BENDING ANALYSIS OF SYMMETRIC LAMINATED COMPOSITE BEAMS USING STACKING SEQUENCE OPTIMIZATION

Fatih Karacam, Taner Timarci

Trakya University, Faculty of Engineering, Department of Mechanical Engineering,
22030, Edirne, Turkey

Abstract

The bending analysis of symmetric laminated composite beams is carried out using stacking sequence optimization for various boundary conditions. Stacking sequences are optimized in order to obtain certain parameters such as minimum deflection, normal and shear stresses. A unified shear deformation theory and a genetic algorithm are used in the analytical method and optimization process respectively. Stacking sequences representing the individuals of each population are used as the optimization parameters. An elitist approach is applied to the algorithm in order to increase the convergency to optimum values.

Key words: *bending of composite beams, symmetric laminated composite beams, stacking sequence optimization, genetic algorithm*

1. INTRODUCTION

Composite materials have become significant in many engineering fields due to the high elasticity modulus-weight, strength-weight ratios and applicability to any complex structures. Composite structures are generally constructed of many thin layers. Due to the mechanical properties changing from layer to layer and difficulties in the manufacturing processes, composite structures are generally constructed symmetrically with respect to the axis passing through the mid-plane.

Along other optimization techniques, genetic algorithm is a common one used in the composite structures. Depending on the type of structure, many design parameters can be taken into account such as number of layers, layer thicknesses, stacking sequences, material and geometrical properties. Owing to the fact that too many design parameters in the optimization process will bring about too many solutions. Instead of dealing with the whole possible solutions, using an evolutionary optimization technique will be inevitable to obtain the optimum values. Genetic algorithm (GA) is widely used in the design of laminated composite structures that have many variables and different fitness functions. Walker and Smith investigated the bending analysis of eight layered composite plates by a finite element method and minimized a fitness function depending on weight and deflection parameters by use of genetic algorithm (Walker and Smith, 2003). Almeida and Awruch used GA in the optimization of composite structures by using genetic operators effectively in the design process and compared the results with the ones obtained by finite element method (Almeida and Awruch, 2009). Lopez et al developed a GA to pursue the optimization of hybrid laminated composite structures. They considered the fiber orientation, material and total number of plies as design variables and investigated the maximum stresses (Lopez et al, 2009). Li et al used a shear-deformable beam theory to model the coupled bending-torsional vibrations of symmetric laminated composite beams. They extensively investigated the influences of Poisson effect, stacking sequences and boundary conditions on the natural frequencies of symmetric laminated beams (Li et al, 2015). Liu searched for an optimization technique to find the optimum width and depth in order to minimize the mass of a composite beam under the constraints of stiffness, strength and delamination failure (Liu, 2016).

In the study, on the basis of a unified shear deformation beam theory (Soldatos and Tımarcı, 1993), bending analysis of symmetric laminated composite beam is carried out for various boundary conditions. Stacking sequences are optimized so as to minimize the targeted parameters in each generation. The minimum deflection, normal and shear stresses and corresponding stacking sequences are obtained for a specific number of population.

2. ANALYTICAL METHOD

The beam is assumed to be constructed of arbitrary number of linearly elastic layers with a rectangular cross-section and subjected to a uniform distributed transverse load "$q(x)$" on its top plane, unit width, length of "L" and thickness of "h" (Figure 1). The coordinate system is placed in the mid-plane where $0 \leq x \leq L$ and $-h/2 \leq z \leq h/2$.

