the transverse shear deformation effect and satisfies the zero-traction boundary conditions on the top and bottom surfaces of a plate (Kim & Thai 2009). A refined shear deformation theory for isotropic plates is developed by Shimpi (2002). In this theory displacement component in thickness direction spares two parts which are bending and shear components of transverse displacement. The theory is used for orthotropic plates by Shimpi & Patel (2006), also used for laminated composite plates by Kim, Thai & Lee (2009). Bending analysis of functionally graded plates and sandwich plates using refined shear deformation theory are presented by Mechab et al. (2010) and Abdellaziz et al. (2011) respectively.

In this paper bending and stress analysis of functionally graded square plates are studied. Various shape functions for displacement fields are compared. Numerical results are presented for thin to thick plates, also deflection, normal and shear stress are illustrated in graphs with varying volume fraction exponent.

2. METHOD

In the study, a functionally graded plate with length of a , width of b and thickness of h is considered. As a special case for squared plate b is equal to a . In Fig. 1 a functionally graded plate is seen. Upper side of plate is metal and lower side is ceramic. Elasticity modulus of plate is varied through thickness direction with a function.

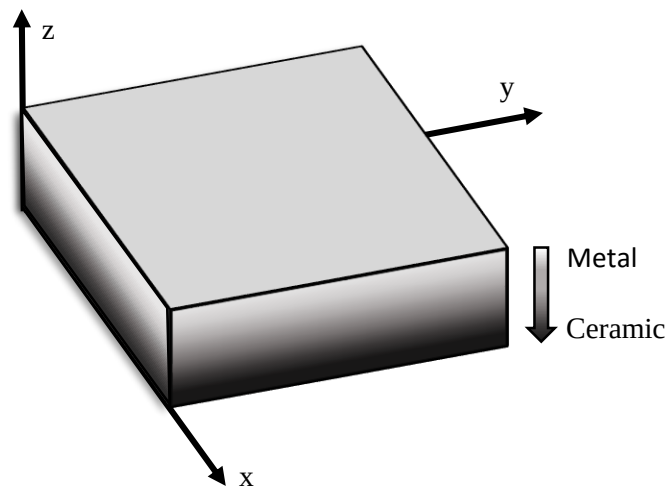


Fig. 1. Functionally graded plate

Elasticity modulus of functionally graded plate is described in Eq. (1). E_c is the modulus of ceramic surface and E_m is the modulus of metal surface. p is the volume fraction coefficient and to be equal zero or above. Changing elasticity modulus with different p value is shown in Fig.2. Poisson ratio is assumed to be constant through thickness.

$$E(z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^p \quad (1)$$

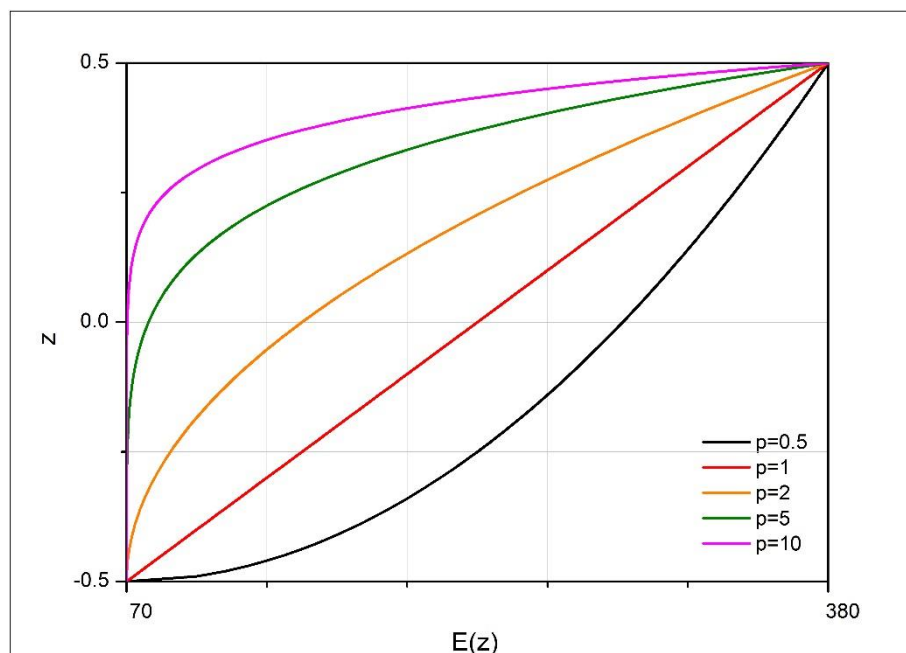


Fig. 2. Elasticity modulus function with different p values ($E_c=380\text{GPa}$, $E_m=70\text{GPa}$)

