BASES OF DEGRESSIVELY PROPORTIONAL DISTRIBUTIONS

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Abstract

Under the Lisbon Treaty, the distribution of mandates to the European Parliament should satisfy the condition of degressive proportionality. A proposition of such distributions, known as the Cambridge Compromise, assumes a proportional adjustment of an initial, degressively proportional, base distribution. First, a fixed base number of seats is allocated, and then the actual solution is determined by the divisor method, adding to the base a number of seats that results from the proportional allocation subject to rounding up. The paper presents some modifications of this idea as regards the choice of a base and its proportional adjustment.

Key words: *degressive proportionality, fair division, European Parliament, voting*

1. INTRODUCTION

The paper deals with a problem of degressively proportional allocation of seats in the European Parliament. Such a principle of mandates' distribution among the member states of the European Union was imposed by the Treaty of Lisbon whose article 9A stipulates that "*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats*" (The Treaty of Lisbon, 2010). In addition, the European Parliament resolution of 2007 pinpoints some statements included in the Treaty, specifically "the larger the population of a country, the greater its entitlement to a large number of seats" and "the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament" (Lamasoure A., Severin A., 2007). As a result, it is possible to formulate the principle of degressive proportionality in a precise, mathematical manner. Assume that $S = (s_1, s_2, \dots, s_n)$ is a sequence of integer, nonnegative terms denoting the numbers of allocated mandates, and the terms of a positive sequence $P = (p_1, p_2, \dots, p_n)$ denote the numbers of populations in respective member states of the European Union. Then we define that the sequence S is degressively proportional with respect to P, where $0 < p_1 \leq p_2 \leq \cdots \leq p_n$, if and only if

(1) $s_1 \leq s_2 \leq \cdots \leq s_n$

and

(2) $\frac{p_1}{s_1} \leq \frac{p_2}{s_2} \leq \cdots \leq \frac{p_n}{s_n}$.

This form of degressive proportionality is called "rounded degressive proportionality" (RDP) in the literature (González V.R. et al., 2012; Delgado-Márquez B. et al., 2013).

The Treaty of Lisbon also provides some additional conditions characterizing the allocation of seats in the European Parliament. They are so-called boundary conditions of division or numbers specifying a minimum and a maximum representation of the EU member states as well as a total size of the assembly (Łyko J., 2012). The numbers are decided as follows:

(3) $m = s_1 = 6,\ M = s_n = 96$

and

(4) $H = s_1 + s_2 + \cdots + s_n = 751.$

The Treaty deals with these numbers as non-strict inequalities, nevertheless the above mentioned resolution of the European Parliament of 2007 (Lamasoure A., Severin A., 2007) indicates that the minimum and maximum numbers set by the Treaty "must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States", which can be understood as a recommended application in the form of equation. A set of all feasible solutions, i.e. a set of all sequences $S = (s_1, s_2, \dots, s_n)$ which satisfy the conditions (1)-(4), comprises over 5 millions elements in case of the set $P = (p_1, p_2, \dots, p_n)$ of populations in 2012 that underlay the allocation of seats for the 2014-2019 term. It means that without some additional conditions, the provisions of the Treaty of Lisbon do not allow for a unique allocation of seats, thus producing a space for political disputes and negotiations.

2. CAMBRIDGE COMPROMISE

In 2011, a joint standpoint regarding the provisions of the Lisbon Treaty was attempted. The Committee on Constitutional Affairs organized a meeting at the University of Cambridge aimed at the presentation of a mathematical basis applicable to the apportionment of mandates for succeeding terms. The solution proposed at Cambridge recommends the apportionment of mandates using the allocation function (Grimmett G. et al., 2011):

$$A(p) = \min\{b + \frac{p}{d}, M\}$$

where a parameter *b* is called the base, and *d* is a free parameter called the divisor.

