

CONSTRUCTIVE EFFECT OF SPECTRAL AND LOGICAL REPRESENTATIONS IN THE ANALYSIS OF NEURAL INTERACTIONS

A. N. Sychev

Candidate of Technical Sciences, Associate Professor,
Embassy of Russia in Latvia, Antonias 2, Riga, Latvia, LV-1010a

Abstract

The excitatory and inhibitory synaptic links of the retinal neurons form a mechanism for linear filtration of the visual signal. Synaptic gradient potentials on the dendrites of neurons in the visual cortex control the traffic of postsynaptic action potential, which is modelled by commuting the logical network. The cascade combination of presynaptic inhibitions in the local parts of the central nervous system is represented by a multilocal filter corresponding to an artificial neuron with threshold activation and a structural vector expressed by the Fibonacci series.

Key words: inhibition synapse, filter, commutation network, affine transformation

1. INTRODUCTION

It is possible to agree with the imperfection and weak functionality of the known models of artificial neurons (AN) in hope of obtaining successful solutions due to the application of a large number of such neurons combined in a network of certain structure. Though, unfortunately, traditionally synthesized networks do not always provide effective solutions in a whole range of tasks, and in some cases solutions are completely absent (Zlobin & Ruchkin, 2011).

The emerging difficulties in tuning (training) the neural networks are conditioned by the predominant orientation toward changing the weights of the AN inputs, despite of the fact that these changes provide only a narrow range of functionality.

To overcome the low efficiency of artificial neurons in the work (Sychev, 2011), it was suggested to perform logical transformations along with arithmetic settings on the AN inputs. This proposition naturally follows from the analysis of neural interactions in biological prototypes, whereby appears an obvious need to attract the means of logical algebra.

It is obvious that it is rational to continue to search for more efficient AN models that provide solutions to a wide range of problems with acceptable technical and computational costs.

The purpose of this work is the subsequent development of the logical model of neuron and the analysis of possibilities provided by the application of spectral and logical representations in neural networks modelling.

2. INTERACTION OF RETINAL NEURONS

Retinal neurons perform a wide range of functions to ensure the perception of visual information. One of these functions is the formation and transmission of signals with the participation of the lateral geniculate body in the visual cortex of the brain for effective pattern recognition. In Hubel's research (Hubel, 1988), the principle of action of a specific neuronal mechanism of the retina, formed by ganglion cells, perceiving exciting and inhibitory signals from bipolar cells associated with receptors was analysed. These ganglion cells, named as cells with "on" and "off" centres, provide recognition of a small illuminated portion of the receptive field against the darkened background and conversely: a darkened area against the illuminated background. Fig.1a presents a slightly modified Figure 24 from the Hubel's research (Hubel 1988) explaining the functioning of the cell with the "on" centre. From the positions of the algebra of logic, the table of the linear function "sum modulo 2" for two variables corresponds to this diagram, $f = x_1 \oplus x_2$, which is shown in Fig. 1; if not consider τ - the time delay

of the signal that is unimportant for subsequent reasoning and which occurs when the receptor fields of ganglion cells are inversely activated.

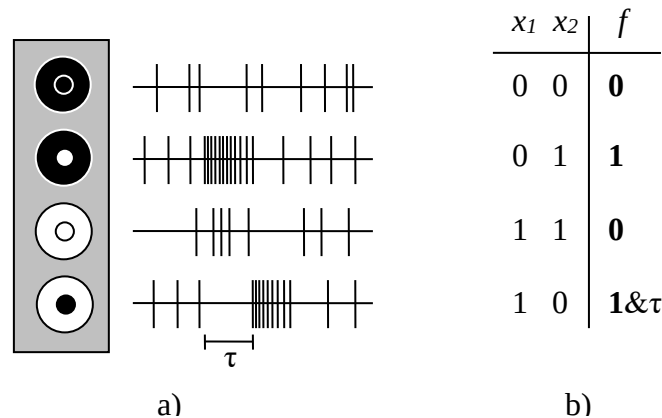


Fig. 1. Functioning of ganglion cells with “on”-centre.