Displacement components u , v and w along the x , y and z directions, respectively, the following displacement field of the high order deformation theory can be written:

$$u = -z \frac{\partial w_b}{\partial x} + f(z) \frac{\partial w_s}{\partial x} \quad (2a)$$

$$v = -z \frac{\partial w_b}{\partial y} + f(z) \frac{\partial w_s}{\partial y} \quad (2b)$$

$$w = w_b + w_s \quad (2c)$$

Strain displacement relation is shown in Eq. (3)

$$\varepsilon_x = \frac{\partial u}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} + f(z) \frac{\partial^2 w_s}{\partial x^2} \quad (3a)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} - z \frac{\partial^2 w_b}{\partial y^2} + f(z) \frac{\partial^2 w_s}{\partial y^2} \quad (3b)$$

$$\varepsilon_z = 0 \quad (3c)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z \frac{\partial^2 w_b}{\partial x \partial y} + 2f(z) \frac{\partial^2 w_s}{\partial x \partial y} \quad (3d)$$

$$\gamma_{yz} = g(z) \frac{\partial w_s}{\partial y} \quad (3e)$$

$$\gamma_{xz} = g(z) \frac{\partial w_s}{\partial x} \quad (3f)$$

It can be derived constitutive relations for isotropic functionally graded materials as follows.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (5)$$

$$Q_{11} = Q_{22} = \frac{E(z)}{1 - \nu^2} \quad Q_{12} = \frac{\nu E(z)}{1 - \nu^2} \quad Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1 + \nu)} \quad (6)$$

Governing equations are derived using virtual displacement principles.

$$\delta(U - W) = 0 \quad (7)$$

$$W = \int_{\Omega} qW d\Omega \quad (8)$$

$$U = \int_{-h/2}^{h/2} \int_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} + \tau_{yz} \gamma_{yz} + \tau_{xz} \gamma_{xz}) d\Omega dz \quad (9)$$

$$\delta W = \int_{\Omega} q \delta W d\Omega = \int_{\Omega} q (\delta w_b + \delta w_s) d\Omega \quad (10)$$

$$\begin{aligned} \delta U = \int_{\Omega} & \left[N_x \delta \left(\frac{\partial u}{\partial x} \right) + N_y \delta \left(\frac{\partial v}{\partial y} \right) + N_{xy} \delta \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - M_x^b \delta \left(\frac{\partial^2 w_b}{\partial x^2} \right) - M_y^b \delta \left(\frac{\partial^2 w_b}{\partial y^2} \right) \right. \\ & - M_{xy}^b \delta \left(\frac{\partial^2 w_b}{\partial x \partial y} \right) - M_x^s \delta \left(\frac{\partial^2 w_s}{\partial x^2} \right) - M_y^s \delta \left(\frac{\partial^2 w_s}{\partial y^2} \right) - M_{xy}^s \delta \left(\frac{\partial^2 w_s}{\partial x \partial y} \right) + S_{yz}^s \delta \left(\frac{\partial w_s}{\partial y} \right) \\ & \left. + S_{xz}^s \delta \left(\frac{\partial w_s}{\partial x} \right) \right] d\Omega \end{aligned} \quad (11)$$

$$\begin{aligned} \delta(U - W) = \int_{\Omega} & \left[N_x \delta \left(\frac{\partial u}{\partial x} \right) + N_y \delta \left(\frac{\partial v}{\partial y} \right) + N_{xy} \delta \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - M_x^b \delta \left(\frac{\partial^2 w_b}{\partial x^2} \right) - M_y^b \delta \left(\frac{\partial^2 w_b}{\partial y^2} \right) \right. \\ & - M_{xy}^b \delta \left(\frac{\partial^2 w_b}{\partial x \partial y} \right) - M_x^s \delta \left(\frac{\partial^2 w_s}{\partial x^2} \right) - M_y^s \delta \left(\frac{\partial^2 w_s}{\partial y^2} \right) - M_{xy}^s \delta \left(\frac{\partial^2 w_s}{\partial x \partial y} \right) + S_{yz}^s \delta \left(\frac{\partial w_s}{\partial y} \right) \\ & \left. + S_{xz}^s \delta \left(\frac{\partial w_s}{\partial x} \right) \right] d\Omega - \int_{\Omega} q (\delta w_b + \delta w_s) d\Omega = 0 \end{aligned} \quad (12)$$

Force and moment resultants can be derived as follows.

