

FRactal Radioelement's, Devices and Systems for Radar and Future Telecommunications: Antennas, Capacitor, Memristor, Smart 2D Frequency-Selective Surfaces, Labyrinths and Other Fractal Metamaterials

Alexander Potapov^{1,2}, Victor Potapov¹

¹V.A. Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Science,
11-7, Mokhovaya str., Moscow, 125009, Russia

²Joint-Lab. of JNU-IREE RAS, JiNan University, 601, Huangpu Avenue, Guangzhou, 510632, China

Abstract

Prospects for the creation of fractal radio systems and fractal devices for various purposes are considered. Physical approaches to fractal capacitor and fractal impedances design are presented. For the purpose of technical implementation of fractal radio electronics methods it is necessary to have the new elements base which allows processing signals in the fractional measure space and simulating fractal objects and processes which have the dynamics following the differential equations of fractional order. Promising elements of fractal radio electronics are also functional elements whose fractal impedances are realized basing on the conductors fractal geometry on a surface (fractal nanostructures) and in space (fractal antennas), fractal geometry of substrates surface microrelief and so on. The research is carried out within the framework of new scientific direction "Fractal Radio Physics and Fractal Radio Electronics: Design of Fractal Radio Systems", offered and developed in IREE RAS (middle of 80th XX).

Key words: *fractal, metamaterial, fractal capacitor, fractal antenna, radar, smart surfaces*

1. INTRODUCTION

The increasing complexity of the modern radio electronic hardware and functions it performs makes it essential to consider opportunities of the new physical principles of creation of new elementary base and the new radio engineering systems. For this, the fractals theory and the deterministic chaos theory are of the great importance. The main trends and directions of designing and developing the new fractal radio elements, antennas, Metamaterials and also fundamentally new radio systems for radio location and telecommunication problems are considered in the paper. The fractal antennas are widespread throughout the world but they pay little attention to other fractal nodes and radio systems. The reason is the quite exotic mathematical apparatus of the fractal analysis and the conception of "fractal" itself. Meanwhile, application of fractal analysis principles and non-standard mathematical branches allows to obtain results (for example, in radio location), which are quite unexpected in practice but at the same time are physically valid.

Antenna devices and various frequency-selective surfaces (FSSs) of the packet/multilayer type are needed in any radio technical system. Various modern antennas and FSSs can be found in [1, 2]. However, the modern trends of science and engineering involve the development of various antennas with different characteristics. The key problems in the antenna theory and technology are a decrease in dimensions, wide-range variation in the electromagnetic parameters, and extension of the working frequency band. The geometrical dimensions of antenna depend on the working wavelength. The dimensions affect the mass and size of the radio system in general.

Proposed in IREE RAS fractal radio systems and fractal radioelements bring many improved capabilities in modern radio electronics and can be widely used in practice in future.

Within the framework of this trend, Professor A. Potapov has published more than 840 papers and 25 monographs.

2. FRACTAL ANTENNAS

Experience in analysis and synthesis of fractal antennas and frequency-selective surfaces (FSSs) proves their broadband and multi-band [3–5]. So, such fractal antennas are extremely effective in development of two-frequency or multi-frequency radar and telecommunication systems. The result achieved by using fractal antennas is caused by geometric scheme of radioelements rather than accumulation of individual components or elements (like in classic antennas). It helps us avoid potential excessive complexity and some points of failure. Fractal antennas also allow to create multiband versions, reduced size, and optimal or “awesome” technology of antennas. Doubtless advantage of fractal antennas (monopoles and dipoles) is that they have lower resonance frequencies comparing with classic (Euclidean) antennas of the same size. Inherent broadband qualities of fractal antennas are perfect for smart application and protection.

Compared to conventional methods based on synthesis of smooth antenna radiation patterns, theory of fractal synthesis contains the idea of implementation of emission characteristics with repeated structure on an arbitrary scale. It gives the opportunity to create new regimes in fractal dynamics and to obtain new properties. Particularly, the presence of fractal elements on the target surface can appreciably spoil the signature and its radar portrait.

Applications of fractal antennas: modern telecommunications, noise radar, nonlinear radar, systems of search, localization and tracing of mobile objects, direction finding in complex urban environments, locating unauthorized radio sources in the fight against terrorism, operative military liaison, markers on various subjects, space communication, modern physics experiment, etc.