Fig. 1a shows record the responses of ganglion cells with “on” - centre; and Fig.1b demonstrates the table of logical functions “sum modulo 2”: variables x_1 and x_2 are respectively the state of periphery and centre of the receptive field of the cell, “1” and “0” the illumination and absence of illumination of the area of the field, “1” and “0” of the function f correspond to the presence and absence of the series of impulses in responses of ganglion cells

Without going into the detailed analysis, abstracting and generalizing this fact for the effectiveness of neural structures modelling, it is possible to assume that neural mechanisms of retina form the elements of linear filters of the pattern of $\lambda(x) = \omega_1 x_1 \oplus \omega_2 x_2 \oplus \dots \oplus \omega_n x_n \pmod{2}$, where $x_i, i = 1 \div n$, is a binary signal that corresponds to the state of the receptor, ω_i - Walsh function index $wal(\omega_i, x)$, corresponding to a certain spatial frequency. With the contribution of these neural mechanisms, along with the other visual functions completion, spatial linear filtration of the visual signal is carried out. The purpose of that filtering is isolation of the dominant in the spectrum of spatial frequencies characterizing some class of images.

In accordance with these ideas in the research performed by Sychev (Sychev 2011) it is proposed to select linear blocks in artificial neural structures (ANS) - the blocks of linear transformation of input signals. This constructive technique allows not only optimizing the ANS expressed in the reduction of the number of neurons and in the simplification of the connections between them, but also ensures the construction of ANS that unmistakably implement any consistent control or recognition algorithm.

The procedure for constructing linear blocks is based on the application of the mathematical apparatus of Walsh spectral representations. In this case, the computational costs are minimized by applying fast Fourier-Walsh transform algorithms.

3. CONDUCTING THE POTENTIAL OF ACTION IN DENDRITES

The propagation of the postsynaptic action potential (PSAP) over the surface of the neuron membrane is not trivial and unvaried. The research conducted by Nichols et al. (Nichols et al., 2001, p.140) presents that the action potentials are adjacent to the synaptic potentials in the dendritic tree. For example, the reliability factor of the backward propagation of the action potential from the soma depends on the input resistance of the various branches; in its turn, the input resistance depends on the

degree of the excitatory and inhibitory synapses activity. Thus, the reverse propagation of excitation into dendrites depends on synaptic activity.

Thereafter, in the conclusions of Nichols et al. (Nichols et al., 2001, p.141) it is stated that the propagation of the action potential depends strongly on the geometric factors associated with the change in the surface area of membrane. Distribution can be interrupted at the points of branching of the nerve end, and the movement of excitation in branched dendrites can have preferred directions. Consequently, for the information function of neuron that is expressed by the action potential, the geometrical features of the dendrites cell membrane and the features of the spatial placement of nerve endings on them are of significant importance, along with the processes of interaction of gradient postsynaptic potentials.

It should be noted that, according to the facts given above, the synaptic contacts play the role of a key in action potential propagation. Therefore, the geometric factors of membrane and the peculiarities of location of the synapses on it are modelled to a certain extent by the commutation logical network (CLN) of the channel switches (CS) (Sychev 2012, 2014). In this case, the CS itself simulates a synaptic effect, and the connection circuits of the CS outputs and inputs are associated with the PSAP traffic on the membrane surface. The common network output, coupled with the outputs of individual CS, simulates the axon function (Fig.2)

