

ACCURACY EVALUATION OF CAMERA CONTROLLED PINTOGRAPH

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Abstract

Pintograph is a lateral (2D) device that uses a number of rods to move a pen or other tool relative to a flat surface. Known from 19th century, our days pintograph became popular alternative to X-Y Plotters and two-arm robot mechanisms because of its simple mechanical implementations using inexpensive actuators (servo motors) controlled by an inexpensive microcontroller. In attempt to improve '{X, Y} positioning' accuracy of the tool, it is proposed to use digital camera to evaluate real '{X, Y} position of the tool by using special markers, and, then, correct tool' '{X, Y} position. Usage of camera can be useful when accuracy is more important than speed. Results of this research can be used in design of inexpensive laser engravers.

Key words: *pintograph, image processing, laser engraver*

1. INTRODUCTION

Pintograph is a lateral/planar (2D) device that uses a number of rods to move a pen or other tool relative to a flat surface. Known from 19th century, our days pintograph became popular alternative to X-Y Plotters and to planar two-arm robot mechanisms because of its simple mechanical implementations using inexpensive actuators (servo motors) controlled by an inexpensive microcontroller (for example: Arduino, Raspberry Pi, etc.). The following sections contain a basic description of X-Y Plotters; description of typical design of a planar two-arm robot mechanism and its mathematical model; description of the specific design of Pintograph and its mathematical model, and proposed design of a camera-controlled Pintograph.

2. X-Y PLOTTER

In the early computer' days a popular device for drawing a continuous vector graphics on a paper was an X-Y Plotter. A simplified exemplary implementation of an X-Y Plotter is presented on Figure 1. All mechanical elements of the X-Y Plotter are mounted on a rigid flat base (1). X-Motor (2) rotates threaded rod (3). Its rotation, by using threaded nut (4), moves in the X-direction a platform consisting on moving member (5), Y-Motor (6), which motor rotates a threaded rod (7), and thus, moves in the Y-direction a threaded nut (8) and a tool assembly (9). Optionally, tool assembly (9) may contain a means to move the tool (for example, pen or laser) in the Z-direction (up and down). Described design enables to move the tool to nearly any point of the base (1) by operating X-Motor (2) and Y-Motor (5).

This design (in case of proper implementation) enables '{X, Y} accuracy better than 0.1 mm. X-Y Plotter can be used to draw on the paper practically any figures by using any available pen. It can be used as an alternative to printer. Control equations and software design are trivial: as X-position of the tool, as Y-position of the tool, are controlled independently by proper operation of the relevant motor. In most designs, easily controlled by digital means stepper motors are used. In case of high accuracy motors and high accuracy threaded rods usage, resulted '{X, Y} position is accurate enough without feedback of any kind. Additional usage of X-Y Plotter is by using laser to engrave some pattern on the different materials like plastics, metals, etc. In this case, accuracy of the X-Y Plotter is expected to be better than 0.1 mm. In order to achieve higher accuracy, high precision mechanical components are to be used. However, the need to use high precision mechanical components leads to high weight and to high cost of the classic X-Y Plotter. When effective drawing/burning size became bigger than A4, this cost became prohibitive at least for the X-Y Plotters designed for small office or for a home usage.