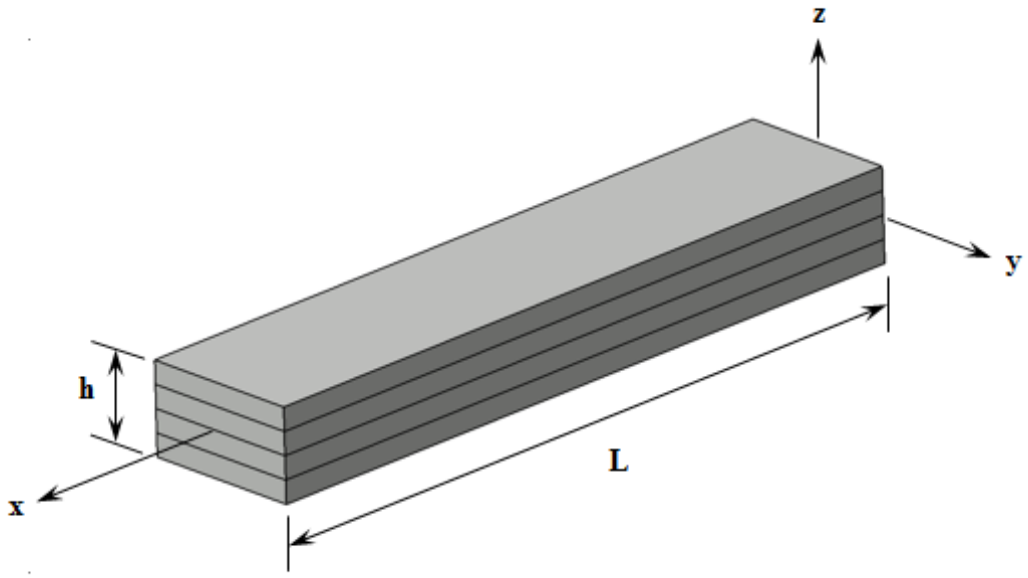


Figure 1. Beam geometry and coordinate system

In the study, a uniform shear deformation beam theory is used at which classical and shear deformation theories can be obtained as a special case by use of a shape function. The displacement fields are used in parallel with general shear deformation theory (Soldatos and Timarci, 1993) and given as

$$\begin{aligned} U(x,z) &= u(x) - z\, w_{,x} + \emptyset(z)\, u_1(x) \\ W(x,z) &= w(x) \end{aligned} \tag{1}$$

where the terms of "U", "W" represent the displacement fields along x and z axis, "u", "u_1" and "w" represent the displacement fields of the mid-plane. ",x" denotes the differentiation with respect to x. In the study a parabolic shape function ($\emptyset(z)$) is chosen that satisfies stress-free conditions at the top and bottom surfaces and continuity of interlaminar stresses through the thickness of the beam.

$$\emptyset\,(z) = z\left(1 - \frac{4z^2}{3h^2}\right) \tag{2}$$

The displacement fields yield to the following kinematic relations.

$$\begin{aligned} \varepsilon_x &= u_{,x} - z w_{,xx} + z\left(1 - \frac{4z^2}{3h^2}\right) u_{1,x} \\ \gamma_{xz} &= \left(1 - \frac{4z^2}{h^2}\right) u_1 \end{aligned} \tag{3}$$

Hooke's Law for the stresses of each k[th] layer is given as follows. The terms of "$Q_{ij}^{(k)}$" are the well-known reduced stiffnesses depending on material properties (Jones, 1975).

$$\begin{bmatrix} \sigma_x^{(k)} \\ \tau_{xz}^{(k)} \end{bmatrix} = \begin{bmatrix} Q_{11}^{(k)} & 0 \\ 0 & Q_{55}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \gamma_{xz} \end{bmatrix} \tag{4}$$

Using stress-strain relations into force and moment definitions (Timarci and Soldatos, 1995), the constitutive equations are obtained as

$$\begin{bmatrix} N_x \\ M_x \\ M_x^a \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & B_{111} \\ B_{11} & D_{11} & D_{111} \\ B_{111} & D_{111} & D_{1111} \end{bmatrix} \begin{bmatrix} u_{,x} \\ -w_{,xx} \\ u_{1,x} \end{bmatrix}, \; Q_x^a = A_{55}\, u_1 \tag{5}$$

where "A_{ij}", "B_{ij}" and "D_{ij}" denote the material rigidities of extensional, coupling and bending respectively. The superscript "a" stands for the shear deformation effects. "Q_x^a" is the resultant transverse shear force. Rigidities with more than two indices correspond to shear deformation theories where the ones with two indices correspond to classical theories. The governing equations of the bending behaviour of laminated composite beams under a uniform distributed load are given as follows:

$$\begin{aligned} N_{x,x} &= 0 \\ M_{\,x,xx} &= q(x) \\ M_{x,x}^a - Q_x^a &= 0 \end{aligned} \tag{6}$$

Using the constitutive equations into governing equations, a set of differential equations and their derivatives are obtained as follows:

$$\begin{aligned} A_{11}u_{,xx} - B_{11}w_{,xxx} + B_{111}u_{1,xx} &= 0 \\ B_{11}u_{,xxx} - D_{11}w_{,xxxx} + D_{111}u_{1,xxx} &= q(x) \\ B_{111}u_{,xx} - D_{111}w_{,xxx} + D_{1111}u_{1,xx} - A_{55}u_1 &= 0 \end{aligned} \tag{7}$$

Simply supported, cantilever and free boundary conditions prescribed at both edges of the beam where $x = 0$ and $x = L$ are given respectively as follows:

$$\begin{aligned} N_x = w = M_x = M_x^a &= 0 \\ u = w = w_{,x} = u_1 &= 0 \\ N_x = M_x = M_{x,x} = M_x^a &= 0 \end{aligned} \tag{8}$$