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz, \quad N_y = \int_{-h/2}^{h/2} \sigma_y dz, \quad N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz \quad (13)$$

$$M_x^b = \int_{-h/2}^{h/2} \sigma_x z dz, \quad M_y^b = \int_{-h/2}^{h/2} \sigma_y z dz, \quad M_{xy}^b = \int_{-h/2}^{h/2} \tau_{xy} z dz \quad (14)$$

$$M_x^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x \left[-\frac{1}{4}z + \frac{5}{3}z \left(\frac{z}{h} \right)^2 \right] dz, \quad M_y^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y \left[-\frac{1}{4}z + \frac{5}{3}z \left(\frac{z}{h} \right)^2 \right] dz \quad (15)$$

$$M_{xy}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} \left[-\frac{1}{4}z + \frac{5}{3}z \left(\frac{z}{h} \right)^2 \right] dz \quad (16)$$

$$S_{xz}^s = \int_{-h/2}^{h/2} \tau_{xz} g(z) dz, \quad S_{yz}^s = \int_{-h/2}^{h/2} \tau_{yz} g(z) dz \quad (17)$$

Rigidity matrix is derived in Eq. (18)

$$\begin{Bmatrix} N_x \\ N_y \\ M_x^b \\ M_y^b \\ M_x^s \\ M_y^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{11}^s & B_{12}^s \\ A_{12} & A_{22} & B_{12} & B_{22} & B_{12}^s & B_{22}^s \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{11}^s & D_{12}^s \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{12}^s & D_{22}^s \\ B_{11}^s & B_{12}^s & D_{11}^s & D_{12}^s & H_{11}^s & H_{12}^s \\ B_{12}^s & B_{22}^s & D_{12}^s & D_{22}^s & H_{12}^s & H_{22}^s \end{bmatrix} \begin{Bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ -\partial^2 w_b / \partial x^2 \\ -\partial^2 w_b / \partial y^2 \\ -\partial^2 w_s / \partial x^2 \\ -\partial^2 w_s / \partial y^2 \end{Bmatrix} \quad (18)$$

Rigidity matrix coefficients can be derived as shown in Eq. (19)

$$\{A_{ij}, A_{ij}^s, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{1, g, z, f, z^2, fz, f^2\} Q_{ij} dz \quad (i, j = 1, 2, 4, 5, 6) \quad (19)$$

Equilibrium equations for plate;

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (20)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \quad (21)$$

$$\frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \quad (22)$$

$$\frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial N_{xz}^s}{\partial x} + \frac{\partial N_{yz}^s}{\partial y} + q = 0 \quad (23)$$

Substituting force and moment equations (Eq.18) into equilibrium equations (Eq. 20-23), differential equations of plate are obtained.

$$\begin{aligned} A_{11} \frac{\partial^2 u}{\partial x^2} + A_{66} \frac{\partial^2 u}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v}{\partial x \partial y} - B_{11} \frac{\partial^3 w_b}{\partial x^3} - (B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \partial y^2} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s \\ + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x \partial y^2} = 0 \end{aligned} \quad (24)$$

$$(A_{12} + A_{66}) \frac{\partial^2 u}{\partial x \partial y} + A_{66} \frac{\partial^2 v}{\partial x^2} + A_{22} \frac{\partial^2 v}{\partial y^2} - (B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x^2 \partial y} - B_{22} \frac{\partial^3 w_b}{\partial y^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x^2 \partial y} - B_{22}^s \frac{\partial^3 w_s}{\partial y^3} = 0 \quad (25)$$

