

HYPERBOLIC LAW OF DUNCAN. COMPARISON BETWEEN SP5 AND EXPERIENCE. TESTS IN SMALL DEFORMATIONS.

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Abstract

The elastic and plastic deformations of loaded materials have aroused the interest of soil mechanics, for an otherwise correct representation of the least appropriate. For well-known soil behavior, it is essential to know the rheological laws governing these bodies considered as a continuous medium.

The rigorous method for solving a problem in soil mechanics must contain a model capable of describing the behavior of soils, a choice which justifies the required characteristic parameters (parameters of deformability and rupture), and at the end of the procedures to apply this model to geotechnical problems.

Theories of elasticity and plasticity are commonly used in soil mechanics, considering the facilities they represent for their application.

With the arrival of informatics, the adoption of more realistic models to describe the behavior of soil, albeit more complex, describes the non-linearity and irreversibility of soil behavior.

In this work, we present the curves q - ϵ_p , e - ϵ_p , e - p , and e - $\log p$ of the NC (Normal Consolidated) soils tested under triaxial drained conditions that have been digitized as Kaolinite (black clay), White clay, Very low-density sand.

Afterwards, we used SP5 software that follows the hyperbolic law of Duncan to plot the previous curves (for each type of soil). Thanks to another software program, we have superimposed the curves obtained by the experiment and by SP5 to make the comparison and seek the validity of the law.

The final part of this publication looks into changes of Poisson's ν coefficient and Young's E module in the field of small deformations, and useful data is output.

Key words: Hyperbolic law of Duncan, SP5, White Clay, Kaolinite (Black Clay), Very low-density sand, small deformations

1. CALCULATION WITH THE AID OF A COMPUTER

We used the SP5 software developed by Kai Wong in early 1977, and revised in 1983.

This program uses two points (70-95%) of the Duncan experimental curves to determine the parameters of the law.

In addition to the determination of the parameters, the code plotted the stress-strain curves, the volume variation as a function of the axial deformation, the variation of the initial tangent modulus as a function of the confinement pressure and the variation of the volume deformation modulus with confinement pressure.

Thus, for sand, the variation of the friction angle is a function of σ_3 (σ_3) and for clay the variation of the shear stress is a function of the average between the two main stresses.

The list of data and results obtained by SP5 is given in the following pages.

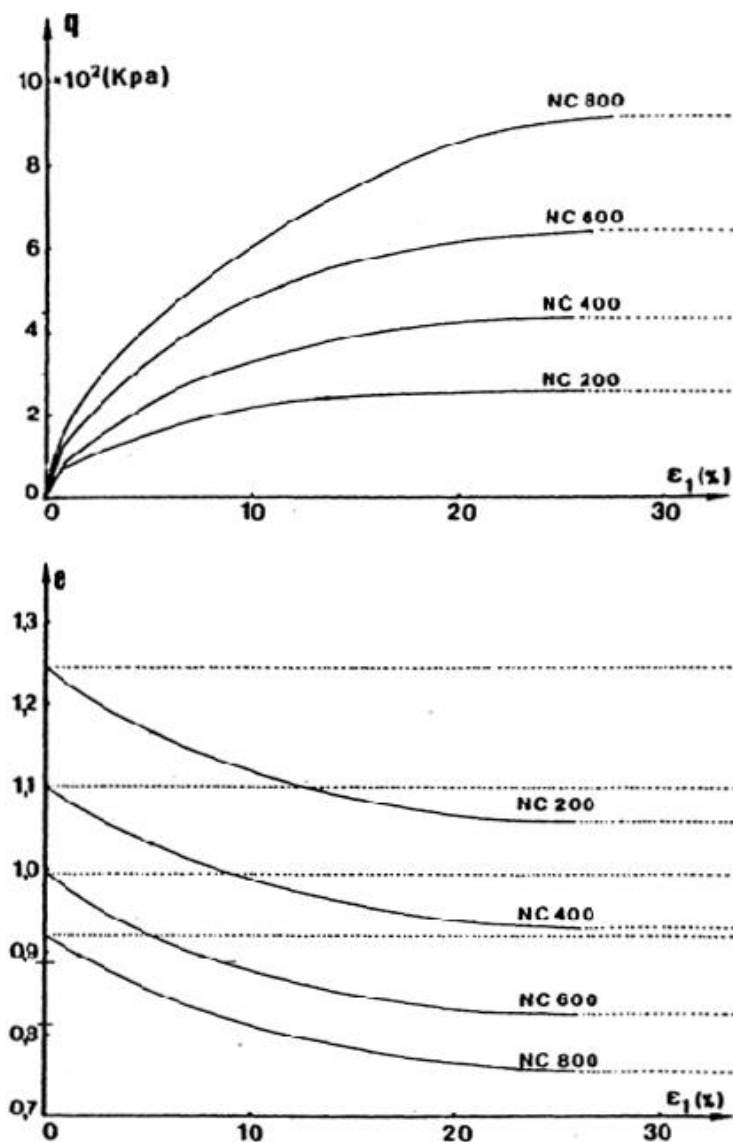


Fig. 1. Stress-strain curves and void index curves as a function of axial deformation for lateral pressures equal to 0.4, 0.6 and 0.8 Mpa.

Table 1. Drained triaxial tests for Kaolinite NC. The two levels (70% -95%) of Duncan are presented.

σ_3 (MPa)	$(\sigma_1 - \sigma_3)_f$ (MPa)	70% Stress level			95% Stress level		
		$(\sigma_1 - \sigma_3)$ (MPa)	ε_a	$\varepsilon_a / (\sigma_1 - \sigma_3)$	$(\sigma_1 - \sigma_3)$	ε_a	$\varepsilon_a / (\sigma_1 - \sigma_3)$
0,4	0,43	0,301	0,092	0,30	0,408	0,188	0,46
0,6	0,638	0,447	0,066	0,15	0,606	0,19	0,31
0,8	0,907	0,635	0,11	0,15	0,861	0,29	0,33

Table 2. Drained triaxial tests for Kaolinite NC. Calculation of law parameters. Comparison between the SP5 software and the measured values.

Drained triaxial tests for Kaolinite NC						
σ_3 (MPa)	a% (MPa)	$E_i=1/a$ (MPa)	b (MPa)	$(\sigma_1-\sigma_3)_{ult}$ (MPa)	$(\sigma_1-\sigma_3)_f$ (MPa)	$R_f\%$
0,4	15	6,67	1,67	0,6029	0,43	0,71
0,6	6,5	15,92	1,29	0,775	0,638	0,823
0,8	8	12,5	0,88	1,136	0,907	0,798

Average=0.78

E_i - σ_3 curve

σ_3 (MPa)	E_i (MPa) from DUNCAN	E_i (MPa) Measured values
0,4	6,67	6,708
0,6	15,38	13,33
0,8	12,5	18,25

Table 3. Drained triaxial tests for Kaolinite NC. Calculation of parameters of the law (Index of voids-deformations).