By integrating and solving the equations (7) simultaneously and applying boundary conditions at both edges, three unknown functions with eight integration constants (C_k) are obtained as below:

$$\begin{aligned} u_1(x) = C_1 e^{-px} + C_2 e^{px} - (qx + C_3)\frac{D_{111}}{A_{55}D_{11}} &= 0 \\ u(x) = -\frac{B_{111}}{A_{11}}u_1(x) + C_7 x + C_8 &= 0 \\ w(x) = \frac{D_{111}}{pD_{11}}\left[-C_1 e^{-px} + C_2 e^{px} - \frac{p}{A_{55}}\left(\frac{qx^2}{2} + C_3 x\right)\right] + \frac{1}{D_{11}}\left(\frac{qx^4}{24} + C_3\frac{x^3}{6}\right) + C_4\frac{x^2}{2} & \\ + C_5 x + C_6 = 0 & \end{aligned} \tag{9}$$

"p" is a coefficient depending on material rigidities and defined as follows:

$$p = \sqrt{\frac{-A_{55}A_{11}D_{11}}{D_{111}^2 A_{11} - D_{1111}A_{11}D_{11}}} \tag{10}$$

3. OPTIMIZATION METHOD

Genetic algorithm is an evolutionary optimization technique using Darwin's principal of survival of the fittest. It's a guided random search technique that works on a population of designs. The principals of GA are firstly proposed by John Holland in optimization problems (Holland, 1995). Early applications of GA to structural optimization are due to Goldberg (Goldberg, 1989) and Hajela (Hajela, 1990). GA starts with a random initial population of possible design alternatives. Genetic operators such as reproduction, crossover and mutation are applied to the algorithm in order to increase the convergency rate and decrease the processing time. Genetic operators are applied until the stopping criteria or the targeted value is obtained. In the study, initially the fiber orientation angles of each layer are coded in order to build up the stacking sequences that correspond to chromosomes of each generation. After coding, a random population of stacking sequences which is also called as the initial population is generated. Reproduction is the first genetic operator at which the population is generated and ranked. Then crossover is applied to generate new individuals by swapping one or more genes which correspond to the fiber orientation angle of a chromosome with another. Crossover ratio may differ related to the number of layers. The last genetic operator used in GA is mutation which is generally used to maintain the genetic diversity from one generation to another. Basically it depends on altering one or more genes of a chromosome among all population. The mutation ratio is chosen between 0,01 and 0,001 in order to prevent the negative influence over the current fitness values in population (Goldberg, 1989). A GA flowchart used in the study is given in Figure 2.

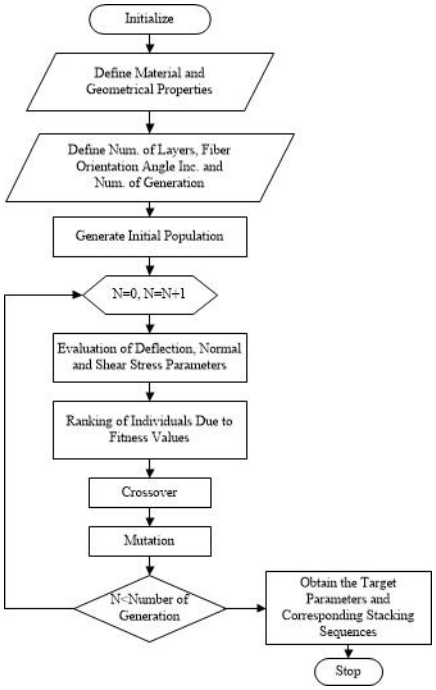


Figure 2. Genetic algorithm flowchart

In the optimization process, the minimization of maximum deflections (w), normal (σ_{xx}) and shear (τ_{xz}) stresses are carried out by chosing the stacking sequences as design parameters where "$\theta^{(k)}$" represents the fiber orientation angles and "k" represents the number of layers respectively. Thus, the optimization problem may be written in a standart form as follows:

$$\begin{aligned} &\text{Find} && [\theta^{(k)}] \\ &\text{Minimize} && Max\ (w, \sigma_{xx}, \tau_{xz}) \\ &\text{Subject to} && 0° \leq \theta \leq 90° \end{aligned} \tag{11}$$

4. NUMERICAL RESULTS

The beam is assumed to be constructed of graphite/epoxy and under a uniform distributed load of $q(x) = 1000$ N/m. The length and thickness of the beam are chosen as $L = 1$ m and $h = 0{,}02$ m respectively. Mechanical properties of graphite/epoxy are given as follows (Karama et al, 2003): $E_{11} = 241{,}5$ GPa, $E_{22} = E_{33} = 18{,}89$ GPa, $G_{23} = 3{,}45$ GPa, $G_{12} = G_{13} = 5{,}18$ GPa, $\vartheta_{23} = 0{,}25$, $\vartheta_{12} = \vartheta_{13} = 0{,}24$.