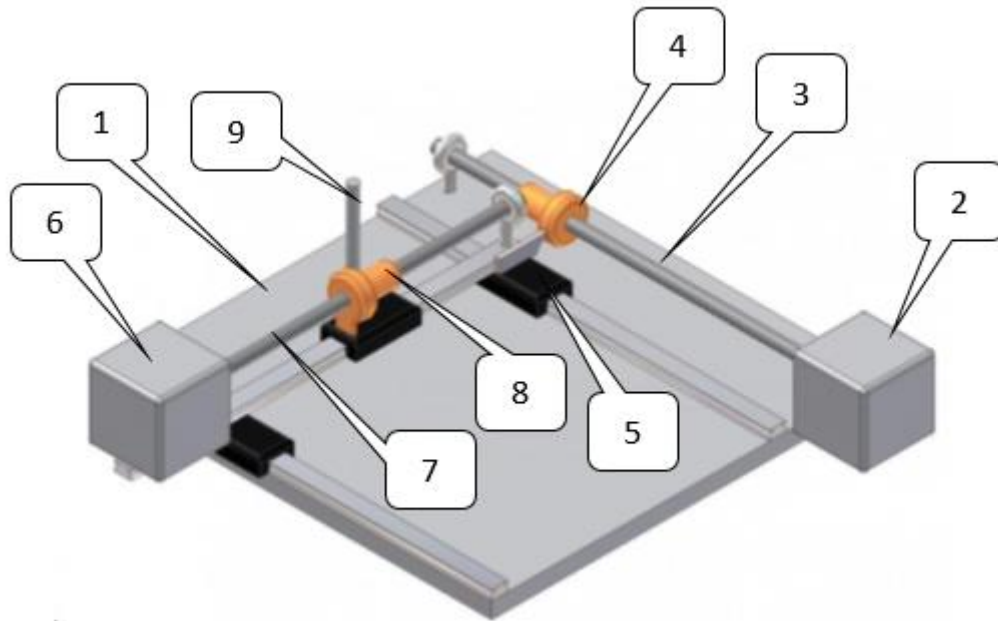


Fig. 1. Simplified exemplary implementation of an X-Y Plotter (Gutierrez 2003)

3. TWO-ARM ROBOT MECHANISM

An alternative to the mechanically sophisticated X-Y Plotter is a “two-arm” robot mechanism (also known as a “Two Link Planar Manipulator”). Operation of the two-arm robot mechanism resembles planar operation of human arm (see Figure 2).

Referring the Figure 2, origin (1) of the planar coordinate system (X-axis is marked as (2), and Y-axis is marked as (3)) operates like an axis of the relevant bones assembly (5) rotation. Real construction of the human arm is not that simple (and its operation uses more degrees of freedom), but, as a result, one can use angle θ_1 (6) between X-axis (2) and bones assembly (5) to find $\{X, Y\}$ position of the joint (4). Additional bone (8) is connected to the joint (4) and to the wrist – “tool” (7). Angle θ_2 (9) between bone assembly (5) and the bone (8) can be used to calculate $\{X, Y\}$ position of the “tool” (7). By changing above angles, human arm can position its wrist.

It is well known, that operation of the human arm, wrist and fingers is expected to be controlled by a visual perception system (containing eyes and brain). By visually comparing real position of the “writing tool” relative to the paper with the desired position of the “writing tool”, human can write on the paper in a very accurate way (as it can be seen on Figure 3). More: by using glasses, or even microscope, humans can position small-size tools with even more accuracy – up to microns.

Simplified operation of the planar two-arm robot mechanism (operation of which in some way resemble operation of the human arm as described earlier) is presented on the Figure 4. Its mathematical description uses X-Y coordinate system (X-axis is marked as (1), whereas Y-axis is marked as (2)). At the origin of the X-Y coordinate system positioned Motor #1 (3). This motor rotates rigid rod (4). Angle α between X-Axis (1) and the rod (4) is marked as (5). At the end of the rod (6) positioned motor #2 (6). This motor rotates another rigid rod (7). At the other end of the rod (7) positioned tool assembly (8) – for example, pen or laser. Position of the tool will be later referred as $\{X, Y\}$. By using auxiliary line (9) – continuation of the rod (4) and line (11) (which is parallel with X-Axis), one can define angles β (10) and γ (12). For the following calculations it must be noted that $\alpha = \beta + \gamma$, and hence, $\gamma = \alpha - \beta$.

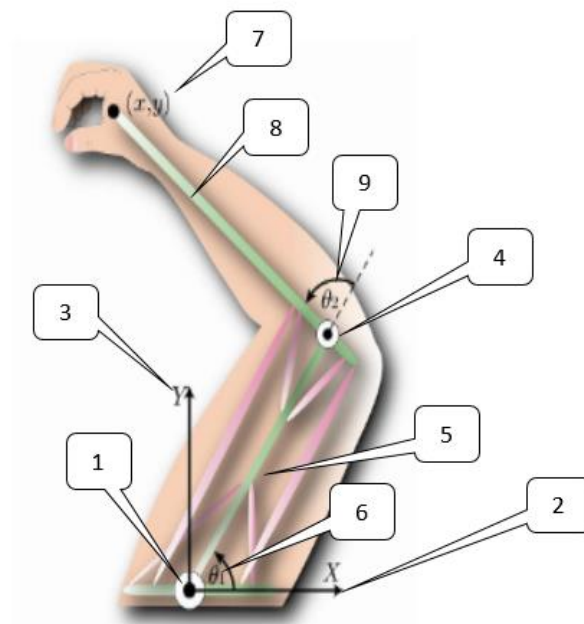


Fig. 2. Simplified planar operation of the human arm. (Kino 2017)



Fig. 3. Visual system (enhanced with glasses) enables to move arm and fingers in a very exact way. (Shutterstock 2017)

In order to control $\{X, Y\}$ position of the tool, angles α and β are to be set by rotating motors. Practically, inexpensive PWM controlled servo motors are used to operate the two-arm robot mechanism. Figure 5 explains operation of the microcontroller' controlled servomotor. Inexpensive servomotor of that kind contains DC Motor, Gear Box, Potentiometer, Pulse Width to Voltage Converter, Comparator and Error Amplifier.