σ_3	$(\sigma_1-\sigma_3)_f$	$(\sigma_1-\sigma_3)_f$ 70%	e_f	e_0	e_v	$(\sigma_1-\sigma_3)_f$	$B=(\sigma_1-\sigma_3)_f/3e_v$
0,4	0,43	0,301	1,0	1,1	0,0476	0,301	2,107
0,6	0,638	0,447	0,89	1,0	0,055	0,447	2,709
0,8	0,907	0,635	0,812	0,924	0,0582	0,635	3,636

Table 4. Drained triaxial tests for very low-density sand. Calculation of parameters of the law. Comparison between the SP5 software and the measured values.

σ_3 (MPa)	E_i (MPa) from DUNCAN	E_i (MPa) Measured values
0,1	2,78	2,12
0,13	3,96	3,32
0,2	6,25	5

Very low-density sand

σ_3	$(\sigma_1-\sigma_3)_f$	$(\sigma_1-\sigma_3)_f$ 70%	e_f	e_0	e_v	$(\sigma_1-\sigma_3)_f$	$B=(\sigma_1-\sigma_3)_f/3e_v$
0,1	0,23	0,161	1,014	1,128	0,053	0,161	1,012
0,13	0,35	0,245	0,974	1,128	0,071	0,245	1,150
0,2	0,42	0,294	0,991	1,10	0,051	0,294	1,92

It should be noted that for each soil type the values of the parameters obtained by the two methods are compared. We note after comparisons that there is not a great difference between the two methods.

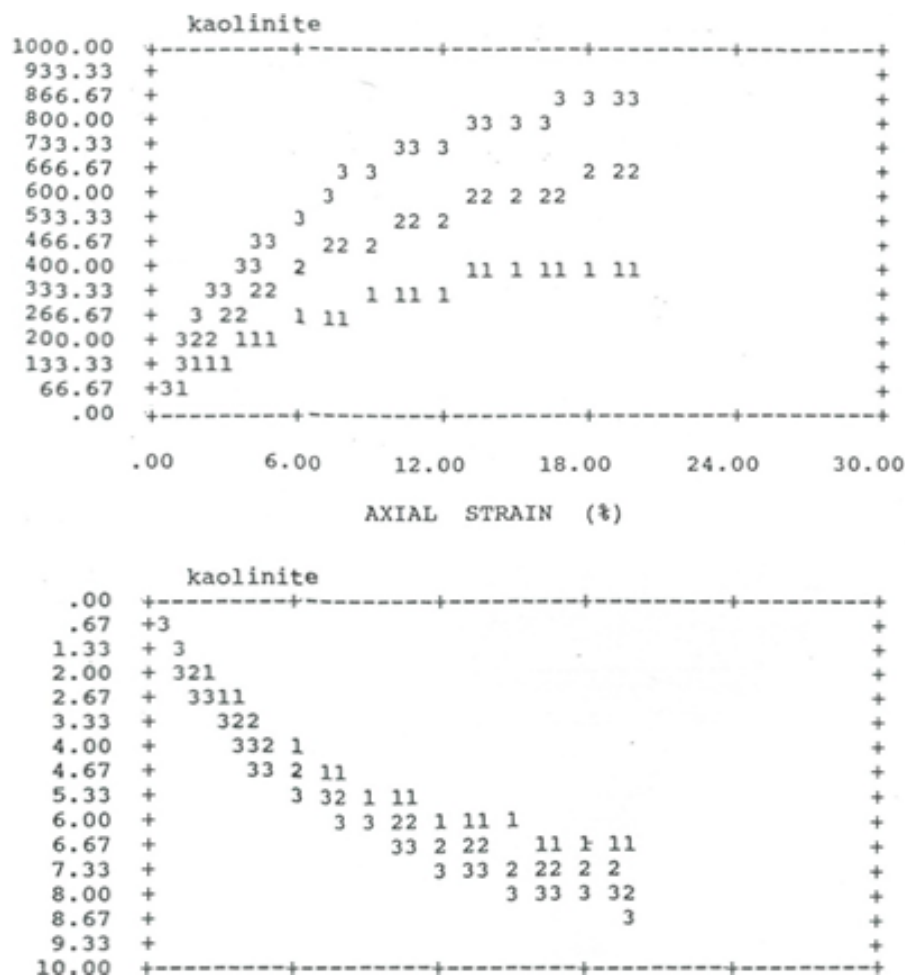


Fig. 2. Stress-strain curves obtained by computer calculation. Kaolinite (Black Clay)

Table 5. Kaolinite (Black Clay). Comparison between the SP5 software and the measured values.

Parameters	SP5 Calculated values	Values calculated by hand
C	0,00	10
Φ	22	20
K	14	17,2
n	1,19	1,00
R_f	0,79	0,78
Kb	7	6,60
m	0,78	0,79