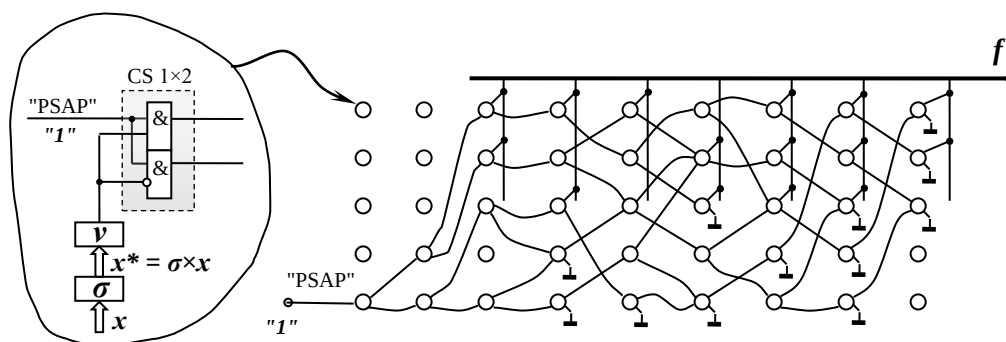


Fig. 2. The approximate scheme of CLN implementing some binary function f .

Fig. 2 demonstrates the approximate scheme of CLN implementing some binary function f ; circles depict neural modules (cells) of CLN containing neurons v and the blocks σ , which carry a matrix of linear convert an argument of the type $\sigma \times x \pmod{2}$, $\sigma = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$, where T is the symbol of transposition.

In general, the network of switches accumulates the signals of the ensemble of control logic devices (neural modules) and forms the reaction of the entire structure. An arbitrary logical function and, consequently, an arbitrary non-contradictory training sequence of a neural network can be implemented by a CLN of a similar type.

The CLN allows an algorithmic construction based on Walsh spectral representations. Preliminary linearization of neurons input signals is a method for the optimization of network as a whole, since the use of multiple linear filters reduces the total number of network elements.

4. PRESYNAPTICAL INHIBITION

To explain the logical construction of an artificial neuron in accordance with the work of Sychev (Sychev, 2017), it is possible to address to biological prototypes. In the research of Nicholls et al. (Nicholls et al., 2001, p.185), the essential importance is given to presynaptic inhibition in the activity of the brain and nervous system. Presynaptic inhibition suggests that the inhibitory axons establish a

synaptic contact with the endings of the excitatory axons. The presence of such axon-axon synapses has been shown in various areas of the central nervous system (CNS) of mammals. Moreover, the inhibitory nerve endings themselves can be influenced at the presynaptic level.

Presynaptic inhibition for one pair of synapses corresponds to the algebra of the logic to the function PROHIBITION. Indeed, let a be a signal of the appearance of postsynaptic action potential (PSAP) in the excitatory axon, b – signal about the appearance of PSAP in the inhibitory axon. In this case, the appearance of the action potential on the ending subjected to presynaptic inhibition is described by the logical function PROHIBITION – $a\bar{b}$. If, however, the inhibitory nerve ending represented by the signal b is subjected to the inhibitory effect of the axon represented by the signal c , then the signal at the end of the excitatory axon a is described by a sequential composition of two functions PROHIBITION – $a\bar{b}\bar{c}$.

Further Nicholls et al. (Nicholls et al., 2001, p.295) notes that GABA (γ -aminobutyric acid) is released in the inhibiting synapses of various regions of the central nervous system. There is no doubt that a large proportion of GABA-containing neurons is presented in the central nervous system. For example, in a number of regions of the cerebral cortex, one of five neurons releases GABA as an inhibitory mediator. It has been established that the conduction blockage (PSAP) is a key moment of integrative processes on the mammalian central nervous system dendrites.