$$B_{11} \frac{\partial^3 u}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u}{\partial x \partial y^2} + (B_{12} + 2B_{66}) \frac{\partial^3 v}{\partial x^2 \partial y} + B_{22} \frac{\partial^3 v}{\partial y^3} - D_{11} \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w_b}{\partial y^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} - D_{22}^s \frac{\partial^4 w_s}{\partial y^4} + q = 0 \quad (26)$$

$$B_{11}^s \frac{\partial^3 w_s}{\partial x^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 u}{\partial x \partial y^2} + B_{22}^s \frac{\partial^3 v}{\partial y^3} - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} - D_{22}^s \frac{\partial^4 w_b}{\partial y^4} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 u}{\partial x^2 \partial y} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} - H_{22}^s \frac{\partial^4 w_s}{\partial y^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - 2(H_{12}^s + 2H_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} + A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + A_{44}^s \frac{\partial^2 w_s}{\partial y^2} + q = 0 \quad (27)$$

Simply supported boundary conditions for the refined theory are;

$$v(0, y) = w_b(0, y) = w_s(0, y) = \frac{\partial w_b}{\partial y}(0, y) = \frac{\partial w_s}{\partial y}(0, y) = 0 \quad (28)$$

$$v(a, y) = w_b(a, y) = w_s(a, y) = \frac{\partial w_b}{\partial y}(a, y) = \frac{\partial w_s}{\partial y}(a, y) = 0 \quad (29)$$

$$N_x(0, y) = M_x^b(0, y) = M_x^s(0, y) = N_x(a, y) = M_x^b(a, y) = M_x^s(a, y) = 0 \quad (30)$$

$$u(x, 0) = w_b(x, 0) = w_s(x, 0) = \frac{\partial w_b}{\partial x}(x, 0) = \frac{\partial w_s}{\partial x}(x, 0) = 0 \quad (31)$$

$$u(x, b) = w_b(x, b) = w_s(x, b) = \frac{\partial w_b}{\partial x}(x, b) = \frac{\partial w_s}{\partial x}(x, b) = 0 \quad (32)$$

$$N_y(x, 0) = M_y^b(x, 0) = M_y^s(x, 0) = N_y(x, b) = M_y^b(x, b) = M_y^s(x, b) = 0 \quad (33)$$

The Navier solution procedure is used to determine the analytical solutions for a simply supported plate. External force is defined with double trigonometric series as Navier's method (Eq. 34).

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin(\alpha x) \sin(\beta y) \quad m = n = 1, \quad q_{11} = q_0 \quad (34)$$

Displacement components are defined as in Eq.(35)

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\alpha x) \sin(\beta y) \\ V_{mn} \sin(\alpha x) \cos(\beta y) \\ W_{bmn} \sin(\alpha x) \sin(\beta y) \\ W_{smn} \sin(\alpha x) \sin(\beta y) \end{Bmatrix} \quad (35)$$

U_{mn} , V_{mn} , W_{bmn} , W_{smn} unknown constants are found with solving the Eq. 36. K is the coefficient matrix, F is the external forces vector.

$$K\Delta = F \quad (36)$$

$$\Delta^T = \{U_{mn}, \quad V_{mn}, \quad W_{bmn}, \quad W_{smn}\} \quad (37)$$

$$F^T = \{0, \quad 0, \quad -q_{mn}, \quad -q_{mn}\} \quad (38)$$

$$K = \begin{bmatrix} A_{11}\alpha^2 + A_{66}\beta^2 & \alpha\beta(A_{12} + A_{66}) & A_{66}\alpha^2 + A_{22}\beta^2 & \\ -\alpha[B_{11}\alpha^2 + (B_{12} + 2B_{66})\beta^2] & -\beta[(B_{12} + 2B_{66})\alpha^2 + B_{22}\beta^2] & -\beta[(B_{12}^s + 2B_{66}^s)\alpha^2 + B_{22}^s\beta^2] & \\ -\alpha[B_{11}^s\alpha^2 + (B_{12}^s + 2B_{66}^s)\beta^2] & -\alpha[B_{11}^s\alpha^2 + (B_{12}^s + 2B_{66}^s)\beta^2] & -\beta[(B_{12}^s + 2B_{66}^s)\alpha^2 + B_{22}^s\beta^2] & \\ D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 & D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4 & H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2 & \end{bmatrix} \quad (39)$$