Extraordinary broadband frequencies coverage together with compact universal form factors of fractal antennas allows to control communications while the signals recognition process, without revealing themselves. Broadband fractal antennas contribute to the appearance of the most advanced mobile tactical communications by combining the bandwidth, interoperability, power control and a compact shape.

The term “Fractal electrodynamics” which came into scientific use in 1990 has currently firmed up abroad [6]. Methods of designing and results of laboratories studies of domestic fractal antennas on the basis of the universal triangular Sierpinsky curve were described in [5, 7].

Strict electrodynamic studies of different fractal monopoles and dipoles on the basis of the universal triangular Sierpinsky curve were described in [7], basing it on the application of algorithms of numerical solution of hyper-singular integral equations – Fig. 1. Calculation of the main parameters of the Sierpinsky fractal antennas with height $H = 0.35$ m was conducted with variation of the opening angle α and different distances to the aperture that was necessary for justification of some special modes of functioning.

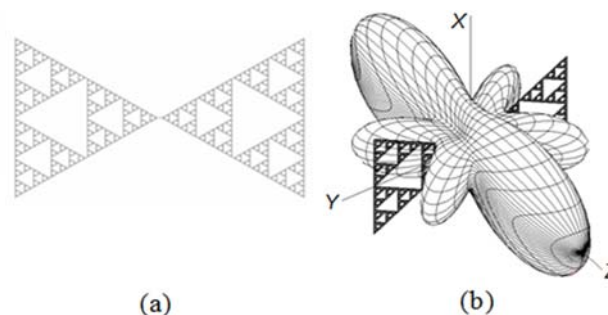


Figure 1. The geometry of the dipole Sierpinski (a) and its radiation pattern (b).

Frequency dependences of the input impedance $Z = \text{Re}Z + i\text{Im}Z$ in frequency range 0.1 - 10.0 GHz were determined, and also all the resonance frequencies of the fractal antennas and FSS under investigation were determined (Fig. 2).