The optimization process is carried out for a population of 500 individuals along 20 generations. In order to ensure that the design parameters are converging to a minimum or not, they are computed for three times by means of changing the fiber orientation angles with an increment of 15°, 30° and 45° between $0° \leq \theta \leq 90°$. The minimization of the deflection parameters is carried out at certain points of the beam for various boundary conditions where the maximum deflections may occur. For clamped-clamped (C – C) and simply supported (S – S) boundary conditions, the deflection parameters are obtained at $x = L/2$ and for clamped-free (C – F) boundary condition, they are obtained at free edge of the beam where $x = L$. The minimization of the stress parameters is carried out at certain sections of the beam where the maximum moments and shear forces may occur. For C – C and C – F, normal stress parameters are obtained at $x = 0$ and $z = -h/2$. They are obtained at $x = L/2$ and $z = -h/2$ for S – S. Shear stress parameters are obtained at $x = L/4$ and $z = 0$ for C – C and C – F and at $x = 0$ and $z = 0$ for S – S respectively. In Table 1, minimum deflection values and corresponding stacking sequences are given for various boundary conditions and number of layers. The subscript "s" in stacking sequences stands for the symmetric case. For all boundary conditions and number of layers, the minimum deflection value is generally obtained in the stacking sequence that has a fiber orientation angle of 0° in all layers.

Table 1. Minimum deflection values and corresponding stacking sequences for various boundary conditions and number of layers

Boundary Condition	Number of Layer	Increment	w ($\times 10^{-5}$) [m]	Stacking Sequence
CC	8	15°	1,7523	$[0°/0°/0°/0°]_s$
		30°	1,7927	$[0°/0°/0°/30°]_s$
		45°	1,7523	$[0°/0°/0°/0°]_s$
CF	8	15°	81,0141	$[0°/15°/0°/15°]_s$
		30°	77,8637	$[0°/0°/0°/0°]_s$
		45°	77,8637	$[0°/0°/0°/0°]_s$
SS	8	15°	10,2789	$[0°/30°/45°/60°]_s$
		30°	12,6005	$[0°/90°/60°/90°]_s$
		45°	8,7860	$[0°/0°/45°/45°]_s$
CC	16	15°	1,7523	$[0°/0°/0°/0°/0°/0°/0°/0°]_s$
		30°	1,7760	$[0°/0°/0°/0°/0°/0°/0°/60°]_s$
		45°	1,7523	$[0°/0°/0°/0°/0°/0°/0°/0°]_s$
CF	16	15°	91,5695	$[0°/0°/30°/30°/15°/30°/15°/0°]_s$
		30°	87,3716	$[0°/30°/0°/0°/0°/0°/30°/0°]_s$
		45°	77,8637	$[0°/0°/0°/0°/0°/0°/0°/0°]_s$
SS	16	15°	12,6216	$[30°/15°/30°/0°/45°/90°/15°/0°]_s$
		30°	12,1338	$[0°/30°/60°/0°/60°/90°/60°/30°]_s$
		45°	11,7914	$[45°/0°/0°/0°/90°/90°/90°/90°]_s$

For a symmetric laminated composite beam with 16 layers, the variation of minimum deflections with respect to the generation are presented in Figure 3-5 for C – C, C – F and S – S boundary conditions respectively. Due to the randomly generated individuals of initial population with better/worse fitness values, the variation of deflection values in earlier generations are bigger than the following ones for all boundary conditions. The minimum values are obtained for an increment of 45° in each cases.