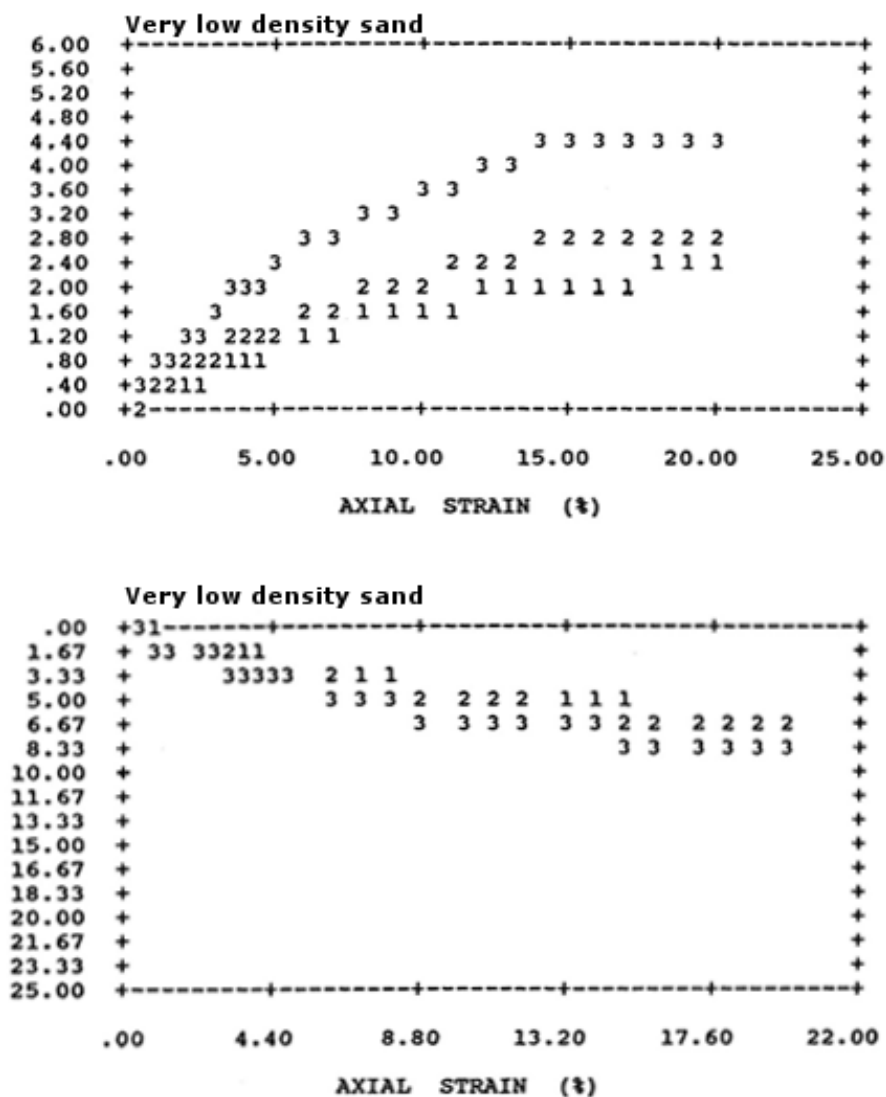


Fig. 3. Stress-strain curves obtained by computer calculation. Very low-density sand.

Table 6. Very low-density sand. Comparison between the SP5 software and the measured values.

Parameters	SP5 Calculated values	Values calculated by hand
C	0,00	0,00
Φ	32,6	41,81
$\Delta\Phi$	4,9	4,77
K	27	28
n	1,34	1,34
R_f	0,57	0,55
Kb	11	5,52
m	0,70	9,2

2. COMPARISON BETWEEN SP5 AND EXPERIENCE.

In this work, we present the curves q-eps, e-eps., e-p, and e-logp of the NC (Normal Consolidated) soils that have been digitized by the book "Laws of Behavior of Reclaimed Soils" ("Lois de comportement des sols remaniés"), (J. Biarez). These involve:

- Kaolinite (black clay, Zervoyanis thesis for $\sigma_3 = 4\text{bar}, 8\text{bar}$)
- White clay (thesis of Trueba for $\sigma_3 = 3\text{bar}, 3.5\text{bar}, 4\text{bar}, 6.5\text{bar}$)
- Very thin sand (Very low-density sand), (Ziani thesis for $\sigma_3 = 0.2\text{bar}, 0.4\text{bar}, 1\text{bar}, 1.3\text{bar}, 2\text{bar}$)

Afterwards, we used SP5 software that follows the hyperbolic law of Duncan to plot the previous curves (for each type of soil). Thanks to another software program, we have superimposed the curves obtained by the experiment and by SP5 to make the comparison and seek the validity of the law. The comparisons between the curves are presented in the figures below.

- It is noted that the law simulates the experiment well (curves q-eps1) but it underestimates the values of initial tangent modulus E_i (Curves eps1 / q-eps1).
- It is clear that the law is far from representing the experiment properly because it reflects the variation in volume.

Then, the variation of the secant modulus is plotted with the axial deformation: it decreases with the axial deformation and is proportional to the lateral pressure.

At the end of this work, the evolution of the initial coefficient of Poisson with the lateral pressure is presented for the case of sand with very low density. It has been found that the variation of ν_1 is presented in the following form:

$$\nu_1 = G - f \times \log_{10} \times \sigma_3$$

where G is the Poisson coefficient value (initial) for the confinement (lateral) pressure of one atmosphere.

The same thing was found by Al-Issa, by performing laboratory sand cap tests, on sand from Loire and sand from Hostun.

Black Clay N.C.