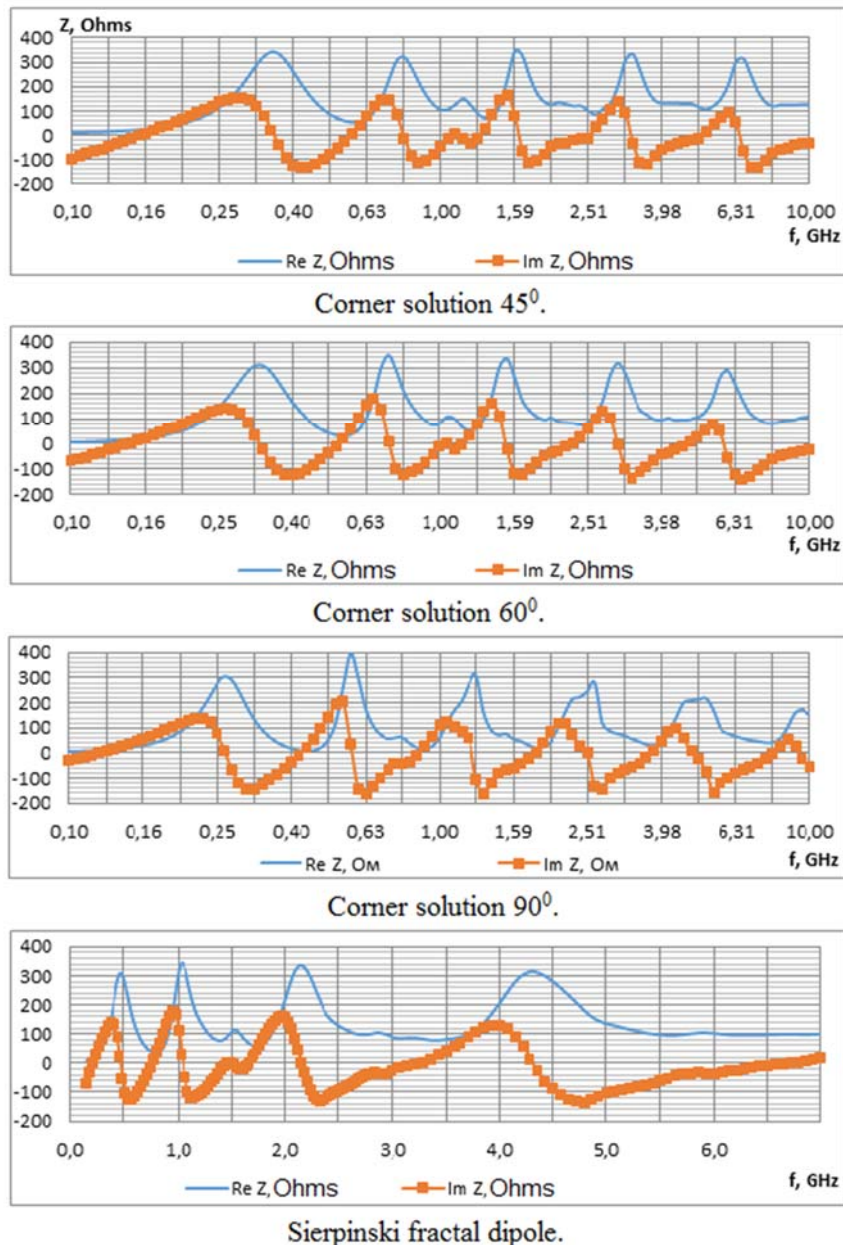


Figure 2. Theoretical dependence of the impedance of fractal antennas.

The active part of the impedance at all values of f_r is several tens ohm that is convenient during its matching with widespread types of feeders. Values of these frequencies f_r for the Sierpinski monopole are $f_1 = 0.53$ GHz; $f_2 = 1.1$ GHz; $f_3 = 2.25$ GHz and $f_4 = 4.70$ GHz. Moreover, a resonance of the same kind as for a half-wave antenna is observed at frequency $f_0 = 0.14$ GHz. Resonance frequencies for the Sierpinski dipole equal 0.76 GHz; 1.66 GHz; 3.21 GHz, and 6.78 GHz.

Always the ratio of two neighbor frequencies approximately equals two that corresponds the scaling coefficient for a topological universal triangular Sierpinski curve. Numerous results of measurements in an anechoic room confirmed the numeric results. Space distribution of currents density on fractal structures was analyzed at resonance frequencies. Also, the field calculation for two values of the working frequency was done at the near-field zone.

3. FRACTAL LABYRINTHS AS SMALL FRACTAL BROADBAND ANTENNAS DEVELOPMENT BASE

Recent several years Fractal labyrinth topology has become fast-growing interest object of scientists. This structures (systems) description can not been presented within traditional derivative equations of integer order. More exactly this processes and objects are quantitatively describing by integration-differentiation operators of fractional order $D_\alpha[f(t)]$, where $-1 < \alpha < 1$ [4, 8, 9]. Presence the fractional derivative in equations is decided to interpret as reflection of specific peculiarity of process or system which is memory or non-Markovian attribute (heredity).

Author's definition: fractal labyrinth $\equiv$ labyrinth fractal is topologically connected structure with fractal dimension $D > 1$ and having scaling character of conducting lines.

Model constructions of L. Cristea [10] are usually employed as examples for mentioned structures investigation.

Wide simulation abilities of stochastic fractal labyrinth require tools development which is to automate needed geometry synthesis operation significantly. The transition of synthesis process from manual to detailed parametric allows to take a step to more complex tasks using the application. These tasks area as noted can be very wide. In particular there is planned to apply fractal labyrinths as miniature UHF radiator geometry which able to become a new class of fractal antennas and fractal antenna arrays.

During our software base scheme development we tried to take into account first of all the primary needs of possible software user, meanwhile trying to complete it by different additional features making development process easier and make results more accurate. The software was named "Fractalizer" [11, 12].

The software (window is shown on Fig. 3) contains first of all graphical plot area to draw a form of fractal curve generator with last action cancel function and choose angle step bar. On the right of generator panel there are tools to parameters specify such as main branch iteration number, branches amount, branches iteration number, width and height of lines, minimum clearance between non-consequent elements (elementary lines) of structure.