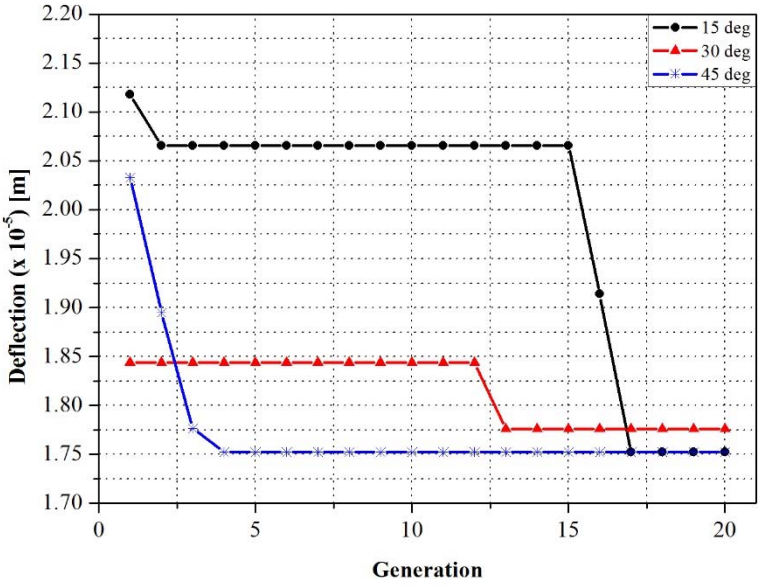


Figure 3. The variation of deflections with respect to the generation for C – C.

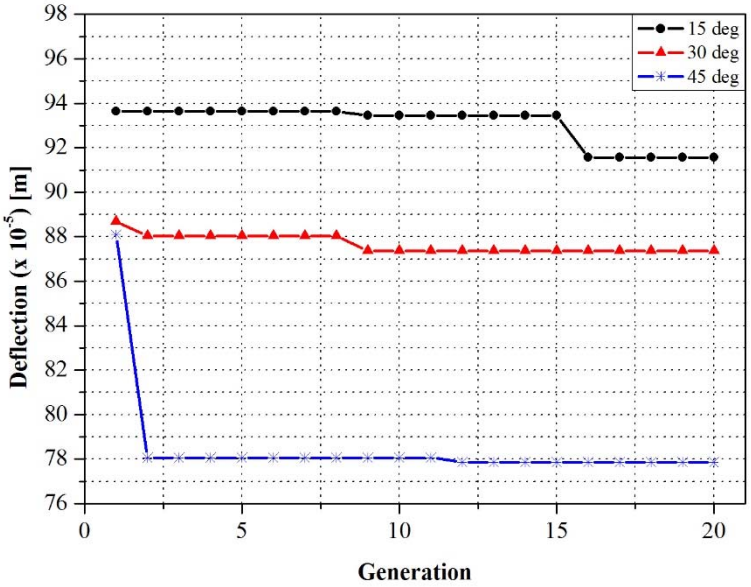


Figure 4. The variation of deflections with respect to the generation for C – F.

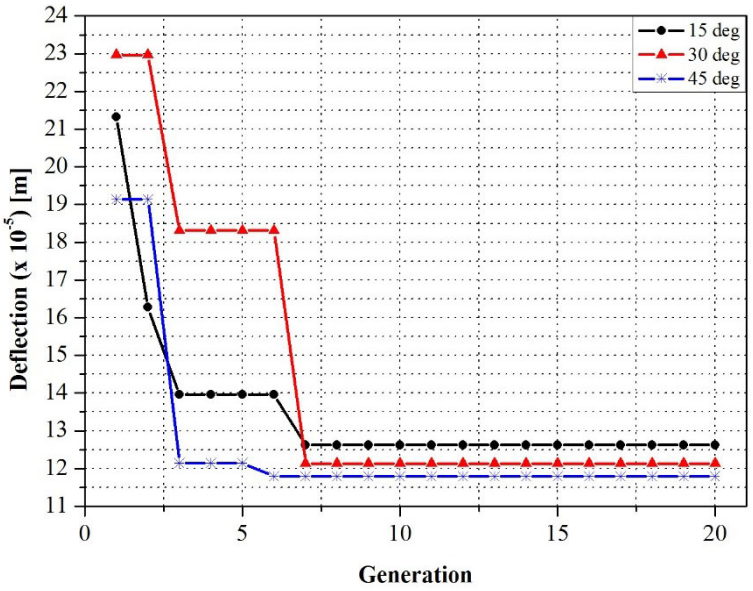


Figure 5. The variation of deflections with respect to the generation for S – S.

In Table 2, minimum normal stress values and corresponding stacking sequences are given for various boundary conditions and number of layers. It's obvious from the table that minimum normal stresses decrease with an increase in the number of layers.