3. NUMERICAL RESULTS

Some representative results of Navier solutions obtained for simply supported square functionally graded plate under sinusoidally distributed load. Following material properties are used;

Ceramic (Al_2O_3) $E_c = 380 \text{ GPa}$, $\nu = 0.3$

Metal (Al) $E_m = 70 \text{ GPa}$, $\nu = 0.3$

Dimensionless displacement and stress parameters used are;

$$\bar{W} = \frac{10hE_0}{q_0a^2} W\left(\frac{a}{2}, \frac{b}{2}\right) \quad \bar{\sigma}_x = \frac{10h^2}{q_0a^2} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right) \quad \bar{\tau}_{xz} = \frac{h}{q_0a} \tau_{xz}\left(0, \frac{b}{2}, 0\right)$$

Table 1 shows that nondimensional deflection results which are computed by using refined plate theory with different shape functions. All results are computed. First shape function of Table1-3 (Polynomial shape function 1) is used by Mechab (2010). Some dimensionless deflection values are obtained by using different shape functions. With comparing the results it is seen that it diverges the reference value with rising plate thickness. Convergent results are obtained by using trigonometric shape functions.

In Fig.3 transverse displacement through x axis for different p value by using exponential shape function are given. With bigger p value the metallic properties of functionally graded plates are rising and transverse displacement shows increment.

Table 1. Non-dimensional deflection for thin, moderate and thick plate various grading parameter

Shape functions	a/h	p=0.5	p=1	p=2	p=5	p=10
Polynomial shape function 1 $f(z) = h \left[\frac{1}{4} \left(\frac{z}{h} \right) - \frac{5}{3} \left(\frac{z}{h} \right)^3 \right]$	5	0,5176	0,6687	0,867	1,0882	1,2271
	10	0,4537	0,5890	0,7573	0,9112	1,0085
	100	0,4326	0,5625	0,7209	0,8527	0,9362
Polynomial shape function 2 $f(z) = z \left[1 - \frac{4}{3} \left(\frac{z}{h} \right)^2 \right]$	5	0,5122	0,6613	0,8529	1,0518	1,184
	10	0,4525	0,5873	0,7541	0,9028	0,9986
	100	0,4325	0,5625	0,7209	0,8526	0,9361
Hyperbolic shape function $f(z) = h \sinh \left(\frac{z}{h} \right) - z \cosh \left(\frac{1}{2} \right)$	5	0,502	0,6478	0,8307	1,0067	1,1268
	10	0,4498	0,5837	0,7481	0,8908	0,9833
	100	0,4325	0,5625	0,7208	0,8525	0,9359
Trigonometric shape function $f(z) = \frac{h}{\pi} \sin \left(\frac{\pi z}{h} \right)$	5	0,5126	0,6618	0,8537	1,0535	1,1856
	10	0,4526	0,5874	0,7543	0,9032	0,999
	100	0,4325	0,5625	0,7209	0,8526	0,9361
Exponential shape function $f(z) = z - z e^{-2 \left(\frac{z}{h} \right)^2}$	5	0,4884	0,6304	0,8058	0,9657	1,0755
	10	0,4464	0,5794	0,7419	0,8806	0,9706
	100	0,4325	0,5624	0,7208	0,8524	0,9358