It can be easily shown that for a given sequence of populations $P = (p_1, p_2, \dots, p_n)$ the values of the function A in points $A(p_1), A(p_2), \dots, A(p_n)$ represent a degressively proportional sequence. The values of this sequence are not necessarily integer, therefore they cannot provide an actual allocation. As a result, it was proposed to relax the requirements of degressive proportionality given in (1) and (2) in such a way that they need to be satisfied before rounding the sequence $A(p_1), A(p_2), \dots, A(p_n)$ to integers. A suitable provision authorizing this solution was included in an amendment to the 2007 resolution that changed the wording of its point 6 as follows: *[The European Parliament] Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State before rounding to whole numbers must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.*

In agreement with the terms adopted in the literature (Delgado-Márquez B. et al., 2013), such a solution is called an "unrounded degressive proportionality" (UDP) and it definitely broadens a range of potential solutions. When solving a problem of allocation of seats in the European Parliament, one can now also consider the sequences $R = (r_1, r_2, \dots, r_n)$ that are degressively proportional, have real values, satisfy the conditions:

(1a) $r_1 \leq r_2 \leq \cdots \leq r_n$ and (2a) $\frac{p_1}{r_1} \leq \frac{p_2}{r_2} \leq \cdots \leq \frac{p_n}{r_n}$,

and, when duly rounded, become the sequences of integers satisfying the conditions (3) and (4). To obtain an actual allocation in agreement with the proposal of the conference at Cambridge and called the Cambridge Compromise, one assumes the value of the parameter *b* equal 5, and adjusts the divisor *d* in such a way that after upward rounding of the sequence terms $A(p_1), A(p_2), \dots, A(p_n)$, its total equals 751, i.e. the full size of the European Parliament. This solution refers to divisor methods of proportional allocation known in the field of fair representation. Nevertheless, the parameter *b* and the rounding method are chosen arbitrarily and without additional justification. Obviously, these two elements must agree so as to attain the value $s_1 = 6$, given the values of the sequence $P = (p_1, p_2, \dots, p_n)$.

3. BASE ALLOCATIONS

Taking the base value $b = 5$ can be perceived as a wish to proportionally allocate the greatest number of mandates in order to comply with a classical Aristotelian principle. With upward rounding, only five seats guaranteed by the Treaty of Lisbon are not subject to the proportional allocation rule. All remaining seats, excluding the upper bound, of course, are allocated proportionally, as commonly accepted and applied in the European cultural area. This approach involves just one rounding method, i.e. an upward rounding, because rounding downwards and rounding to the nearest integer lead to $s_1 = 5$.

With no essential modification of the key idea of the Cambridge Compromise, one can also assume $b = 6$ as a base level. In this case the rounding methods can be rounding downwards or rounding to the nearest integer. Otherwise the value s_1 exceeds six and such an allocation does not satisfy the condition (3). From the viewpoint of actual applications, this solution seems as good as those previously indicated. The main idea is to proportionally divide remaining seats after the base division apportioned the same, fixed number of mandates to each member state. Put differently, such values must be proportionally added to the sequence $s_1 = s_2 = \cdots = s_n = b$, that a resulting sequence satisfies the conditions (3) and (4). With current populations of the European Union member states, the divisions obtained in this way are equal and it makes no difference which rule is actually applied. However, it does not have to be so in general. It must also be emphasized that choosing neither b nor a rounding method is universal. The relevant factors are the structure of populations, i.e. the sequence $P = (p_1, p_2, \dots, p_n)$, and adopted assumptions about division expressed by the conditions (3) and (4). For the sake of consistency of succeeding presentation, the fixed, base sequence will be considered with $b = 6$, and the rounding method will be downward rounding.

The constructs presented above are possible due to the degressive proportionality of the resulting sequence $A(p_1), A(p_2), \dots, A(p_n)$ in the sense of the conditions (1a) and (2a). This is a consequence of the fact that adding a proportional sequence to a constant sequence yields a degressively proportional sequence. It can be easily shown that also a sequence satisfying the conditions (1a) and (2a) added to a proportional sequence yields a degressively proportional sequence. In fact, let $R = (r_1, r_2, \dots, r_n)$ satisfy the conditions (1a) and (2a), and let $Q = (q_1, q_2, \dots, q_n)$ satisfy $\frac{p_i}{q_i} = q > 0$ for all i. Both these sequences are nondecreasing, hence their sum is also a nondecreasing sequence. To prove that this sum is a degressively proportional sequence, it suffices to demonstrate that if $p_i \le p_j$ holds, then $\frac{p_i}{r_i+q_i} \le \frac{p_j}{r_j+q_j}$ holds, which is equivalent to $p_i r_j + p_i q_j \le p_j r_i + p_j q_i$. This is because the condition of degressivity satisfied by the sequence R results in $p_i r_j \le p_j r_i$, and the condition of proportionality results in $Q\ \ p_i q_j = p_j q_i$.