The presence of a large number of inhibitory presynaptic interactions in the central nervous system gives reason to assume the existence of successive chains of neurons with these interactions. The description of these chains can be represented by successive compositions of the PROHIBITION

functions, for example: $f = x_1 x_2 x_3 x_4 x_5 x_6$. Expanding this expression in accordance with the rules of the logical algebra, it is obtained a parenthesis $f = x_1(\bar{x}_2 \vee x_3(\bar{x}_4 \vee x_5 \bar{x}_6))$. This formula, if to ignore the signs of inversion over variables, is $f = x_1(x_2 \vee x_3(x_4 \vee x_5 x_6))$; describing a threshold function with structural vector $A = (p; a_1, a_2, a_3, a_4, a_5, a_6)$ with a threshold p and a set $a_1 \div a_6$ of input weights, is represented by Fibonacci series: $A = (13; 8, 5, 3, 2, 1, 1)$. Indeed, if to assign in accordance with the minor members x_5, x_6 the equal and minimal weights $a_5 = a_6 = 1$, then the weight should be determined as $a_4 = a_5 + a_6 = 2$. Since the variable x_4 is combined within the second brackets by the OR operation with the conjuncture of $x_5 x_6$ and therefore is in equilibrium with it. Furthermore, since for the function f in the first brackets takes place the conjunction $x_3 \bar{x}_4 x_5 x_6$, but not a conjunction of $\bar{x}_3 x_4 x_5 x_6$, it is clear that $a_3 \succ a_4$; consequently, $a_3 = 3$. It is also clear that $a_2 = a_3 + a_4 = 5$. Continuing these arguments with respect to the remaining component a_1 , the Fibonacci sequence received. Threshold value p is determined by weights a_1, a_2 : $p = a_1 + a_2 = 13$.

It follows from these considerations: n – the concatenated function PROHIBITION could be interpreted by a threshold function n variables, which structure vector A containing $n+1$ member of the Fibonacci series.

Simulation of the chains of presynaptic inhibitions is feasible both by the chains of consecutive connections of the PROHIBITION elements and by the chains of combinations of NOT elements and assembly connections. Constructions similar to those shown in Fig. 3 a), b), will be referred to here as Φ -neurons (phi-neurons).