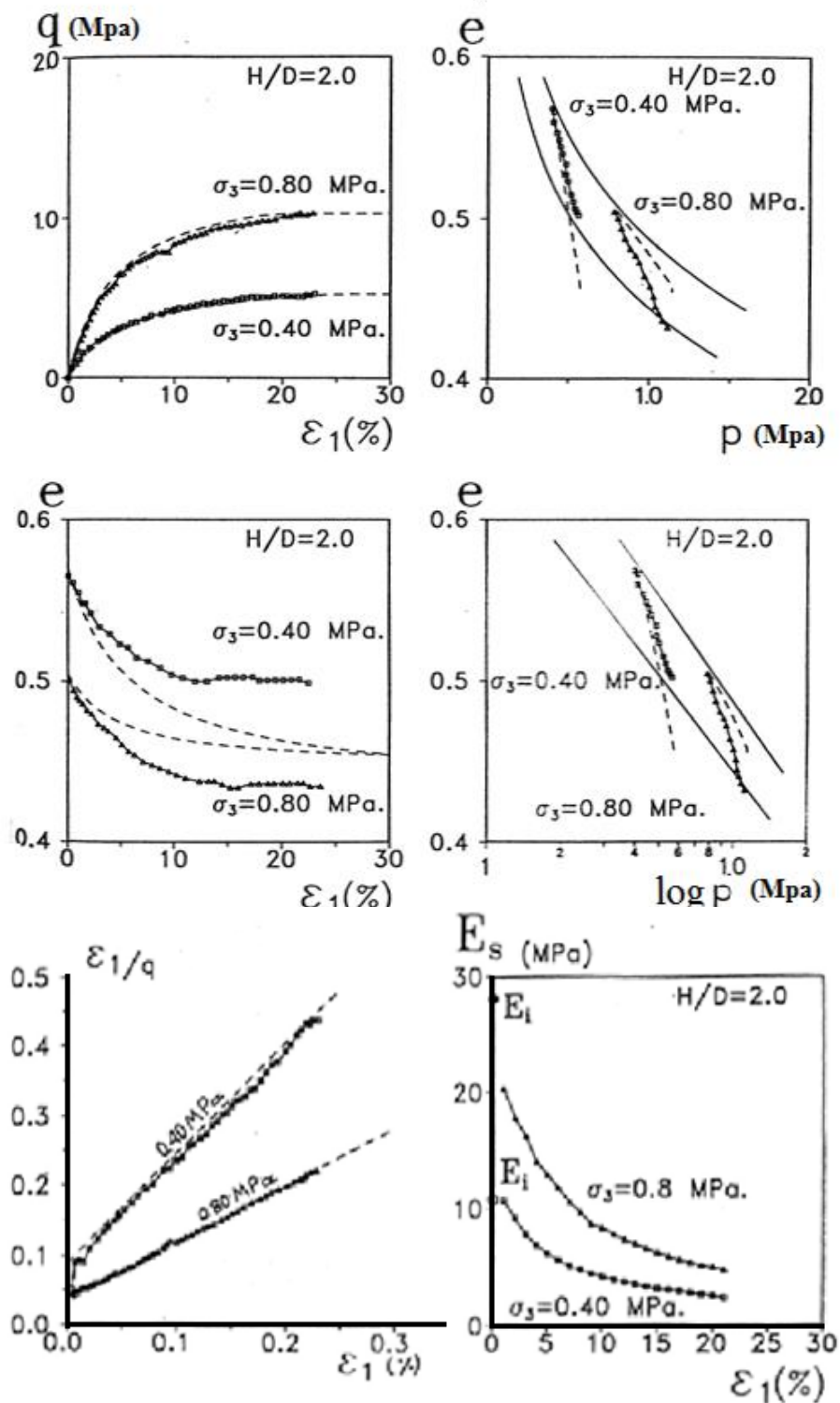


Fig. 4. Curves q - ε ., e - ε ., e - p , e - $\log p$ and E_s - ε for black clay NC

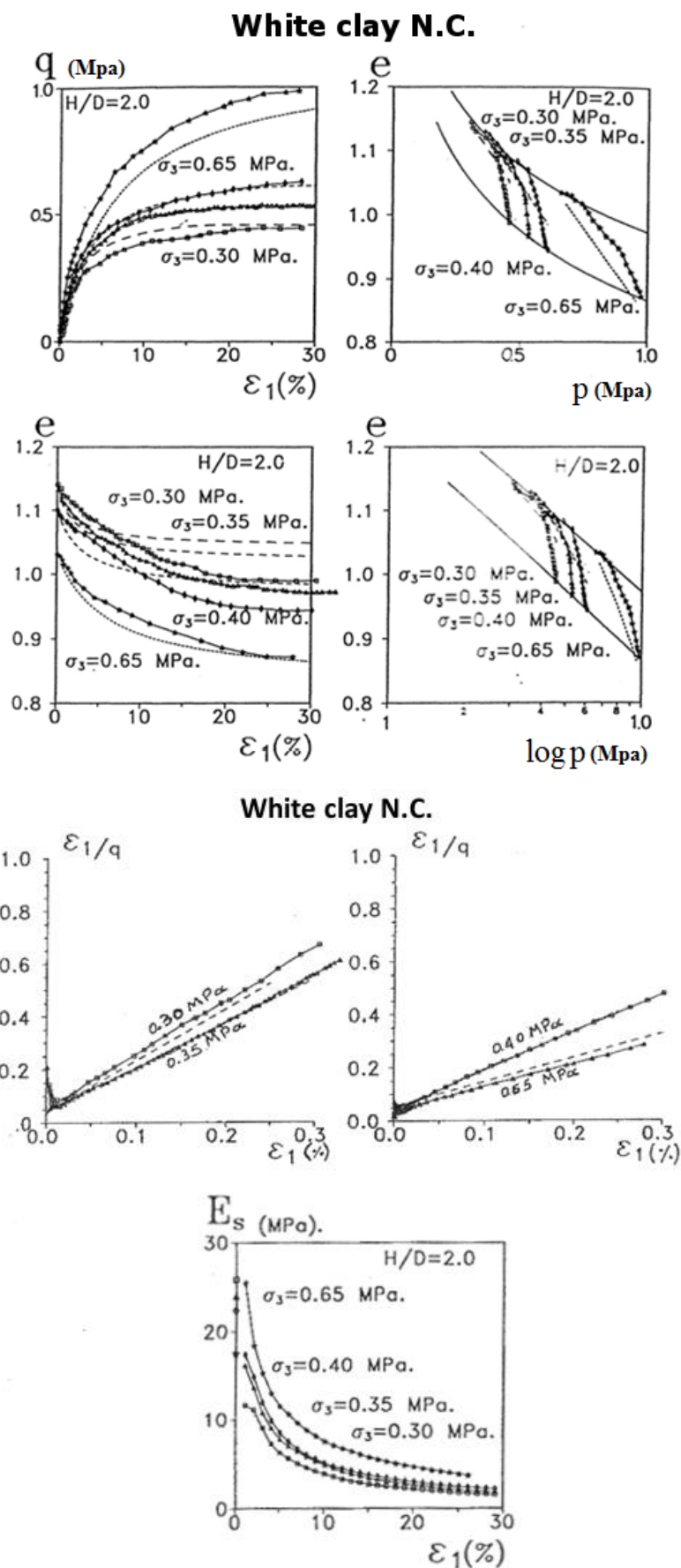


Fig. 5. Curves q -eps, e -eps., E_p , e -logp and E_s -eps for white clay NC.