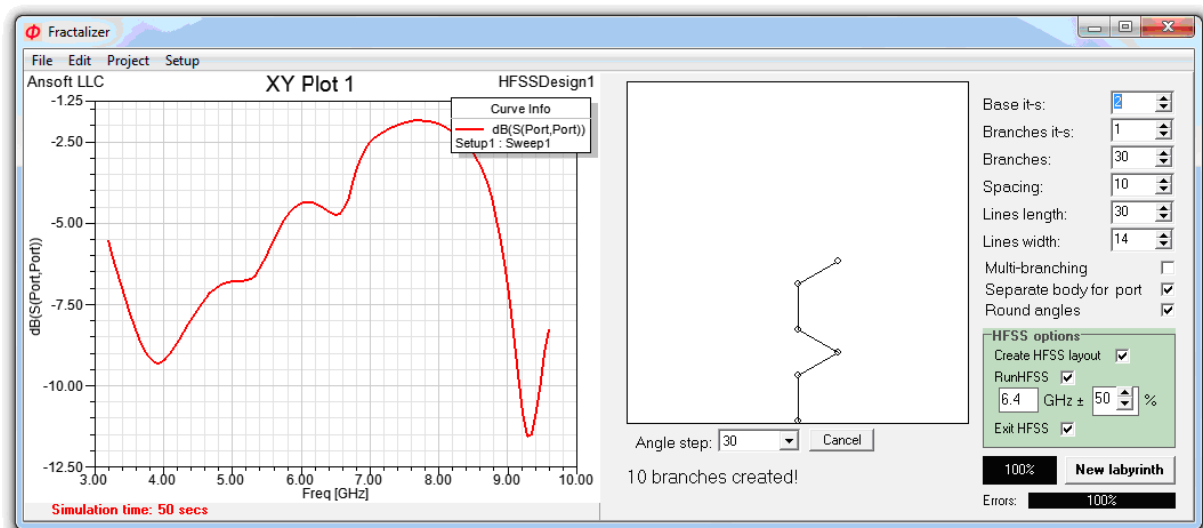


Figure 3. Window for the software.

The software is able to save fractal structures in universal and well-known drawing format Autodesk DXF. DXF file can be imported as geometry in most modern computer designing and simulating software such as ANSYS, Solid Works, etc. Moreover, “Fractalizer” software has settings panel for launch Ansoft HFSS as simulation software for generated fractal structure.

It is shown on Fig. 4 that “Fractalizer” software creates the curves of finite width (b case) instead of zero-width one (case a). Moreover by using angles rounding structure keeps constant width everywhere.

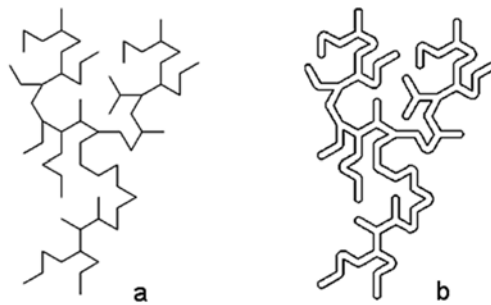


Figure 4. An example of the generated curves.

Today the software is able to create up to several hundreds of branches with satisfactory performance. Every branch here can contain up to 40 elementary lines.

However a kind of fractal antennas which takes the specific form of stochastic (or nondeterministic) fractals still has little presented. In most cases this kind has branching form and presents a fractal, which form is trending to self-similarity only statistically but generally one stochastic branched fractal can have many random realizations. Therefore the assumption is seems to be rightly that stochastic labyrinth fractal antennas would become to new generation of fractal antennas.

Operation principle and features of “Fractalizer” are improving now. In particular the process is automated for electrodynamical simulation of fractal antennas. An operator labor is not necessary in this stage by default. The exchange scheme is shown on Fig. 5 [11, 12].

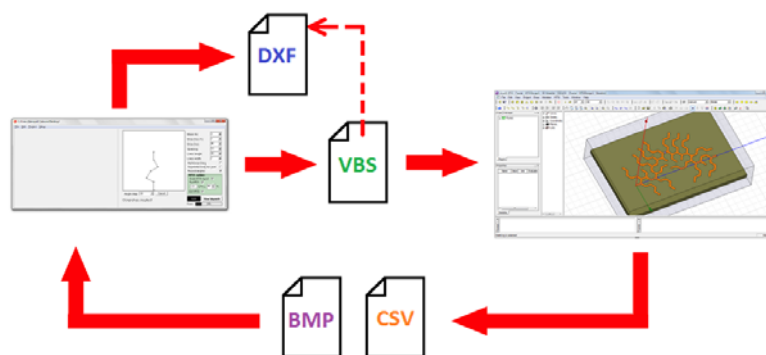


Figure 5. The exchange scheme.

“Fractalizer” operation principle contains several following steps. First the program builds main curve basing on user given generator which is complete user specified iteration of fractal. Next the program calculates amount of curve break points (number of angles) and remembers they as points of possible branch base. Then by using random number generator the program in cycle selects the branch base

point and builds this branch before user specified iteration is complete or before an obstacle (another curve) is reached. The number of cycles must be specified by user. If a point is already busy (as other branch base) then random number generator will be re-launched. The branch direction is between two lines forming an angle and from external side of the angle. After structure is ready the software reports quantity of successfully created branches and save the structure as an DXF file. Examples of these resulting structures are shown on Fig. 6.