Shaft of the Gear Box is mechanically coupled with the shaft of the potentiometer, so that resistance between middle pin of the potentiometer and bottom pin of the potentiometer linearly encodes angle of the shaft of the Gear Box. When upper pin of the potentiometer is connected to some available voltage (for example, +5V), voltage of the middle pin (V_p) linearly encodes angle of the shaft of the Gear Box, so that potentiometer serves as a Position Sensor. Additionally, shaft of the Gear Box can be mechanically coupled with some load – for example, it can be coupled with rigid rod of the two-arm robot mechanism. To control position of the shaft of the Gear Box, microcontroller generates PWM signal – sequence of pulses, duty cycle of which linearly decodes angle of the Gear Box. PWM Signal by using a Pulse Width to Voltage Converter is converted to a voltage V_{pwm} . Comparator compares above two voltages. In case difference between the voltages is close to the zero, DC motor is not rotated.

When above difference is not zero, DC motor rotates in the direction depending of the sign of this difference, until this difference become zero – classical control with negative feedback.

Obviously, proper positioning of the shaft of the servomotor takes some time. Some designs using servomotors exploits additional pin connected to the middle pin of the potentiometer (Presa 2017). In this case microcontroller can utilize information about “current” position of the shaft of the Gear Box to issue next PWM command only when shaft of the Gear Box is close to the requested position.