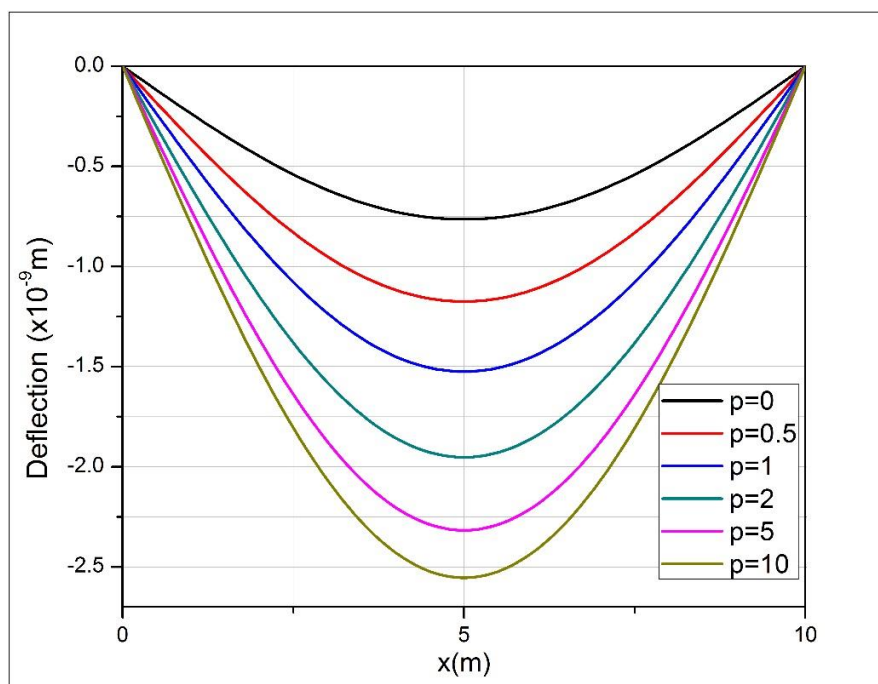


Fig. 3. Deflection of functionally graded plate (a/h=10, a/b=1)

Fig. 4 shows that normal stress distribution through plate thickness by using hyperbolic shape function for displacement fields. It is seen the figure with zero value of p the plate is isotropic and the distribution of stress is linear.

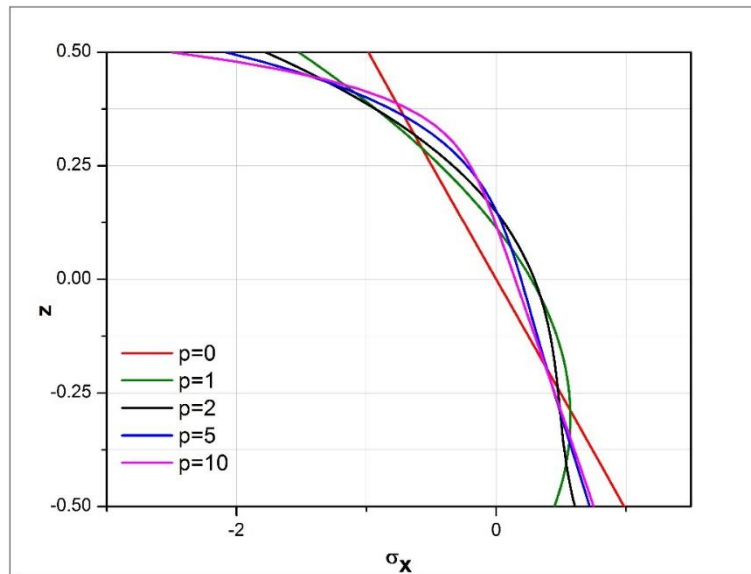


Fig. 4. Normal stress distribution ($a/h=10$, $a/b=1$)

In Fig.5. shear stress distribution through thickness is seen. Maximum shear stress value is obtained $p=2$. With increasing p maximum shear stress value is decreased.

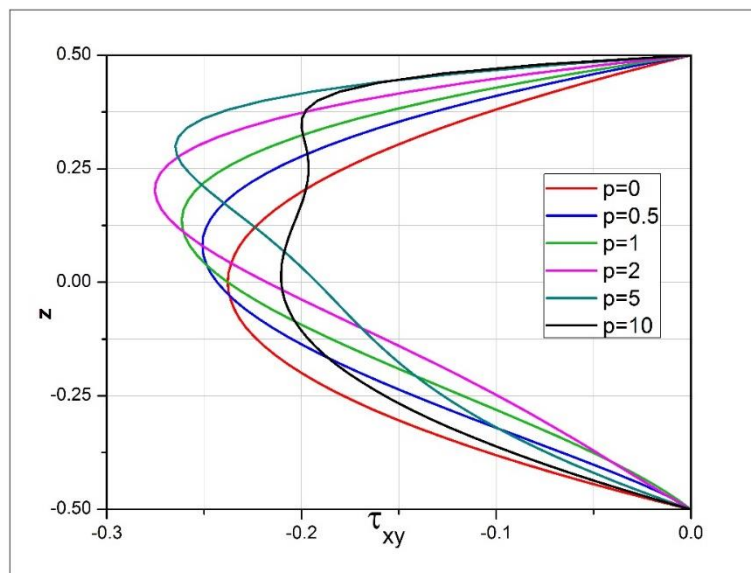


Fig. 5. Shear stress distribution ($a/h=5$, $a/b=1$, Trigonometric shape function)

In Table 2. Nondimensional normal stress value is presented. The results are convergence with reference values for thin plates. With increasing p value the difference between results and reference tend to rise. Nearest value with references is gained by using trigonometric shape function.