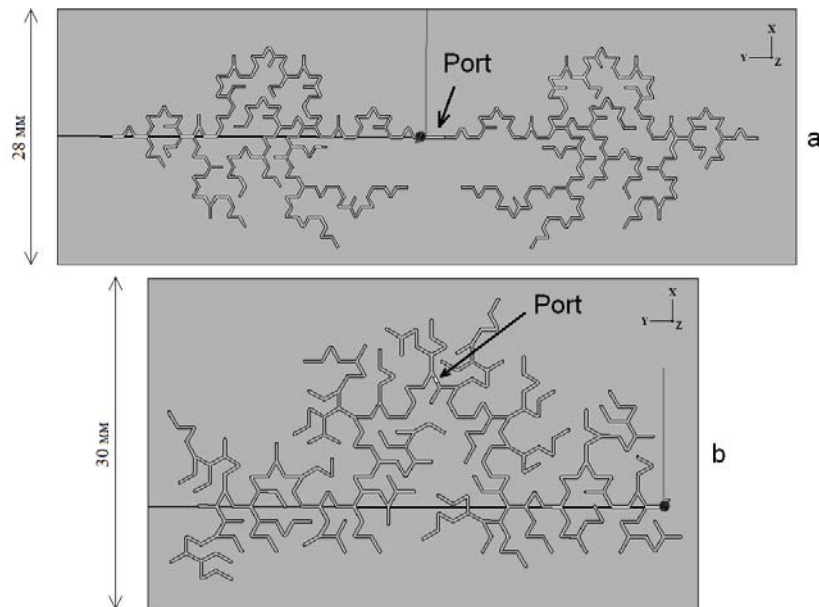


Figure 6. Results structures for fractal antennas.

Results obtained by using the software was automatically imported by it to Ansoft HFSS 12 simulation environment based on finite element method (FEM).

Simulation results of device from Fig. 6b namely reflection coefficient graph in frequency domain and 3D radiation pattern are shown on Fig 7a and Fig. 7b, correspondingly.

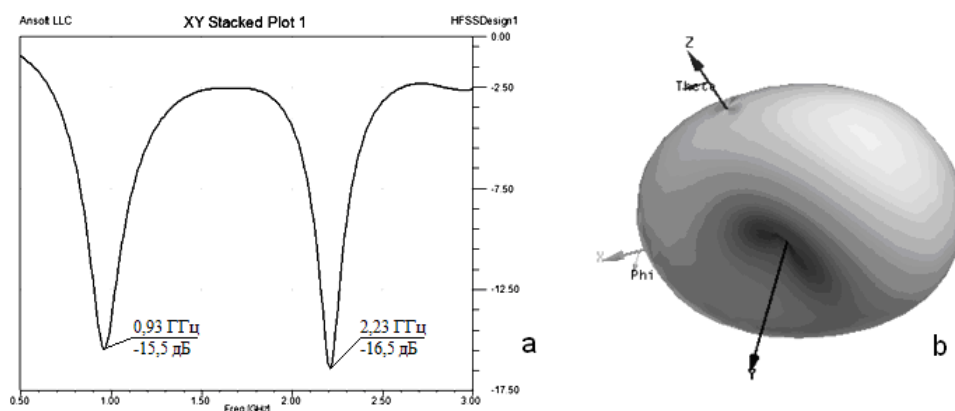


Figure 7. Reflection coefficient in frequency domain (a) and 3D radiation pattern (b).

As it can be seen, the antenna has two resonances. The first one corresponds to frequency lower than 1 GHz. It means that the antenna is able to receive the wave of 0.32 meter length.

The structure which shown on Fig. 6b was synthesized stochastically and therefore it is reasonable to assume better results if this one will be optimized by some algorithm. Labyrinth fractals able to be genetically optimized can become universal tool for achievement of different required parameters such as gain, multi-range and wide range as well as radiation pattern form. But the main requirement is to decrease antenna size while its operating frequency is constant. Fig. 8 below represents several UHF fractal antennas and their S11 graph in frequency domain. In a base of layouts there are geometry which synthesized by “Fractalizer” [11, 12].