Table 2. Minimum normal stress parameters and corresponding stacking sequences for various boundary conditions and number of layers

Boundary Condition	Number of Layer	Increment	σ_{xx} [MPa]	Stacking Sequence
CC	8	15°	0,3358	$[75°/0°/0°/0°]_s$
		30°	0,3361	$[90°/0°/0°/0°]_s$
		45°	0,3361	$[90°/0°/0°/0°]_s$
CF	8	15°	1,5084	$[75°/0°/0°/0°]_s$
		30°	1,5101	$[90°/0°/0°/0°]_s$
		45°	1,5101	$[90°/0°/0°/0°]_s$
SS	8	15°	0,3911	$[75°/15°/45°/90°]_s$
		30°	0,6071	$[60°/0°/30°/90°]_s$
		45°	0,3634	$[90°/0°/0°/45°]_s$
CC	16	15°	0,2442	$[75°/0°/30°/0°/15°/0°/0°/90°]_s$
		30°	0,2318	$[90°/0°/0°/0°/0°/0°/30°/60°]_s$
		45°	0,2190	$[90°/0°/0°/0°/0°/0°/0°/0°]_s$
CF	16	15°	1,0152	$[75°/0°/0°/0°/0°/0°/15°/75°]_s$
		30°	1,2699	$[90°/0°/30°/30°/30°/0°/0°/60°]_s$
		45°	1,0006	$[90°/0°/0°/0°/0°/0°/0°/0°]_s$
SS	16	15°	0,3156	$[75°/15°/30°/90°/90°/0°/75°/45°]_s$
		30°	0,2847	$[90°/0°/0°/90°/30°/90°/90°/60°]_s$
		45°	0,8173	$[90°/45°/90°/90°/0°/90°/90°/45°]_s$

In Figure 6-8, the variation of normal stresses with respect to the generation are presented for various boundary conditions. The variation of parameters mostly depends on the efficient use of genetic operators in the algorithm. The minimum values are generally obtained for an increment of 45°.

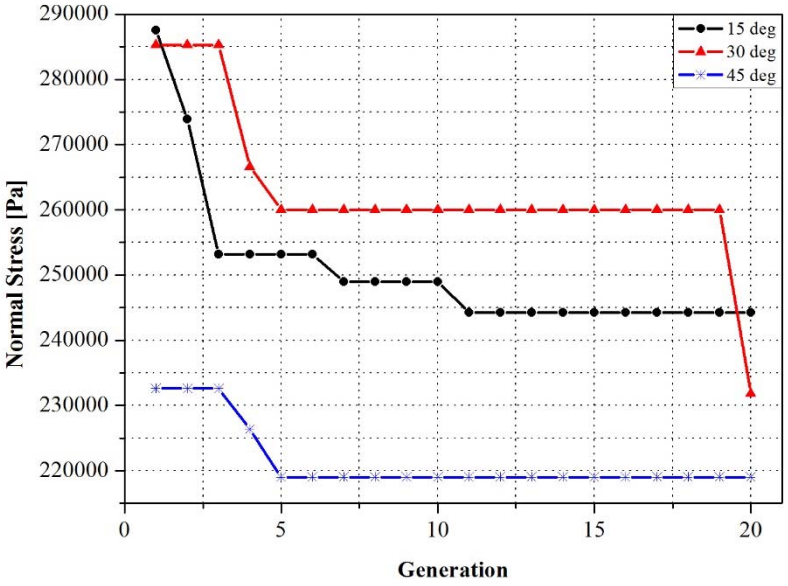


Figure 6. The variation of normal stresses with respect to the generation for C – C.

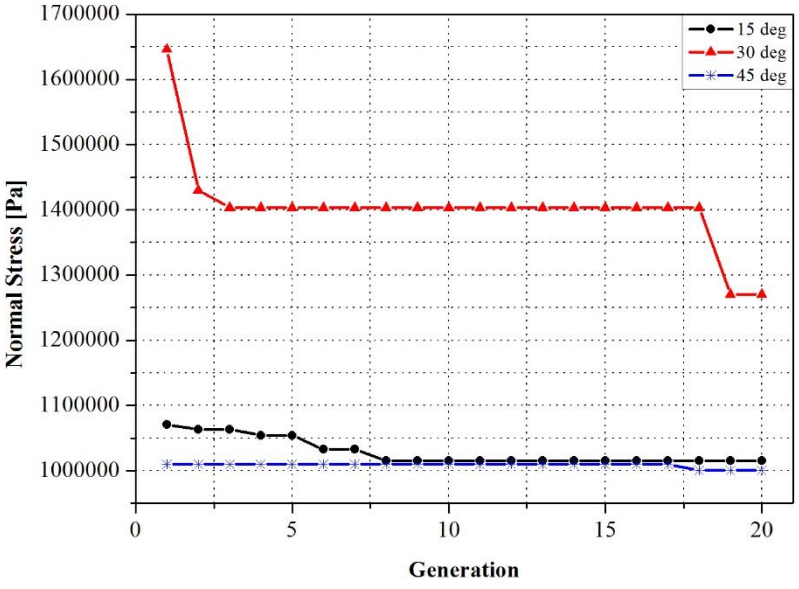


Figure 7. The variation of normal stresses with respect to the generation for C – F.