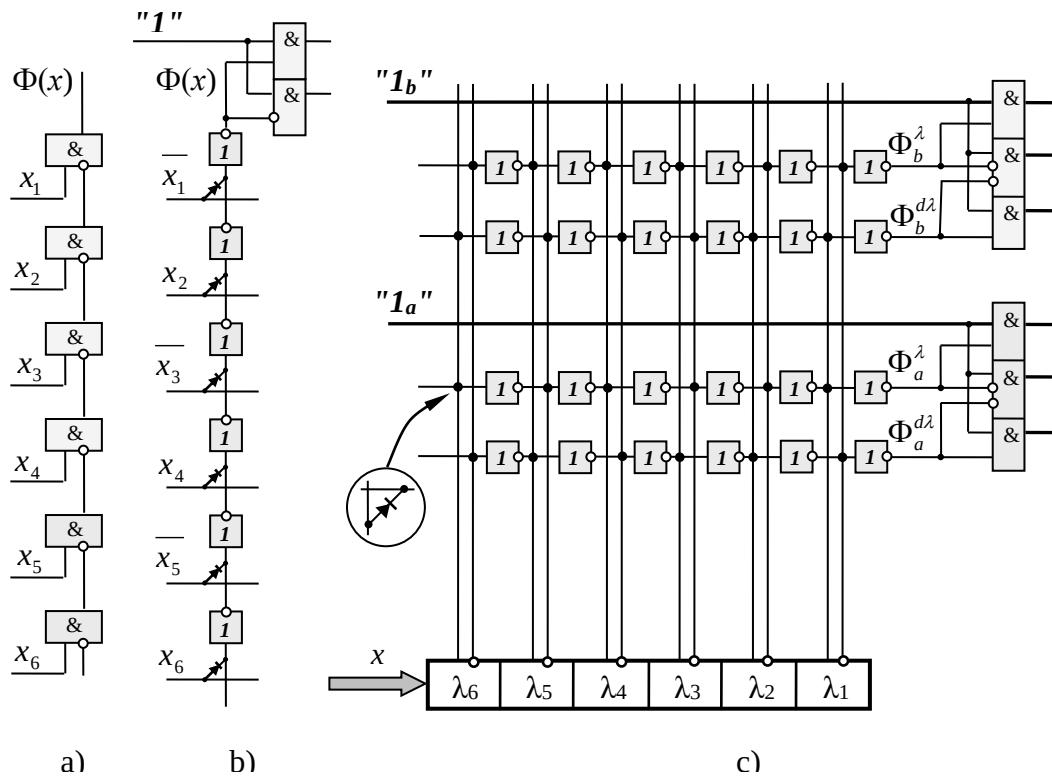


Fig. 3. Design and application of Fibonacci neurons

Fig.3a shows the model of Φ -neuron in the form of compounds of elements of the PROHIBITION; and Fig.3b demonstrates the diagram of switch channels 1×2 and managed Φ -neuron, $\Phi(x) = x_1(x_2 \vee x_3(x_4 \vee x_5 x_6))$ is the connection of diodes and inverters; Fig. 3c relates to the scheme of the switches of the channels 1×3 , matrix-managed of dual pairs $(\Phi_a^\lambda, \Phi_a^{d\lambda}), (\Phi_b^\lambda, \Phi_b^{d\lambda})$ Φ -neurons:

The variant in Fig. 3b easily admits the combination in one matrix of the set Φ^λ neurons controlling the switches of one vertical column of the CLN and receiving the signals from one common linear block " $\lambda_1 \div \lambda_n$ ", Fig.3c. The linear block here serves as the input decoder of the matrix; the signal inversions of some neurons, required for affine transformations, in this case are programmed and implemented by the settings of commutations of the corresponding (direct or inverse) output lines of linear blocks. Thus, in Fig. 3c

$$\Phi_a^\lambda(x) = \bar{\lambda}_1(\bar{\lambda}_2 \vee \lambda_3(\lambda_4 \vee \lambda_5 \lambda_6)), \quad \Phi_a^{d\lambda}(x) = \lambda_1(\lambda_2 \vee \bar{\lambda}_3(\bar{\lambda}_4 \vee \bar{\lambda}_5 \bar{\lambda}_6)),$$

$$\Phi_b^\lambda(x) = \lambda_1(\bar{\lambda}_2 \vee \bar{\lambda}_3(\lambda_4 \vee \bar{\lambda}_5 \bar{\lambda}_6)), \quad \Phi_b^{d\lambda}(x) = \bar{\lambda}_1(\lambda_2 \vee \lambda_3(\bar{\lambda}_4 \vee \lambda_5 \lambda_6)),$$

Where $\lambda_i = \lambda_i(x) = \lambda_{i1}x_1 \oplus \lambda_{i2}x_2 \oplus \dots \oplus \lambda_{in}x_n, i = 1 \div 6, \bar{\lambda}_i$ - is its inversion

As a consequence, it becomes possible to build a CLN based on channel switches of the " 1×3 " type controlled by the pair of dual Φ^λ neurons.

Figure 4 shows the combined Karnaugh table for the functions of the dual pair (Φ, Φ^d) –neurons. It should be noted that each of the fragments $X^\Phi = \{x : \Phi(x) = 1\}$, $X^{\Phi^d} = \{x : \Phi^d(x) = 1\}$ occupies the third part of the total domain of definition X . Similarly, for functions with n variable power of fragments $|X^\Phi| = |X^{\Phi^d}| = \lfloor 2^n / 3 \rfloor$, where $\lfloor t \rfloor$ is a maximal integer that does not exceed t . This

positively distinguishes them from fragments $X^\varphi = \{x : \varphi(x) = 1\}$ of local filters $\varphi(x) = \prod_{i=1}^k \lambda_i(x)$ with the multiplicity factor k , as $|X^\varphi| = 2^{n-k}$.