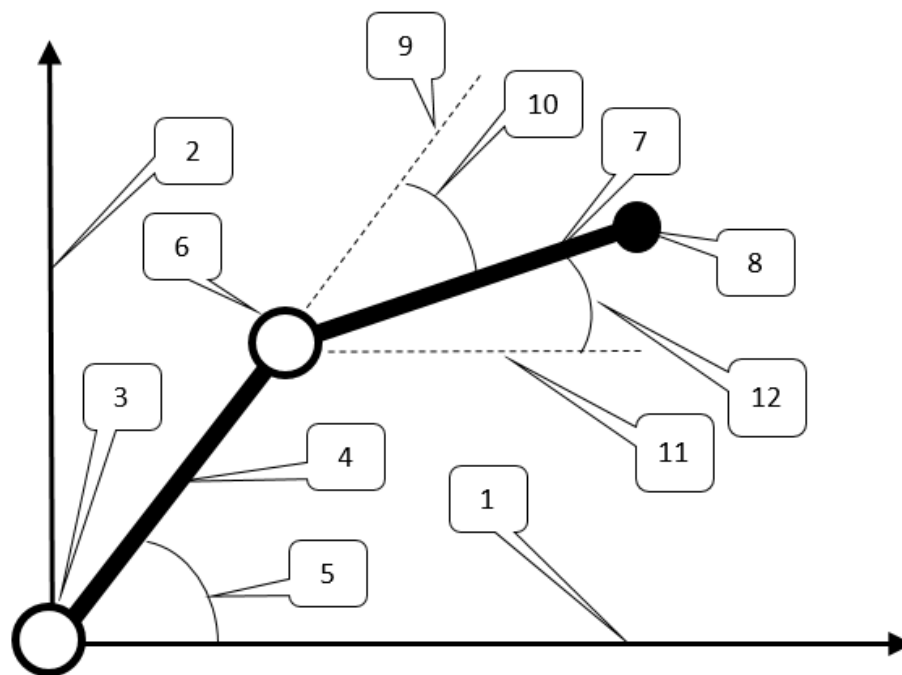


Fig. 4. Planar two-arm robot mechanism

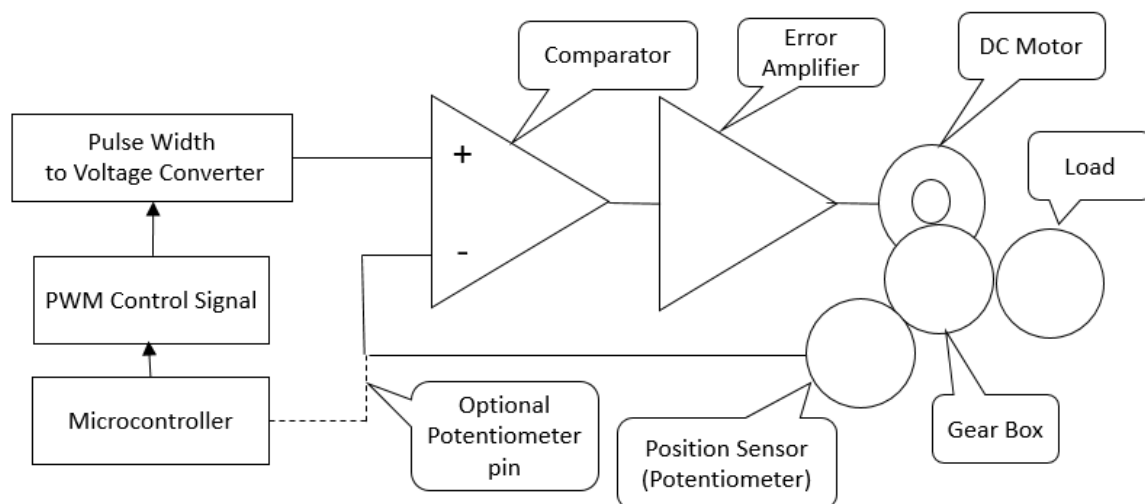


Fig. 5. Operation of servomotor.

Mathematical model of the planar two-arm robot mechanism (as presented on Figure 4) is relatively simple. Equations enabling to calculate $\{X, Y\}$ position of the tool by using two servomotors positions defined as angles (α and β) are presented on Figure 6.

For simplicity of the following discussion the case when $L1 = L2 = L$ (see Figure 7) will be considered later.

$$\begin{aligned}Xl &:= L1 \cos(\alpha) \\Yl &:= L1 \sin(\alpha) \\X &:= L1 \cos(\alpha) + L2 \cos(\alpha - \beta) \\Y &:= L1 \sin(\alpha) + L2 \sin(\alpha - \beta)\end{aligned}$$

Fig. 6. Calculating {X-Y} Position of the tool as a function of two servomotors angles (α (5) and β (10)). $L1$ is a length of rod (4) and $L2$ is a length of rod (7) as referred at Figure 4.

$$\begin{aligned}Xl &:= L \cos(\alpha) \\Yl &:= L \sin(\alpha) \\X &:= L \cos(\alpha) + L \cos(\alpha - \beta) \\Y &:= L \sin(\alpha) + L \sin(\alpha - \beta)\end{aligned}$$

Fig. 7. Simplified equations of Figure 6 for the case $L1 = L2 = L$.

In order to control a planar two-arm robot mechanism in accordance with block-chart presented on Figure 5, we must derive reverse equation: to calculate angles (α and β) that are to be set in order to position tool at the point {X, Y}. To get those equations symbolic software MAPLE was used. MAPLE script used to get requested equations is presented at Figure 8.

```
> Xl := L*cos(alpha);
                                Xl := L cos(alpha)
> Yl := L*sin(alpha);
                                Yl := L sin(alpha)
> Equ1 := X=Xl + L*cos(alpha - beta);
                                Equ1 := X=L cos(alpha) + L cos(alpha - beta)
> Equ2 := Y=Yl + L*sin(alpha - beta);
                                Equ2 := Y=L sin(alpha) + L sin(alpha - beta)
> Solution1 := solve( {Equ1, Equ2}, {alpha, beta});
> assign(Solution1);
> alpha := convert(alpha, radical);
> beta := convert(beta, radical);
```

Fig. 8. MAPLE script used to get α and β as a functions of X and Y

Results of the execution of the script presented on Figure 8 are presented at Figure 9 and at Figure 10. By using those results one can program microcontroller to translate a 2D rout (sequence of requested {X, Y} points) into sequence of angles α and β . By converting this set of angles to sequence of PWM signals operating two servomotors, one can expect that the tool of the planar two-arm robot mechanism presented at Figure 4 will follow the requested 2D route.