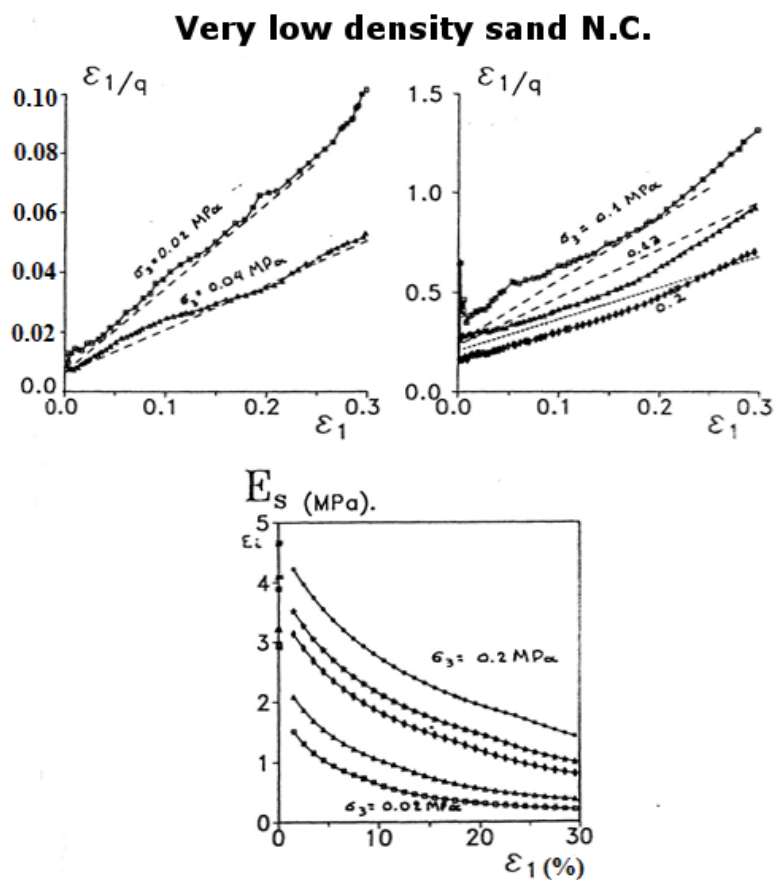
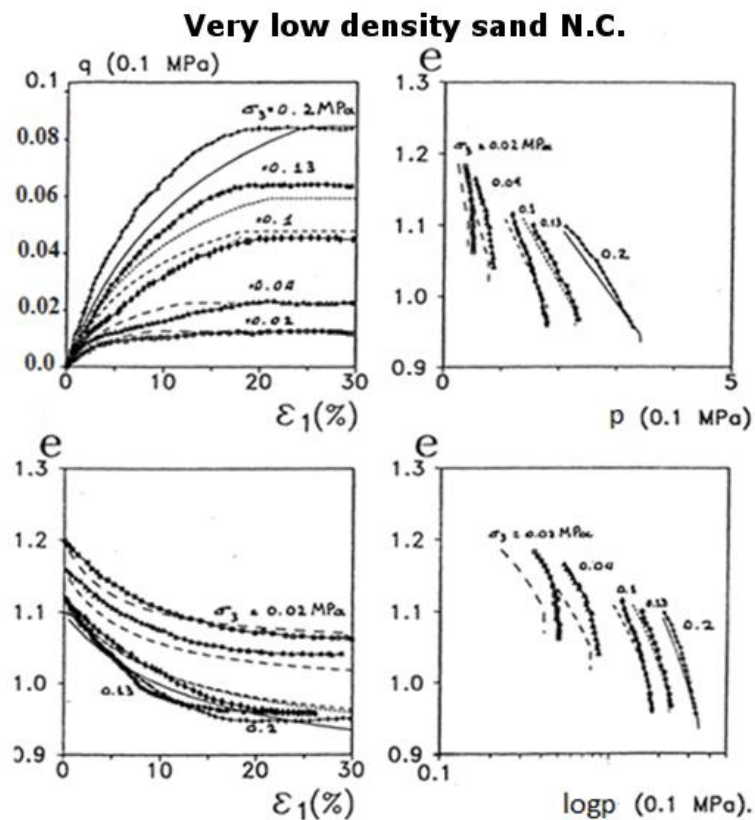


Fig. 6. q-eps, e-eps., e-p, e-logp and E_s -eps for very low-density sand.

Comparison between SP5 modules and experience

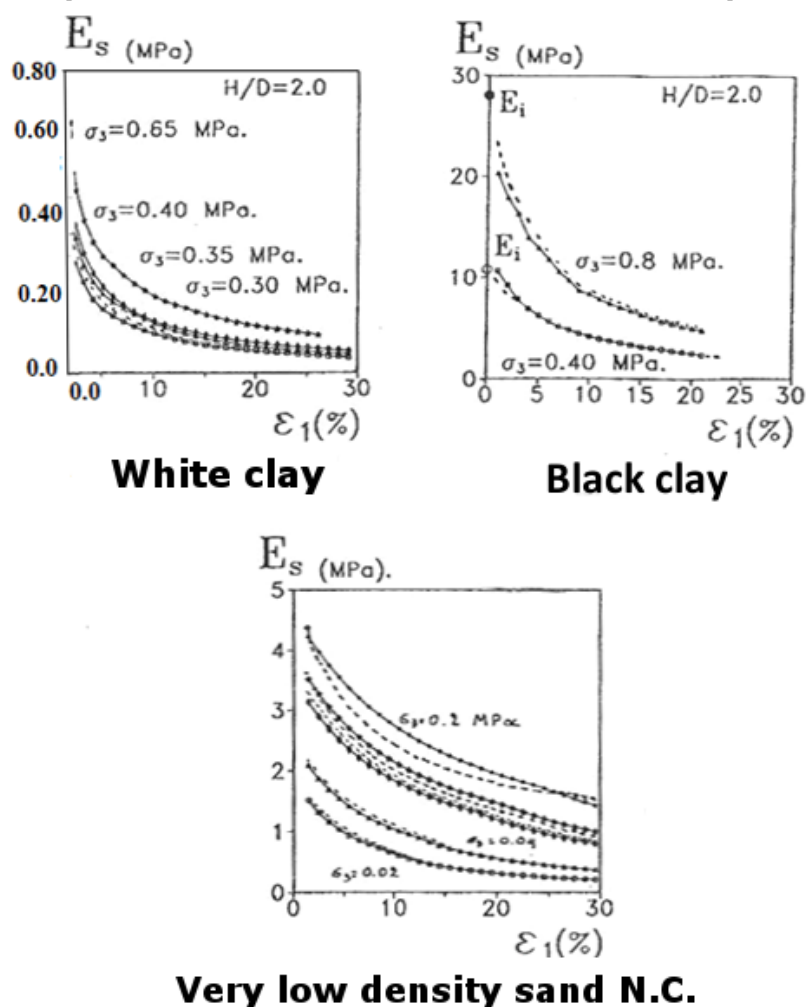


Fig. 7. Comparison of white clay, black clay, and very low-density sand.

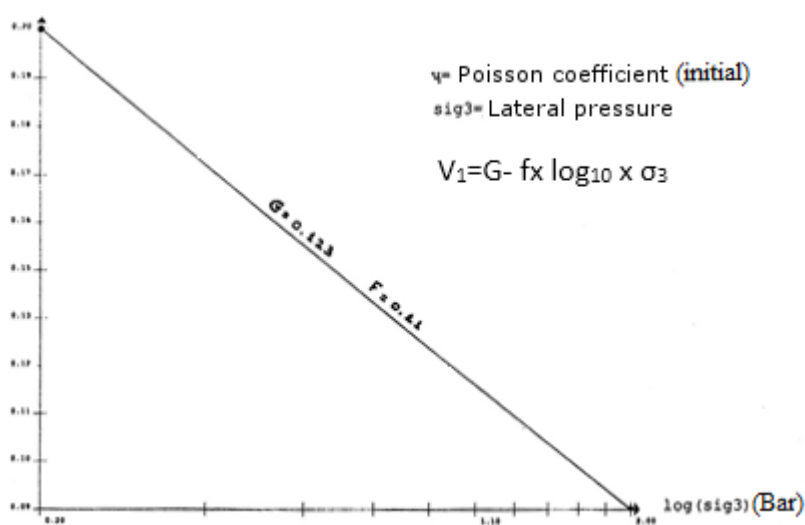


Fig. 8. Evolution of initial Poisson coefficient with lateral pressure for the case of very low-density sand.