Essentially, if to expand the parenthesized form of the record $\Phi^\lambda(x)$, linearized Φ^λ neuron is a *multi-local filter* $\psi(x) = \bigvee_{i=1}^k \varphi_i(x)$, where the local filter is $\varphi_i(x) = \prod_{j=1}^m \lambda_j(x)$. For example, in case $n = 6$,

$$\Phi^\lambda(x) = \psi(x) = \varphi_1(x) \vee \varphi_2(x) \vee \varphi_3(x) = \lambda_1(x)\lambda_2(x) \vee \lambda_1(x)\lambda_3(x)\lambda_4(x) \vee \lambda_1(x)\lambda_3(x)\lambda_5(x)\lambda_6(x)$$

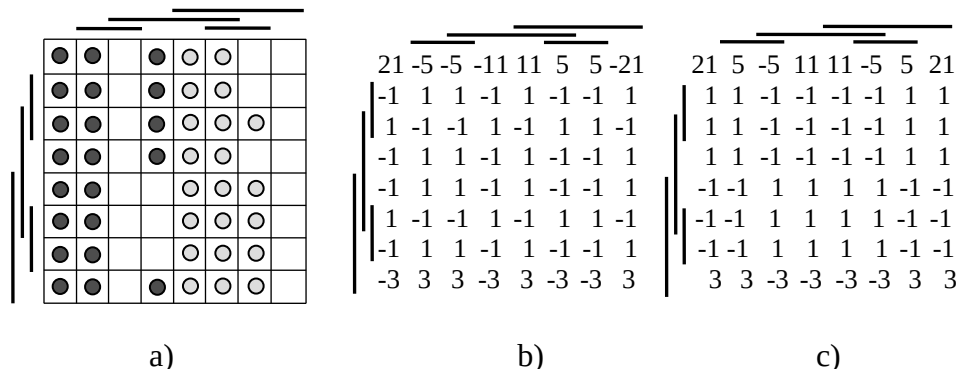


Fig. 4. Tabular representations of functions and spectra of Fibonacci neurons

Fig.4a shows combining tables of Karnaugh functions of Φ and Φ^d neurons: $\Phi(x) = x_1(x_2 \vee x_3(x_4 \vee x_5 x_6))$, $\Phi^d(x) = \bar{x}_1(\bar{x}_2 \vee \bar{x}_3(\bar{x}_4 \vee \bar{x}_5 \bar{x}_6))$ the values of the function $\Phi(x)=1$ is represented by grey circles, functions of a dual neuron $\Phi^d(x)=1$, by black ones;

Fig. 4b demonstrates the table of Karnaugh spectrum $S_\Phi(\omega)$ of the function Φ ;

Fig. 4c demonstrates the table of Karnaugh spectrum $S_{\Phi^d}(\omega)$ of the function Φ^d ;

Spectra S_Φ, S_{Φ^d} of dual functions $\Phi(x), \Phi^d(x)$ are related by $S_{\Phi^d}(\omega) = (-1)^{|\omega|} S_\Phi(\omega)$, where $|\omega|$ is the number of unit bits of the argument ω .

5. CONCLUSIONS

The above presented examples demonstrate the effectiveness of the spectral-logical concept of constructing the artificial neural structures. The functional capabilities of ANS increase over time, the mathematical apparatus of analysis and synthesis becomes more accurate and adequate. The structures themselves acquire an easily perceived modular form, they are close to configurable homogeneous environments and they are convenient both in hardware and in software appliances.

In connection with the rejection of the arithmetic model in calculating the weighted sum of the input signals of neurons, it is possible to avoid performing numerous summation operations with accumulation. The centre of gravity in the calculations of the neuron response is transferred to linear filtering operations for which the summation is performed in the residue field modulo 2, where no accumulation of the result is required. As a result, the action speed of the ANS increases.

The speed of ANS also increases in connection with the use of commutation system for the formation of the ensemble of paralleled neurons reaction. A high degree of parallelism increases the reliability of the ANS as a whole.

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