> alpha;

$$\arctan \left(-\frac{1}{2} \frac{X \left(X^3 + XY^2 + \sqrt{4L^2 X^2 Y^2 + 4L^2 Y^4 - X^4 Y^2 - 2X^2 Y^4 - Y^6} \right) - X^2 - Y^2}{LY}, \right. \\ \left. \frac{1}{2} \frac{X^3 + XY^2 + \sqrt{4L^2 X^2 Y^2 + 4L^2 Y^4 - X^4 Y^2 - 2X^2 Y^4 - Y^6}}{(X^2 + Y^2)L} \right)$$

Fig. 9. α as a functions of X and Y

> beta;

$$\arctan \left(\left(\left(\frac{1}{2} \frac{1}{X^2 + Y^2} \left((-2X^3 - 2XY^2) (X^3 + XY^2 + \sqrt{4L^2 X^2 Y^2 + 4L^2 Y^4 - X^4 Y^2 - 2X^2 Y^4 - Y^6}) \right) - 4L^2 Y^2 + X^4 + 2X^2 Y^2 + Y^4 \right) (X^2 + Y^2) \right) \right. \\ \left. \right) / \left(L^2 Y (X^3 + XY^2 + \sqrt{4L^2 X^2 Y^2 + 4L^2 Y^4 - X^4 Y^2 - 2X^2 Y^4 - Y^6}) \right), \\ \left. \frac{-2L^2 + X^2 + Y^2}{L^2} \right)$$

Fig. 10. β as a functions of X and Y

By using above calculations, it must be taken into account, that the same {X, Y} position of the tool can be achieved by an alternative motors position (see Figure 11). Alternative position of rod (4) is marked as (8), alternative position of rod (6) is marked as (9), and alternative position of the second motor (5) is marked as (9). To exploit this alternative configuration, term $(\alpha - \beta)$ in the above equations must be replaced into term $(\alpha + \beta)$.

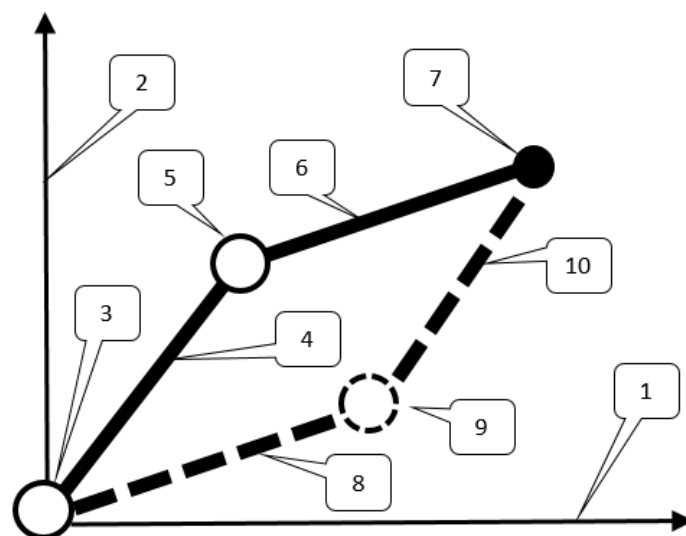


Fig. 11. Ambiguity in the planar two-arm robot mechanism

Serious drawback of the two-arm robotic mechanism is that the first motor, additionally to moving two rods, must move relatively heavy second motor. To prevent this drawback, Pintograph can be used.

4. PINTOGRAPH: MECHANICAL DESIGN AND MATHEMATICAL MODEL

A number of Pintograph designs are known. In this contribution 4-rods design (see Figure 12) will be discussed. Obvious advantage of this design is that two motors are mounted in a static way – only light weighted rods and tool are to be moved.