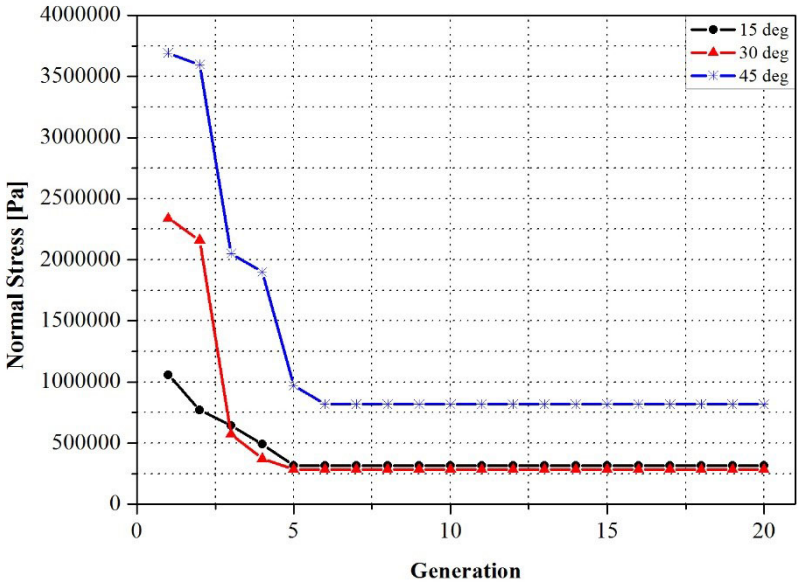


Figure 8. The variation of normal stresses with respect to the generation for S – S.

In Table 3, minimum shear stress values and corresponding stacking sequences are given for various boundary conditions and number of layers.

Table 3. Minimum shear stress values and corresponding stacking sequences for various boundary conditions and number of layers

Boundary Condition	Number of Layer	Increment	τ_{xz} [kPa]	Stacking Sequence
CC	8	15°	10,6814	$[15°/0°/0°/45°]_s$
		30°	10,9374	$[0°/0°/0°/60°]_s$
		45°	10,5589	$[0°/0°/0°/45°]_s$
CF	8	15°	31,6768	$[0°/0°/0°/45°]_s$
		30°	32,8122	$[0°/0°/0°/60°]_s$
		45°	31,6768	$[0°/0°/0°/45°]_s$
SS	8	15°	30,3662	$[30°/75°/75°/15°]_s$
		30°	27,6088	$[0°/0°/30°/60°]_s$
		45°	23,6599	$[90°/90°/0°/90°]_s$
CC	16	15°	9,8267	$[0°/15°/90°/45°/0°/0°/0°/45°]_s$
		30°	10,3398	$[0°/60°/0°/0°/30°/0°/0°/60°]_s$
		45°	9,0288	$[0°/0°/0°/0°/0°/0°/0°/45°]_s$
CF	16	15°	29,0612	$[0°/0°/90°/0°/15°/0°/15°/45°]_s$
		30°	28,2693	$[0°/0°/0°/0°/0°/0°/0°/60°]_s$
		45°	27,6020	$[0°/90°/0°/0°/0°/0°/0°/45°]_s$
SS	16	15°	22,1249	$[0°/90°/60°/15°/60°/15°/15°/15°]_s$
		30°	20,4539	$[0°/0°/30°/0°/60°/60°/90°/60°]_s$
		45°	22,0681	$[90°/0°/90°/0°/45°/0°/0°/90°]_s$

In Figure 9-11, the variation of shear stresses with respect to the generation are presented for various boundary conditions. Unlike from C – C and C – F boundary conditions, there is not a sharp decrease in shear stress parametres in S – S boundary condition. The minimum shear stress parameters are generally obtained between 5th and 10th generations for all cases. The convergency rate can be increased by a larger population but it will also increase the processing time.