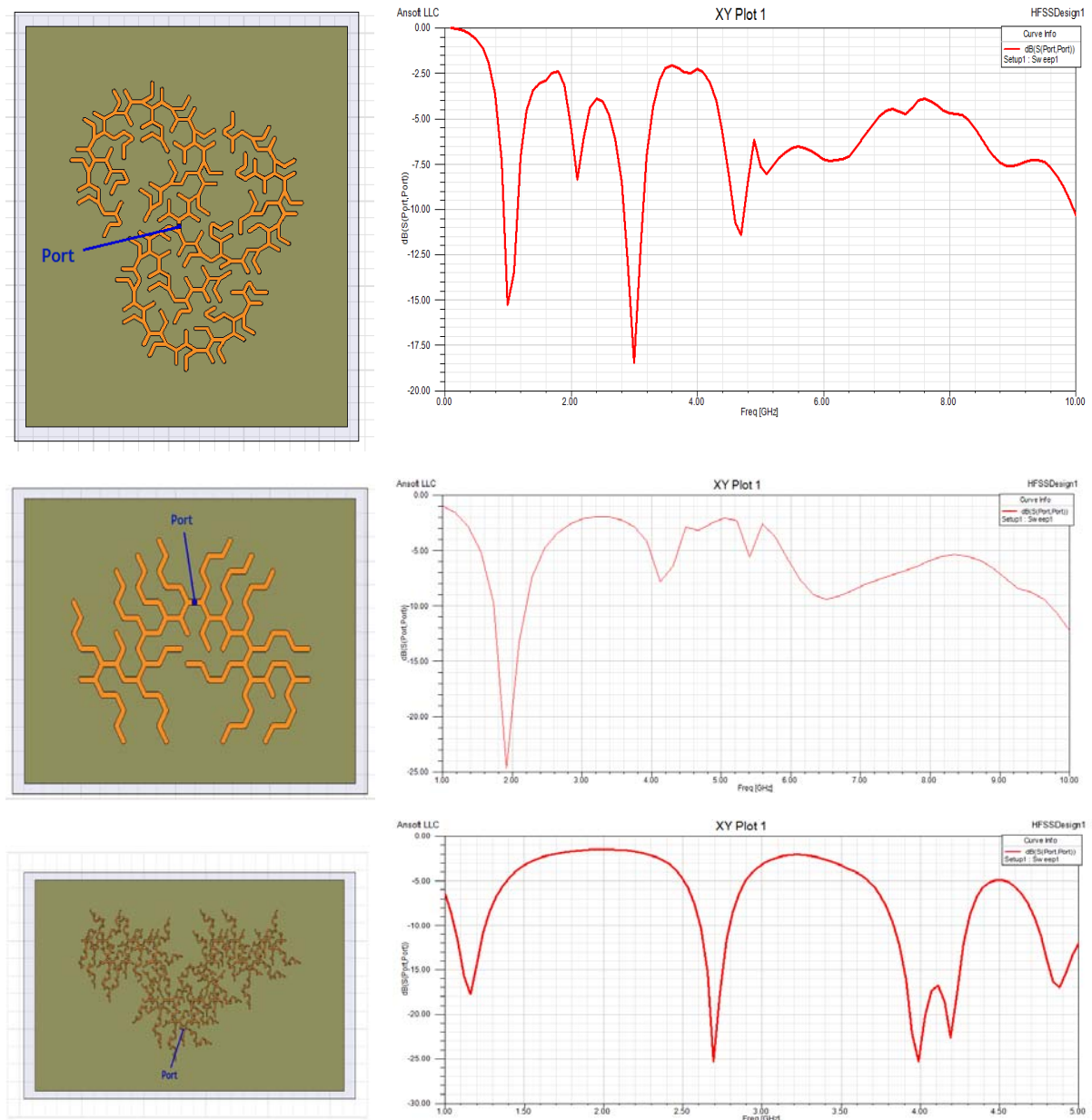


Figure 8. UHF fractal antennas and their S11 graph in frequency domain.

4. FRACTAL LABYRINTHS AND GENETIC ALGORITHMS IN SYNTHESIS OF BIG ROBUST ANTENNA ARRAYS AND NANO – ANTENNAS

Antenna arrays is one of main components of modern radiosystems. The theory of fractals and fractional calculus application allows to unite accomplishments of amplitude and stochastic arrays.

The first type of antenna arrays has relatively small side lobes but it is sensitive to elements placement errors and excitation currents values. The second type of arrays is robust to elements placement errors and they's failures, but the type characterized by relatively high side lobes level. The scaling principle application for antenna arrays allows more flexible control of radiation pattern in the lobes area.