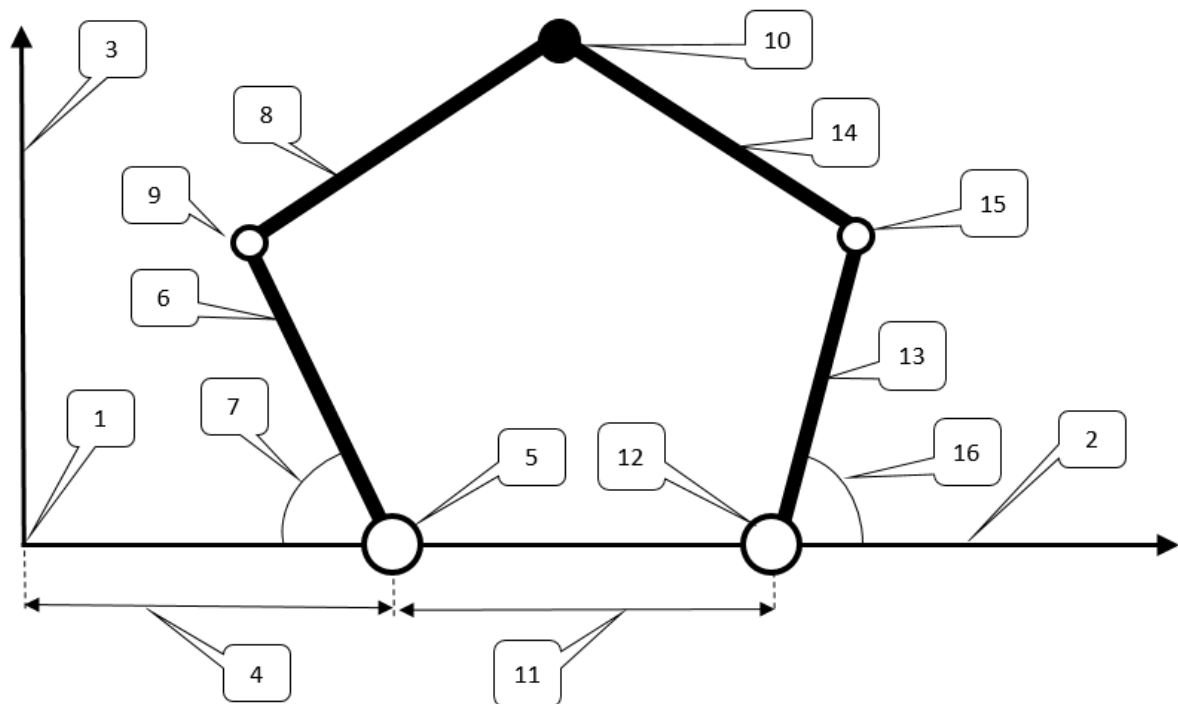


Fig. 12. Exemplary design of Pintograph

Referring to the Figure 12, the following calculations will be executed by using planar X-Y coordinate system relative to an origin point $\{0, 0\}$ marked as (1). X-axis is marked as (2), whereas Y-axis is marked as (3). Motor #1 (5) and Motor #2 (12) are statically mounted on the X-axis (2) – that is, both motors are not moving during operation of the pintograph. Design utilizes four rigid rods marked as (7), (8), (13) and (14). Rods (7) and (8) are connected by using joint (9), whereas rods (13) and (14) are connected by using joint (15). Tool assembly is marked as (10).

In order to enable evaluation of design' accuracy, length of every rod can differ from the length of other rods. This software trick enables to simulate production tolerance and shaft in the motors and in the joints by changing in a pseudo-random way relevant parameters. Parameters of the Pintograph presented at the Figure 12 are summarized in a Table 1. For example, by changing parameters L_1 and L_6 in a pseudo-random way around their designed values, shaft in the motor #1 in the X and Y directions can be simulated (albeit in some indirect way). Modifying in the same way value other parameters, one can simulate shafts in other components of the pintograph.

By using MAPLE symbolic software, mathematical model of the Pintograph was created. In order to get formulae calculating position of the tool $\{X, Y\}$ by the angles "a" and "b" equations presented at Figure 13 can be used (Kosolapov 2016).

Table 1. Parameters of the Pintograph described at the Figure 12.

#	Symbol	Meaning
1	{0, 0}	Origin of the planar X-Y coordinate system
2	X-Axis	X-Axis of the planar X-Y coordinate system
3	Y-Axis	Y-Axis of the planar X-Y coordinate system
4	L1	Distance from origin (1) to the shaft of the Motor #1 (5)
5	(L1, 0)	Position of the shaft of the Motor #1
6	L3	Length of the left bottom rod (6) from the shaft of the Motor #1 to the joint (9)
7	a	Angle between X-Axis (2) and left bottom rod (6)
8	L5	Length of the left upper rod (8) from the joint (9) to the joint of the tool (10)
9	{X1, Y1}	Position of the joint (9)
10	{X, Y}	Position of the tool (10)
11	L2	Distance between shafts of the Motor #1 (5) and Motor #2 (12)
12	{L2, 0}	Position of the shaft of the Motor #2
13	L4	Length of the right bottom rod (13) from the shaft of the Motor #2 to the joint (15)
14	L6	Length of the right upper rod (14) from the joint (15) to the joint of the tool (10)
15	{X2, Y2}	Position of the joint (15)
16	b	Angle between X-Axis (2) and right bottom rod (13)