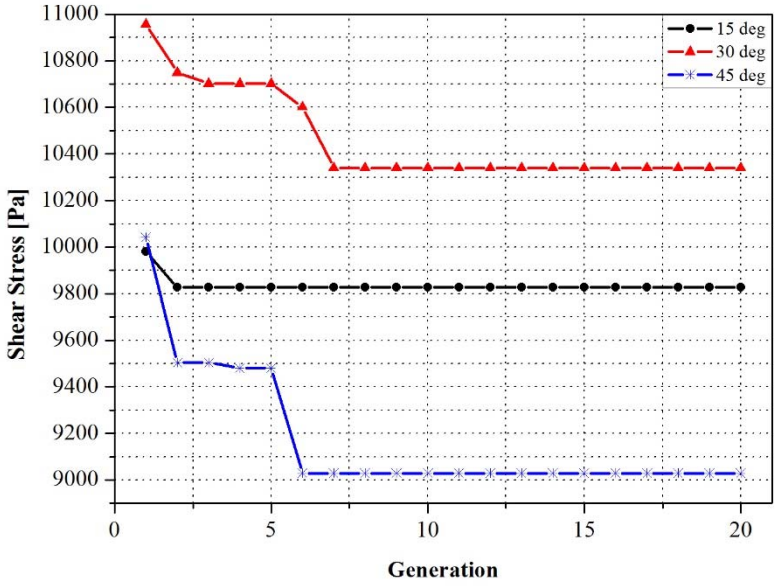


Figure 9. The variation of shear stresses with respect to the generation for C – C.

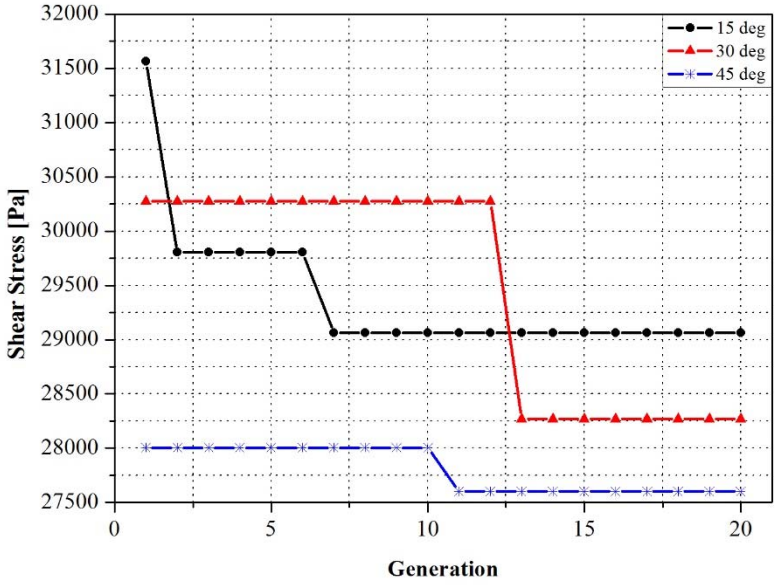


Figure 10. The variation of shear stresses with respect to the generation for C – F.

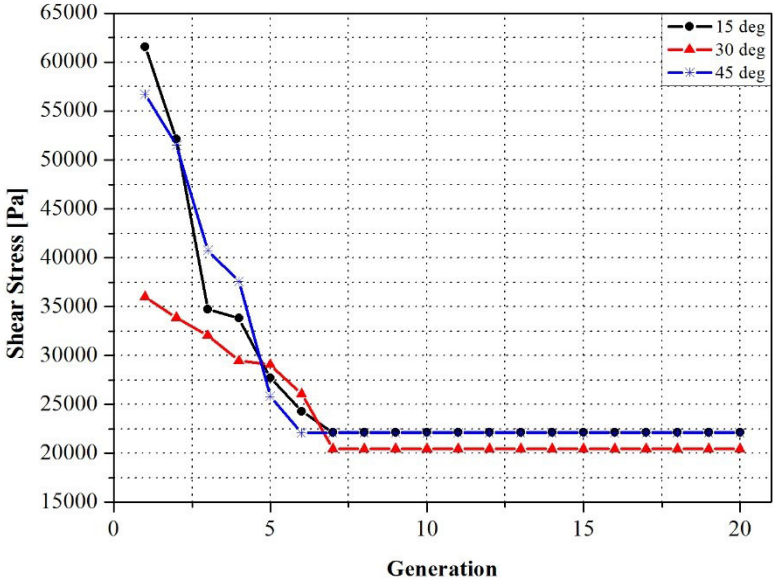


Figure 11. The variation of shear stresses with respect to the generation for S – S.

5. CONCLUSIONS

In the study, bending analysis of symmetric laminated composite beams is carried out by use of stacking sequence optimization for various boundary conditions. GA is used as an optimization technique and elitism is included in order to increase the convergency rate. The optimization problem is considered as a minimization problem of the maximum deflection, normal and shear stress parameters along the number of generation and the stacking sequences are optimized in order to obtain the minimum among all. It is concluded that an improved algorithm by use of elitism will lead to the minimum parameters in earlier generations. Although the variation of design parameters with respect to number of generation vary for each cases, the minimum parameters are generally obtained between 5^{th} and 10^{th} generations with a fiber angle increment of 45°. The convergency rate of the algorithm can be increased by use of a bigger population and number of generation.

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