Autors suggest to synthesize the stochastic robust antenna arrays by dint of fractal labyrinths properties use. It will enable to control of radiation pattern side lobes energy. The second step is union of several fractal labyrinths clusters with different fractal dimension D in antenna array synthesis space. Therefore by natural way we comes to adaptive fractal antennas. Here it is necessary to use of genetic algorithms for optimize the spatio-temporal big antenna apertures corresponding to specified criteria.

New methods are suggested for adaptive and robust fractal antenna arrays synthesis. The same principles can be used for synthesis of nano-antennas. New methods abilities to creation of topological (not energetic!) multi-dimensional signal detectors and they's further processing are shown [3-5, 13].

5. NANOSTRUCTURES AND FRACTALS

It is getting possible nowadays to create planar and three-dimensional nanostructure modeling «fractal» radioelements and radio devices of microelectronics based on nanophase materials [3-5, 13, 14]. In other words, we are talking about the construction of the element base of new generation based on fractal effects and properties. In particular, elementary extension of the Cantor set to the physical level allows us to go to so-called *Cantor blocks* in planar technology of molecular nanostructures. The percolation synthesis suggested by the authors in 2007 for nanostructured composites is also possible.

Recursive process also allows to create self-similar hierarchical structure, down to separate conductive tracks on the chip and nanostructures. It is necessary to take into account and learn how to calculate the mutual and collective influence of electromagnetic fields with all components of the chip: the conductive tracks, semiconductor, insulator, etc.

Considerable attention of the most advanced experts is currently paid to modeling of fractal objects with the complex dynamics by various dissipative systems. The most natural way of modeling is to use the Feigenbaum scenario of transition to chaos through period doubling. In the context of our work, Julia, Fatou and Mandelbrot sets are interesting objects of development of new types and forms of fractal antennas and others fractal nanostructures and Metamaterials based on them [3-5, 13-15].

6. FRACTAL RADIO ABSORB MATERIALS AND SURFACES

Modern and prospective absorb materials and surfaces should be provided with wide range of absorption of electromagnetic radiation at arbitrary angles of sensing and polarization of incident light. From this viewpoint, the use of fractal Metamaterials and artificial composites that can be attributed to «smart» system is the most promising way [3-5]. In addition to the direct use, they may have a variety of functionality. The calculation of reflection and transmission coefficients of these materials can be made by developed techniques. Then the inverse problem has to be solved. It means the estimation of the effective dielectric and magnetic permeability coefficients of a multilayer fractal environment that may be tensors in the case of anisotropic materials. It is necessary to constantly use multiple reference to the direct problem.

7. FRACTAL PHOTON AND MAGNON CRYSTALS

Conventional materials with photon band gaps uses Bragg scattering to create band gaps [4, 16]. As a result of Bragg scattering mechanism, width and transverse sizes of photon crystals should be several wavelengths. Systems based on photon band gaps and frequency-selective surfaces usually works in a single frequency band with a suitable wavelength in the volume damped periodically arranged basic functional blocks. Fractal photon and magnon crystals have a number of advantages over their classical counterparts, and are essentially new media for transmitting information [4, 17, 18].

An N -order fractal should theoretically have N inherent resonances. Each resonance is defined by the excitation current in wire circuits of a certain order of iterations and this current flows towards structures of a higher order. The fractal which interacts with an electromagnetic wave at normal incidence is pretty exactly simulated by a thin homogeneous plate of the same thickness with the effective permittivity:

$$\varepsilon_{eff}(f) = \varepsilon_0 + \sum_l \frac{\beta_l}{f_l^2 - f^2}, \quad (1)$$

where f is the frequency, index l defines the resonances number, ε_0 , f_l and β_l are parameters obtained from the calculated spectrum. It follows from (1) that when changing from one resonance frequency f_l to another f_{l+1} there is always a point which it is $\varepsilon_{eff}(f) = 1$ in and consequently certain transmission bands exist. It is always desirable that the reflection/transmission coefficient of a fractal structure was *controllable* using an external "control knob". Each line segment in a fractal is connected with each other. The external electric current which is supplied to the center of the first level line with a certain phase can be a *secondary source*. Modulation of the transmittance is determined by the phase shift (or the time lag τ of the sensed signal with respect to the main incident beam) between the incident wave and the feeding current.