Therefore, other sequences may be considered as base sequences, in particular the sequences that are degressively proportional. With the key concept of the Cambridge compromise in mind, i.e. dividing a maximum number of seats in a proportional manner, one can think about indicating a degressively proportional sequence that will perform a minimum allocation satisfying the condition (3). The concept of minimum allocation can be explained at least twofold, for that reason there are at least two ways leading to such a sentence.

When applying the first manner, one considers all possible integer divisions satisfying the conditions (1)-(4) and indicates a minimum number of mandates allocated to a given country (Łyko J., Rudek R., 2013). This is the actual minimum number of seats that can be given to the state under conditions (1)-(4). The number can be seen as a guarantee of a minimum representation for a given country under fixed boundary conditions. The sequence of these numbers is degressively proportional for the sequence of populations of the European Union countries in 2012 (see table 1, columns B and P/B). In general, it does not have to be so, which results in loss of generality, therefore its application is not always possible.

Table 1. Base divisions and "proportionally adjusted" divisions generated by

$$\lfloor A(p_i) \rfloor = \left\lfloor \min\left\{ b_i + \frac{p_i}{d}, M \right\} \right\rfloor$$

	Country	Population in thous.	bases						base + proportional					
			A	P/A	B	P/B	C	P/C	A′ 840	P/A′	B′ 5560	P/B′	C′ 5260	P/C′
1	Malta	416.1	6	69.4	6	69.4	6	69.4	6	69.4	6	69.4	6	69.4
2	Luxembourg	524.9	6	87.5	6	87.5	6	87.5	6	87.5	6	87.5	6	87.5
3	Cyprus	862.0	6	143.7	6	143.7	6	143.7	7	123.1	6	143.7	6	143.7
4	Estonia	1339.7	6	223.3	6	223.3	6	223.3	7	191.4	6	223.3	6	223.3
5	Latvia	2041.8	6	340.3	6	340.3	6	340.3	8	255.2	6	340.3	6	340.3
6	Slovenia	2055.5	6	342.6	6	342.6	6	342.6	8	256.9	6	342.6	6	342.6
7	Lithuania	3007.8	6	501.3	8	376.0	7	429.7	9	334.2	8	376.0	7	429.7
8	Croatia	4398.2	6	733.0	10	439.8	9	488.7	11	399.8	10	439.8	9	488.7
9	Ireland	4582.8	6	763.8	10	458.3	9	509.2	11	416.6	10	458.3	9	509.2
10	Finland	5401.3	6	900.2	11	491.0	10	540.1	12	450.1	11	491.0	11	491.0
11	Slovakia	5404.3	6	900.7	11	491.3	10	540.4	12	450.4	11	491.3	11	491.3
12	Denmark	5580.5	6	930.1	11	507.3	10	558.1	12	465.0	12	465.0	11	507.3
13	Bulgaria	7327.2	6	1221.2	14	523.4	13	563.6	14	523.4	15	488.5	14	523.4
14	Austria	8443.0	6	1407.2	15	562.9	14	603.1	16	527.7	16	527.7	15	562.9
15	Sweden	9482.9	6	1580.5	15	632.2	15	632.2	17	557.8	16	592.7	16	592.7
16	Hungary	9957.7	6	1659.6	15	663.8	15	663.8	17	585.7	16	622.4	16	622.4
17	Czech Rep.	10505.4	6	1750.9	15	700.4	15	700.4	18	583.6	16	656.6	16	656.6
18	Portugal	10541.8	6	1757.0	15	702.8	15	702.8	18	585.7	16	658.9	17	620.1
19	Belgium	11041.3	6	1840.2	15	736.1	15	736.1	19	581.1	16	690.1	17	649.5
20	Greece	11290.9	6	1881.8	15	752.7	15	752.7	19	594.3	17	664.2	17	664.2
21	Netherlands	16730.3	6	2788.4	22	760.5	22	760.5	25	669.2	25	669.2	25	669.2
22	Romania	21355.8	6	3559.3	27	791.0	27	791.0	31	688.9	30	711.9	31	688.9
23	Poland	38538.4	6	6423.1	47	820.0	47	820.0	51	755.7	53	727.1	54	713.7
24	Spain	46196.3	6	7699.4	56	824.9	56	824.9	60	769.9	64	721.8	64	721.8
25	Italy	60820.8	6	10136.8	73	833.2	73	833.2	78	779.8	83	732.8	84	724.1
26	UK	62989.6	6	10498.3	75	839.9	75	839.9	80	787.4	86	732.4	86	732.4
27	France	65397.9	6	10899.7	77	849.3	77	849.3	83	787.9	88	743.2	89	734.8
28	Germany	81843.7	6	13640.6	96	852.5	96	852.5	96	852.5	96	852.5	96	852.5
	Sumary	508077.9	168		689		681		751		751		751	