3. TESTS IN SMALL DEFORMATIONS.

When the mechanical behavior of a soil is idealized as being linear, homogeneous and isotropic elastic, it is necessary to know two elastic coefficients to characterize such behavior. These being:

- The Poisson's coefficient ν
- The Young's module E

Since these two parameters are highly influenced by the level of deformation to which they are determined, the conventional laboratory tests or the deformation levels concerned are generally greater than 10^{-3} , and do not lead to a good knowledge of such parameters.

It is for this reason that an investigation has been undertaken to determine the mechanical properties in the truly elastic domain using the experimental results of Bollée and Rivera.

4. DYNAMIC TESTS: EVOLUTION OF THE POISSON'S COEFFICIENT WITH THE MEAN PRESSURE Σ_0

In the following figures, the variation of the Poisson coefficient as a function of the mean pressure for the Hostun sand and for the different isotropic pressure cycles is shown.

For the Hostun sand (Rivera tests), a slight decrease in the Poisson coefficient (from 0.20 to 0.15 for $e = 0.664$) is observed following the application of the isotropic pressure cycles (between 25 Kpa and 600 Kpa)

The decay of the Poisson coefficient for the Hostun sand can be explained from the theoretical results established by Deresiewicz (1957) for an assembly of identical elastic grains of the same material, subjected to an initial isotropic stress state, then to a low-intensity deviatoric stress.

Deresiewicz shows that the Poisson's coefficient does not depend on the initial isotropic stress, but is a function of the ratio between the tangential stiffness and the normal stiffness of the assembly. More precisely, the Poisson's coefficient decreases when the tangential stiffness of the assembly increases.

In the case of the Hostun sand, the application of the isotropic pressure cycles will then increase the number of contacts between soil particles, thus also increasing the tangential stiffness.

At the Bollée tests for the same index of voids $e = 0.66$ and for the same material, the starting values of the Poisson's coefficient 0.10 and 0.12 are noted.

The experimental results of the Hostun sand show the non-dependence of the Poisson coefficient on the average isotropic stress.

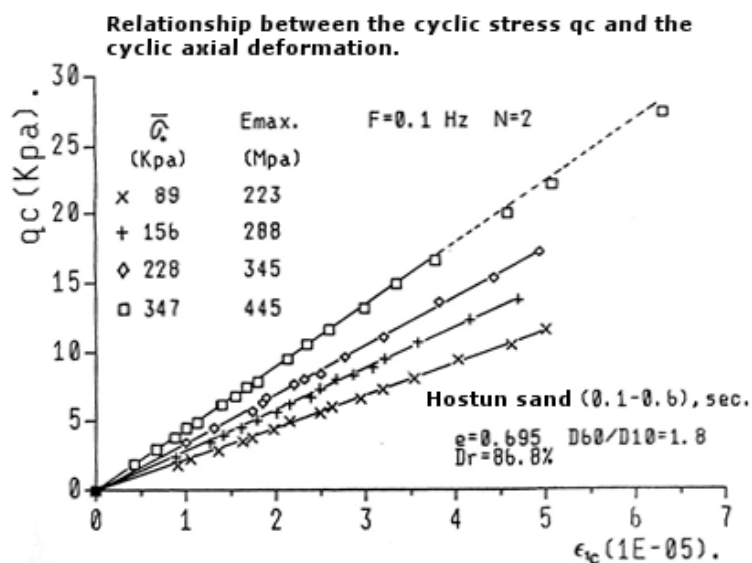


Fig. 9. Relationship between the cyclic stress q_c and the cyclic axial deformation.

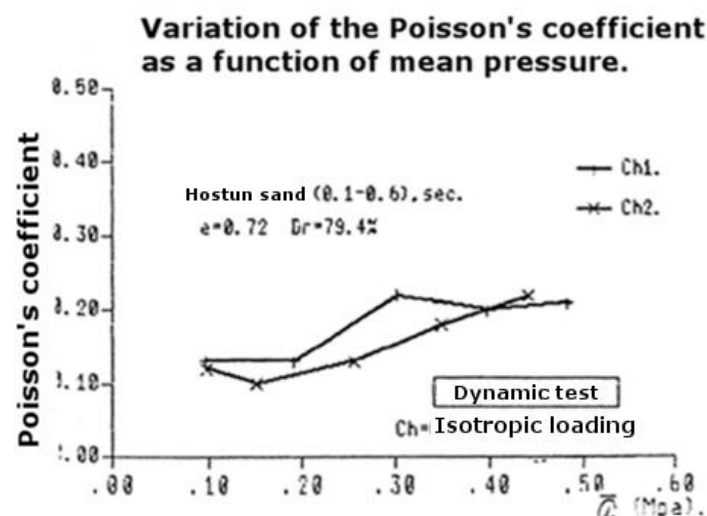
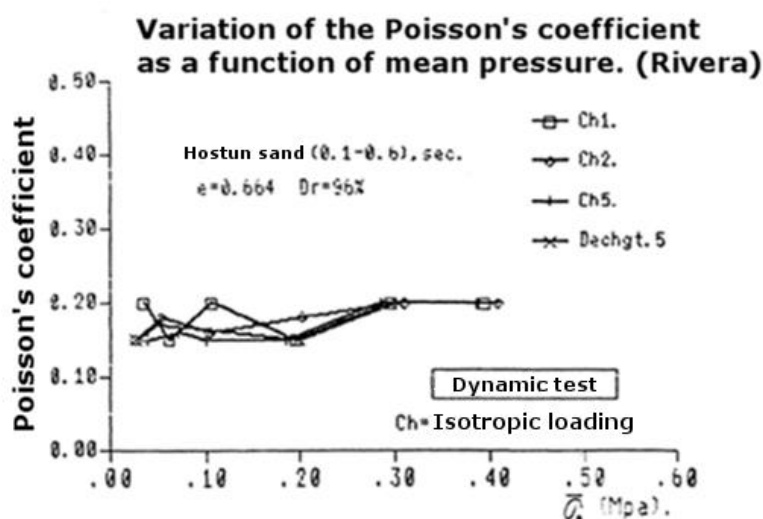


Fig. 10. Variation of the Poisson's coefficient as a function of mean pressure.

The following figures show the stress-strain curves for the Hostun sand samples tested at the same density and under a confinement (lateral) stress of 105 KPa and 200 KPa respectively.

By observing the stress - strain curves, it is observed that the relationship between the stresses and the deformations remain approximately linear for an axial deformation between 3×10^{-5} and 5×10^{-5} . Once this limit is exceeded, the stress-strain curves are no longer linear. This result is in complete agreement with that found for the cyclic tests on the same material (Rivera).