$$Equl := (x - X1)^2 + (y - Y1)^2 - L5^2 - (x - X2)^2 - (y - Y2)^2 + L6^2 = 0$$

$$X1 := L1 - L3 \cdot \cos(a);$$

$$X1 := L1 - L3 \cos(a)$$

$$Y1 := L3 \cdot \sin(a);$$

$$Y1 := L3 \sin(a)$$

$$X2 := L1 + L2 + L4 \cdot \cos(b);$$

$$X2 := L1 + L2 + L4 \cos(b)$$

$$Y2 := L4 \cdot \sin(b);$$

$$Y2 := L4 \sin(b)$$

Fig. 13. Equations for the Pintograph described at Figure 12

However, while creating the model, ambiguity of the Pintograph design must be taken into account (see Figure 14). It can be seen that for the same angles of the shafts of the motors (5) and (12), two positions of the tool: (10) and (19) are possible. Coping with this ambiguity in the math model is not trivial – forced rejection of the possible solution was required. Fortunately, if, during assembly of the Pintograph upper rods were assembled in such a way that Y of the tool was greater than Y1 and Y2, then Pintograph will never enter into the alternative configuration marked by dash lines.

Because of the big number of parameters used, resulted equations became too long to be presented here. Even when those equations were converted into c# code, printout of this very dense code requires more

than 2 pages, hence resulted equations and the c# code will not be presented here. The c# code was used to create Pintograph simulator. Simulator enable to set parameters listed in the Table 1 and test working zone for the selected set of parameters. Additionally, simulator takes into account discrete nature of the servomotors. As a result, not all points inside the working zone can be achieved. Examples of the working zone will be presented in the RESULTS section.

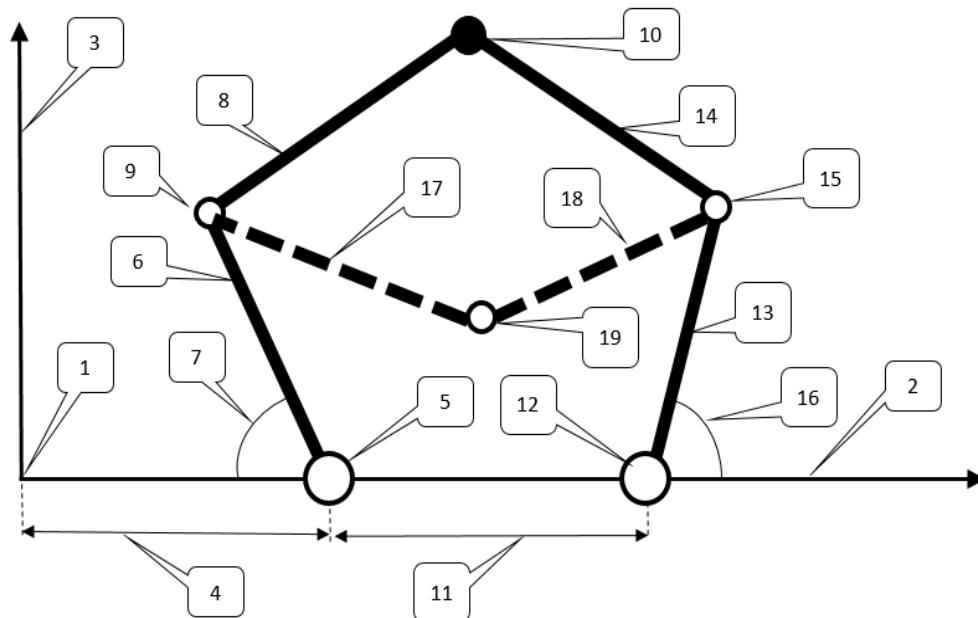


Fig. 14. Ambiguity in the design of Pintograph

5. CAMERA-CONTROLLED PINTOGRAPH

Rationale to use Pintograph instead of X-Y Plotter is that much less precise mechanical elements can be used. However, when extremely inexpensive servomotors and acrylic rods are used, final $\{X, Y\}$ position of the tool may differ from the requested position. In attempt to cope with this problem, it was proposed to use camera-controlled modified servomotors (see Figure 15). It is assumed that shafts of the motors, joints and tool will bear markers clearly distinguished by the digital camera, so that coordinates of centers of the above objects can be evaluated after proper camera calibration. So, during the Pintograph operation two potentiometers of two motors still used to evaluate angles “a” and “b”. However those values are considered only as a rude evaluation. Real-life coordinates of the markers obtained by the camera, connected to the microcontroller, are used to generate proper correction signal for the DC motor. Considering that microcontroller already present in the system, there is no rationale to use PWM, so that microcontroller can directly control DC motor by using DAC and H-Bridges. Additionally, output of the potentiometer is now connected to the analog input of the microcontroller.