Source: own works.

The other manner consists in applying a sequence that follows the consecutive allocation of a minimum number of mandates to all countries, beginning with the largest, subject to conditions (1)-(3). To ensure the rounded degressive proportionality, a recursive sequence is constructed beginning with the largest country in a decreasing order. The largest country is allocated $s_n = M = 96$ seats, with each next country allocated a minimum number of mandates still satisfying the degressive proportionality condition, bound by the minimum feasible value $m = 6$ (see table 1, columns C and P/C). The recurrence is described by the formula

$$s_n = 96,\ s_i = \max\{\lceil s_{i+1} p_i / p_{i+1} \rceil, 6\} \quad \text{for} \quad i = n-1, n-2, \ldots, 1.$$

This is a minimum sequence characterized by rounded degressive proportionality and satisfying the conditions (1)-(3) in the sense that a sum of its terms is less than a sum of terms of any other sequence of this type (Łyko J., Maciuk A., 2016).

4. CONCLUSION

The proportional adjustment of a fixed base division proposed by the Cambridge Compromise can be generalized allowing other degressively proportional divisions as base divisions. The rule of "unrounded degressive proportionality" is not violated, whereas both inequalities in the condition (3) can be employed when building the base which means that the boundary conditions are more satisfied. The two proposals presented in the paper also embrace, in a maximally feasible way, the proportional allocation that is a classical principle in terms of fair representation. In each case, a base division is minimal in one way. Except for the proportional algorithm, no country is allocated more mandates than it is indicated by the degressively proportional rule and the boundary conditions, i.e. the number of seats in the European Parliament legally guaranteed by the Treaty of Lisbon.

Columns A', B' and C' in table 1 show divisions generated by a proportional adjustment of respective base divisions A, B and C. In order to preserve the condition $s_1 = 6$, downward rounding is used in the proportional divisor method, i.e. allocations are determined by the formula $\lfloor A(p_i) \rfloor = \left\lfloor \min\left\{ b_i + \frac{p_i}{d}, M \right\} \right\rfloor$. Except for Malta and Germany, whose numbers of mandates result from the condition (3), each of the three methods leads to the same result for the two countries only, namely Luxembourg and the Netherlands. The modification of the Cambridge proposal, i.e. replacing a constant base sequence by another minimum (in a different sense), degressively proportional sequence, gives preference to more populated countries. The largest difference equals six mandates and occurs, depending on the adopted base, in case of Italy, the UK and France. These differences occur primarily at the cost of no diversification in case of the six smallest countries of the Union. Such result is evident in view of the fact that the proportional division practically concerns, respectively, 62 and 70 mandates, and with the downward rounding the smallest countries just get the numbers of seats allocated by the base division and nothing more. However, the advantage of the proposed method to employ different base divisions than those proposed by the Cambridge Compromise for a given problem of allocation, especially the proposal shown in column C, table 1, consists in accounting for both inequalities written in the condition (3), not just one, where only $s_1 = 6$ is included. Hence, the solution achieved is closer to the one generated by less arbitrary factors, and the emphasis of indicating the division is shifted towards more serious consideration of boundary conditions imposed by the Treaty of Lisbon.

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