The initial tangent modules (E_0), were determined from the slope of the linear part of the stress-strain curves. The results obtained are presented in the table which follows, along with the values of the Young modulus (E_{max}), determined from its cyclic tests carried out on the same material and with identique initial density and confinement (lateral) pressure conditions.

Table 7. Sand from Hostun. The initial tangent modules (E_0), as well as the values of the Young's modulus (E_{max}).

Material	(e)	σ_0 (Kpa)	E_0 (Mpa)	E_{max} (Mpa)
Sand from Hostun	0,664	105	265	268
		200	408	369

This table shows that for the lowest confinement pressure (105KPa) there is a relative agreement between the module (E_0) and (E_{max}). On the other hand, with regard to the test carried out on the Hostun sand under a confinement stress of 200 KPa, the initial tangent modulus (E_0) appears to be larger (by about 10%) than that obtained by the cyclic test (E_{max}). Rivera put this difference down to technical reasons.

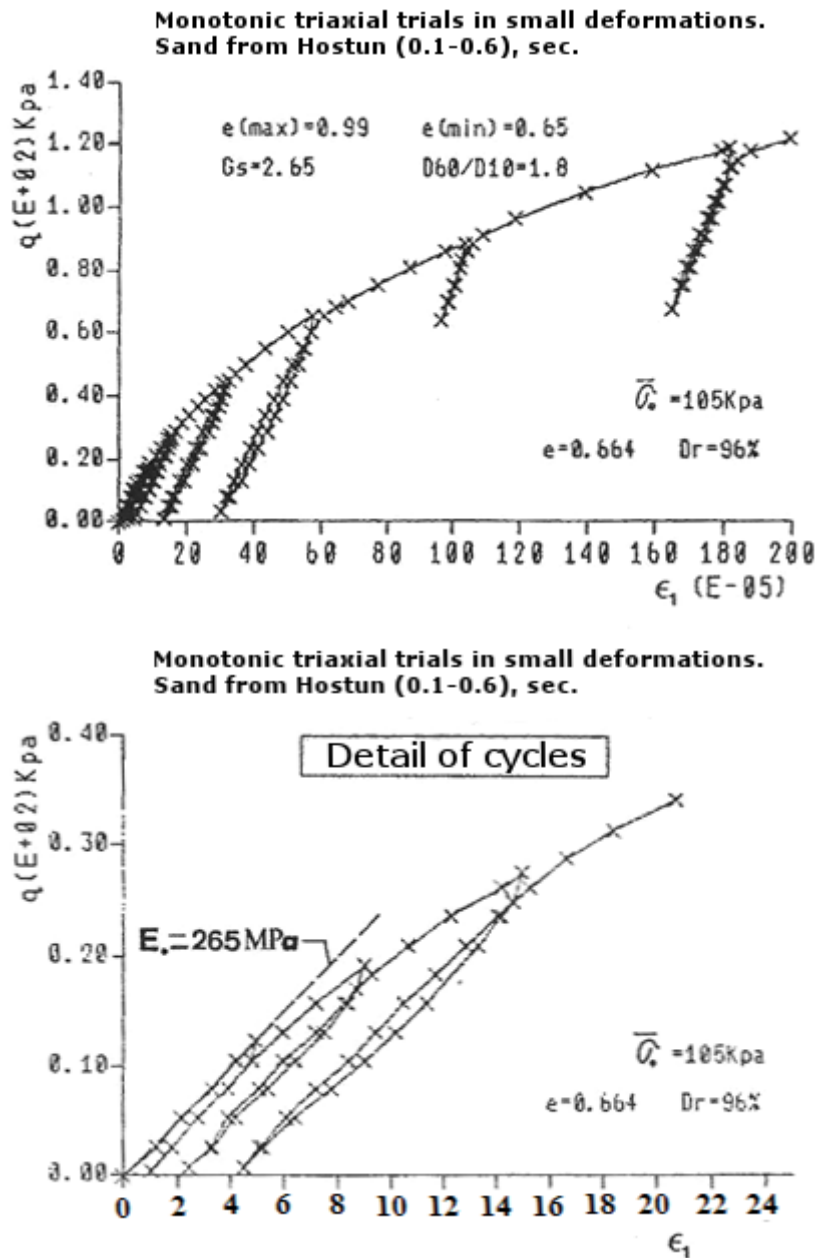


Fig. 11. Monotonic triaxial trials in small deformations (Sand from Hostun, $\sigma_0=105$ KPa).