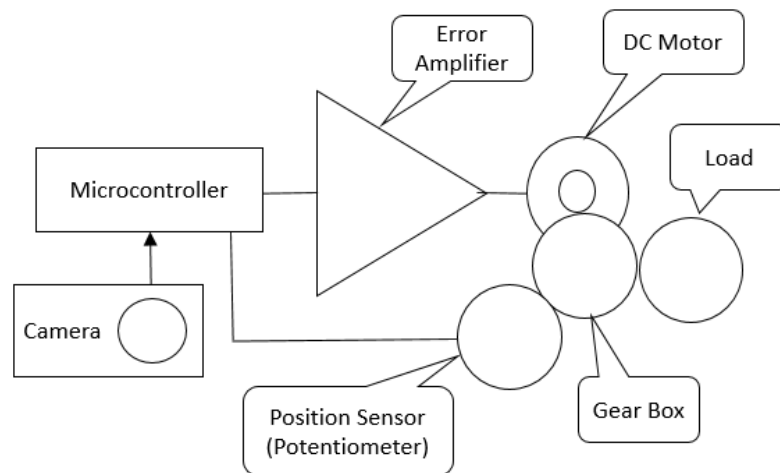


Fig 15. Operation of Camera-controlled modified servomotor.

As a result of above changes, one can expect that accuracy of the Pintograph assembled from the inexpensive and inaccurate mechanical elements, will approach accuracy of the X-Y Plotter assembled by using high-precision mechanical elements.

6. RESULTS

Exemplary simulation results are presented on Figure 16. By selecting different parameters of the Pintograph, working zone and its granularity (see zoomed pattern in the middle) can be evaluated. By operating special Monte-Carlo mode, simulator can evaluate resulted {X-Y} accuracy in case of “from the shelf components” usage.

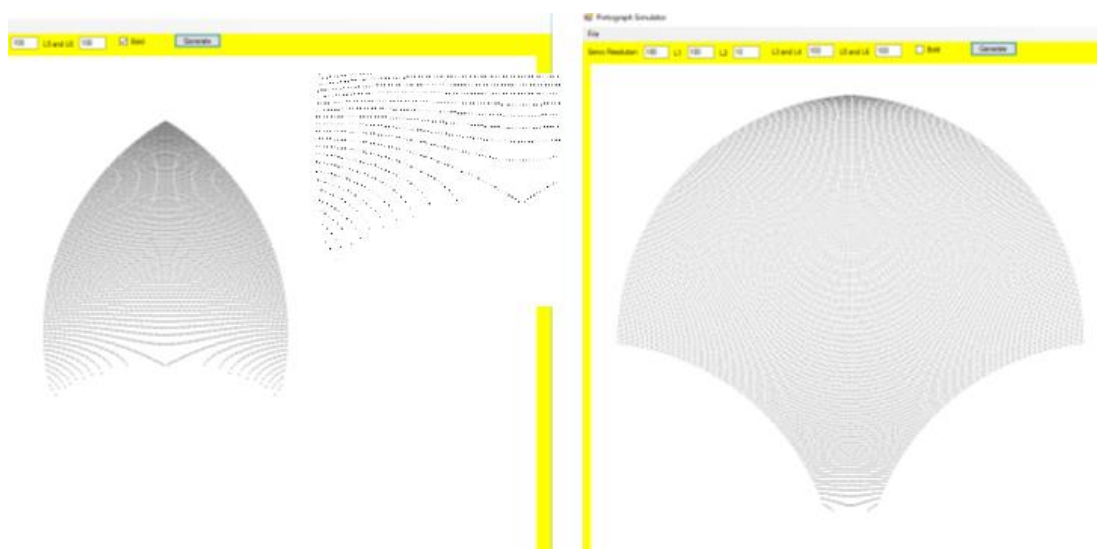


Fig 16. Working Zone Maps obtained by software simulator.
Left: $L1 = L2 = L3 = L4 = L5 = 100$ mm. Right: $L2 = 10$ mm

In the frames of this R&D work preliminary prototype using two servomotors, Paspberry Pi3 microcontroller and digital camera was assembled and tested. Camera control enables to improve {X, Y} tool' positioning accuracy from 2 mm to 0.5 mm.

7. SUMMARY

Developed software simulator enable to test different sets of the pintograph parameters and thus, save time needed to the specific design. Preliminary results obtained by a prototype using camera connected to Raspberry Pi3 reveal that camera-controlled pintograph is feasible. Obtained results can be used in the design of low cost laser engravers.

Camera controlled pintograph described here can be useful when accuracy, and not the speed of an operation, is a factor. However more R&D works are needed in order to create real-life product.

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