Table 2. Nondimensional Normal stress value

Shape functions	a/h	p=0.5	p=1	p=2	p=5	p=10
Polynomial shape function 1 $f(z) = h \left[\frac{1}{4} \left(\frac{z}{h} \right) - \frac{5}{3} \left(\frac{z}{h} \right)^3 \right]$	5	1,34737	1,58933	1,86462	2,21167	2,64322
	10	2,61815	3,085	3,60668	4,24468	5,08486
	100	25,9279	30,53976	35,6607	41,8542	50,1797
Polynomial shape function 2 $f(z) = z \left[1 - \frac{4}{3} \left(\frac{z}{h} \right)^2 \right]$	5	1,31763	1,55269	1,81542	2,13658	2,55871
	10	2,60333	3,06674	3,58217	4,20725	5,04273
	100	25,9264	30,53794	35,6582	41,8504	50,1755
Hyperbolic shape function $f(z) = h \sinh \left(\frac{z}{h} \right) - z \cosh \left(\frac{1}{2} \right)$	5	1,29149	1,52114	1,77596	2,0838	2,49898
	10	2,59014	3,05081	3,56222	4,18051	5,01246
	100	25,9251	30,53634	35,6562	41,8477	50,1725
Trigonometric shape function $f(z) = \frac{h}{\pi} \sin \left(\frac{\pi z}{h} \right)$	5	1,31872	1,554	1,81703	2,13867	2,56086
	10	2,60388	3,0674	3,58298	4,20831	5,04381
	100	25,9265	30,53801	35,6583	41,8505	50,1756
Exponential shape function $f(z) = z - z e^{-2 \left(\frac{z}{h} \right)^2}$	5	1,27329	1,49984	1,75171	2,05605	2,46823
	10	2,58102	3,04013	3,55005	4,16658	4,99702
	100	25,9242	30,53527	35,65498	41,84634	50,17093

In Table 3. Nondimensional shear stress values are given. With increasing plate thickness and increasing p value the results are divergence the reference values. In shear stress nearest value with references is obtained by using trigonometric shape function.

Table 3. Nondimensional Shear stress value

Shape functions	a/h	p=0.5	p=1	p=2	p=5	p=10
Polynomial shape function 1 $f(z) = h \left[\frac{1}{4} \left(\frac{z}{h} \right) - \frac{5}{3} \left(\frac{z}{h} \right)^3 \right]$	5	0,24331	0,23787	0,21782	0,19213	0,21041
	10	0,24393	0,23852	0,21851	0,1929	0,21128
	100	0,24413	0,23873	0,21873	0,19315	0,21156
Polynomial shape function 2 $f(z) = z \left[1 - \frac{4}{3} \left(\frac{z}{h} \right)^2 \right]$	5	0,20341	0,19684	0,1736	0,14231	0,15653
	10	0,20563	0,1991	0,17587	0,14456	0,15903
	100	0,20638	0,19986	0,17664	0,14532	0,15988
Hyperbolic shape function $f(z) = h \sinh \left(\frac{z}{h} \right) - z \cosh \left(\frac{1}{2} \right)$	5	0,15586	0,14861	0,12527	0,09491	0,10353
	10	0,15596	0,14871	0,12535	0,09498	0,10361
	100	0,15599	0,14874	0,12538	0,09501	0,10364
Trigonometric shape function $f(z) = \frac{h}{\pi} \sin \left(\frac{\pi z}{h} \right)$	5	0,20721	0,20061	0,17725	0,14576	0,16005
	10	0,20943	0,20286	0,17952	0,14801	0,16254
	100	0,21018	0,20361	0,18028	0,14877	0,16338

Exponential shape function $f(z) = z - ze^{-2(\frac{z}{h})^2}$	5	0,10346	0,09732	0,07894	0,05633	0,06108
	10	0,10371	0,09756	0,07915	0,0565	0,06127
	100	0,10379	0,09764	0,07922	0,05656	0,06134

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