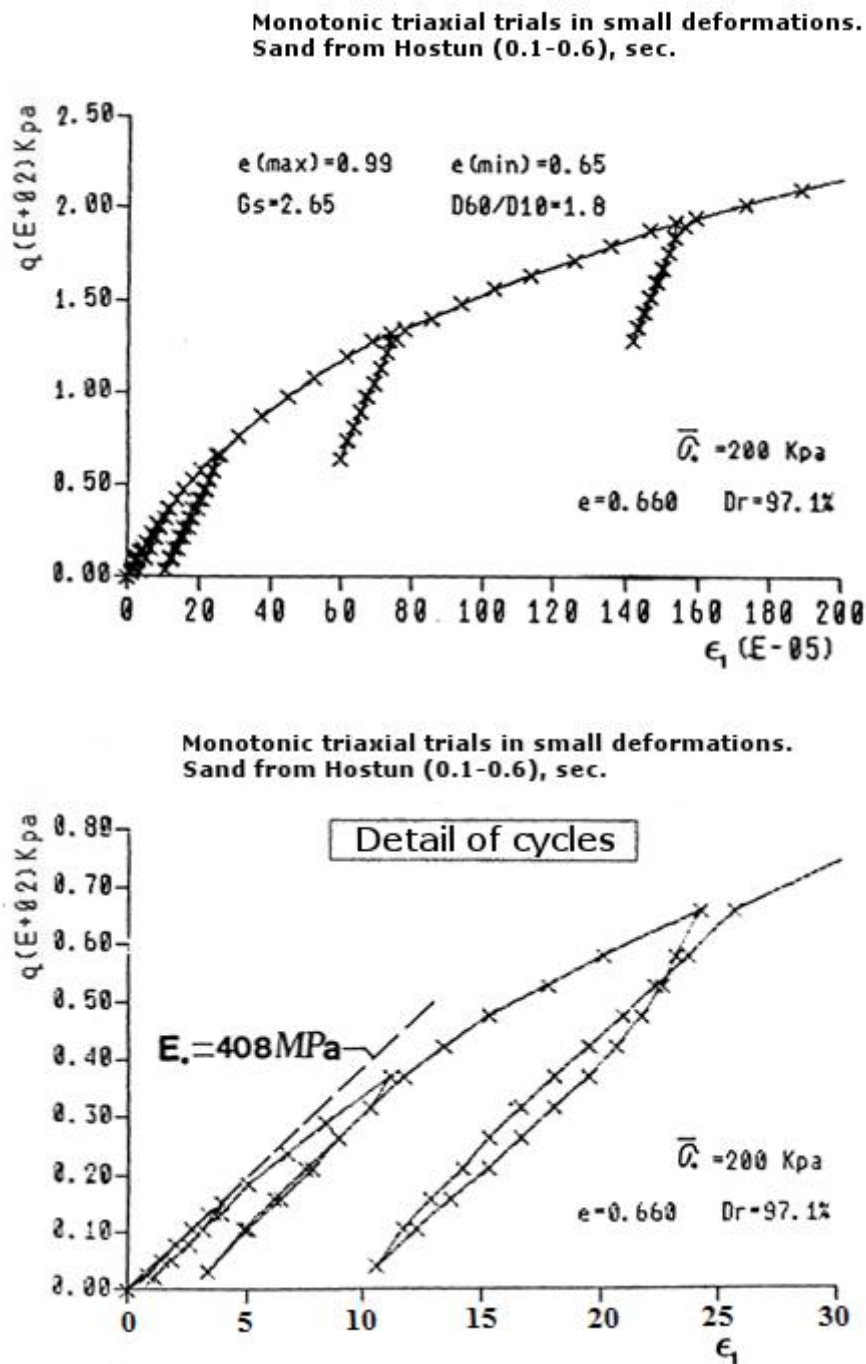


Fig. 12. Monotonic triaxial trials in small deformations (Sand from Hostun $\sigma_0=200 \text{ KPa}$).

5. CONCLUSIONS

In this work, we tried to assess the validity of Duncan's hyperbolic law. We compared the parameter values for each soil type (Kaolinite (Black clay), White Clay and Very Low-Density Sand) obtained by the two methods (SP5-experiment).

It has been observed after comparisons that there is no great difference between the two methods (SP5-experiment) as regards the variation of the initial tangent modulus as a function of the confinement (lateral) pressure and the variation of the volume deformation modulus with the confinement pressure.

After using the SP5 software that follows Duncan's hyperbolic law, the curves obtained by the experiment (using other software) and SP5 are superimposed to make the comparison and continue the research on the validity of the law.

It is noted that the law simulates the experiment well (curves q-eps1) but it underestimates the values of initial tangent modulus E_i (Curves eps1 /q-eps1).

It is clear that the law is far from representing the experiment properly because it reflects the variation in volume.

Then, the variation of the secant modulus is plotted with the axial deformation: it decreases with the axial deformation and is proportional to the lateral pressure.

The Poisson initial coefficient evolution with lateral pressure is presented for the case of very low-density sand. It has been found that the variation of v_1 is presented in the following form:

$$V_1 = G - f_x \log_{10} x \sigma_3$$

where G is the Poisson coefficient value (initial) for the confinement pressure of one atmosphere.

Finally, an investigation undertaken to determine the mechanical properties in the real elastic range or the deformation levels involved, found them to be less than 10^{-3} (small deformation domain). To characterize such behavior, it appears necessary to know two elastic coefficients:

- The Poisson's coefficient ν and
- The Young E module

For the Hostun sand (Rivera tests), a slight decrease in the Poisson coefficient (from 0.20 to 0.15 for $e = 0.664$) is observed following the application of the isotropic pressure cycles (between 25 Kpa and 600 Kpa)

The decay of the Poisson coefficient for the Hostun sand can be explained from the theoretical results established by Deresiewicz (1957) for an assembly of identical elastic grains of the same material, subjected to an initial isotropic stress state, then to a low-intensity deviator stress.

Deresiewicz shows that the Poisson's coefficient does not depend on the initial isotropic stress, but is a function of the ratio between the tangential stiffness and the normal stiffness of the assembly. More precisely, the Poisson's coefficient decreases when the tangential stiffness of the assembly increases.

In the case of the Hostun sand, the application of the isotropic pressure cycles will then increase the number of contacts between soil particles, thus also increasing the tangential stiffness.

The experimental results of the Hostun sand show the non-dependence of the Poisson coefficient on the average isotropic stress.

REFERENCES

- Agnelo I. (1975), "Law of Hyperbolic Soil Behavior." *Thesis ECP 1975*.
- Alamanis N. (1991), "Hyperbolic Law of Duncan. Comparison between SP5 and experience." *ECP Internship Report 1991*.
- Al-Issa. (1973), "Research on stress-strain law of granular materials. Analysis of the validity of hyperbolic laws, application to piles and dams ". *Thesis ECP 1973*.
- Biarez J. and Hicher PY (1989) "Laws of behavior of reworked soils and granular materials", *ECP Paris August 1989*.
- Biarez J. and Hicher P.Y. (1994). *Elementary Mechanics of Soil Behavior-Saturated Remolded Soils. A.A. Balkema, Rotterdam*.

- Boelle J.L. (1983) "Dynamic measurements of the mechanical properties of soils with low deformations", *Thesis ECP 1983*.
- Darbour B., (1975) "Introduction of a hyperbolic law in SAP4." *ECP Internship Report 1991*.
- Duncan J.M and Ghang C.Y, (1970) "Nonlinear analysis of stress and strain in soils", *Journal of the soil mechanics and foundation division, ASCE, vol. 96, No SM5 September 1970*.
- Duncan J.M., Byrne P., Wong K.S., Mabry P. (1980) "Strength, stress – strain and Bulk modulus parameters for finite element analyses of stresses and movements in soil masses". *Department of civil engineering, University of California Berkeley, August 1980*.
- Duncan J.M. and Wong K.S. (1981) "SP5: a computer program for the evaluation of hyperbolic stress – strain parameters" *Department of civil engineering, University of California Berkeley, 1981*.
- Laloui AZ (1989) "Application of the hyperbolic law to black clay. (Kaolinite) " *Internship report ECP 1988-1989*.
- Rivera R. (1988) "Determination of the mechanical properties of sands and clays in dynamic and cyclic regime with weak deformations", *Thesis ECP 1988*.
- Trueba V. (1988) "Study of the mechanical behaviour of saturated clays under three-dimensional stresses", *Thesis ECP 1988*.
- Zervoyanis Ch. (1982) "Synthetic study of the mechanical properties of saturated clays and sands on an oedometric and triaxial path of revolution". *Thesis ECP 1982*.
- Ziani F. "Contribution to the study of soil compaction: a particular case of the behavior of very low density sands". Faculty of Agricultural Sciences of Gatoux, Belgium. *Thesis ECP 1987*.