In this case we can talk about modelling intellectual surfaces with focused control of its scattering characteristics or a field of the transmitted wave in a wide frequency band. When imposing two identical fractal samples if one is turned on 90° with respect to another one can obtain a structure which is invariant under rotation. Thus such an "active" fractal structure can simulate the total reflection which does not depend on the incidence angle and polarization state and what is usually the characteristic of 3D photonic crystals.

It should be noted that sizes of a traditional 3D photonic crystal must form at least several wavelengths before it will be able fully show its photonic bandgap properties. Thus for wave 1GHz the structure thickness must be about 1 meter. On the other hand plain fractal structures are such as their transmission band $\Delta f/f_0$ determined by the similarity law ($\Delta f/f_0 \rightarrow \text{bandgap/middle of the bandgap}$ and $\Delta f/f_0 \sim 5\%$ for one fractal plate) can be significantly enhanced using imposition of identical fractals on each other. Increase of the fractal plates thickness leads to the growth of steepness of transmission bands bounds. Attenuation bands can also be extended using wider metallic conductors of the fractal plates.

The resonance wavelengths may be much larger than the sample sizes. It happens because the low frequency resonance is determined by the longest metallic line in a fractal and such a line is just much longer than linear dimensions of the fractal itself. It gives a fractal its "Superwave" properties that is a fractal plate can effectively reflect electromagnetic waves with lengths much larger than the lateral dimensions. "Superwave" properties imply that the fractal plate can act as a compact reflector. For such technologies authors have developed algorithms and programs which allow calculate different configurations of fractal structures of the crystals under consideration. As an example on Fig. 9 there are samples of some drafts on the basis of Sierpinski curve (a) and Cayley tree (b) respectively. Thus the fractal structures always have a self-similar series of resonances leading to the logarithmic periodicity of working zones.

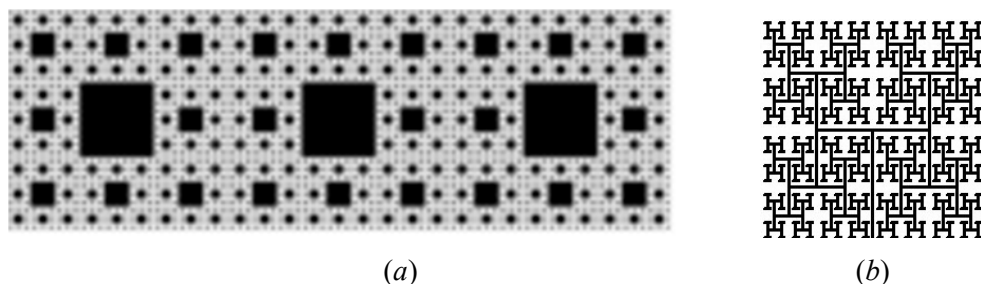


Figure 9. Samples of the first (a) and second (b) fractal drafts.

The linked topological fractal structure makes it possible to modulate the transmittance coefficient of electromagnetic waves. The lowest attenuation frequency corresponds to wave lengths which may significantly exceed external dimensions of a fractal plate making such fractal structures to be superwave reflectors. For controllable intellectual surfaces one may also use the principle of reconfigurable fractal arrays with electronic switching of sub arrays which was described in detail in [3-5].

8. FRACTAL SIGNATURES IN PROBLEMS OF ESTIMATION OF MICRORELIEF OF PROCESSED SURFACES

Basing on the conducted experiments we were the first who proposed estimation methods using different fractal characteristics of *the quality of articles surface and properties of microrelief of modern structural materials* [3-65, 19]. Due to intensive development of methods of processing of concentrated energy streams – CES (laser, plasmic, electro erosive), as well as nanotechnologies (chemical assembly, sol-gel processes, metals vapor-phase deposition, atomic layered epitaxy) significant difficulties in description and estimation of the roughness with a profile method arise.

In these cases the roughness elements form, elements distribution over the processing square strongly differs from its conventional conception which was formed in the framework of the processing by cutting as a periodic alternation of “juts” and “cavities” which are described in the framework of the Euclidean geometry – Fig. 10 [19].

Name	Relief elements	Name	Relief elements
<i>Mushroom-shaped</i>		<i>Dimples</i>	
<i>T-shaped</i>		<i>Globules</i>	
<i>Spades</i>		<i>Whispers (globulous-whisperous)</i>	
<i>Splats</i>		<i>Ridges</i>	
<i>Botiroidal</i>		<i>Moire</i>	

Figure 10. The types of micro surfaces relief elements.

Consequently, now the problems of forming the surface quality including such an important quality characteristics like roughness become particularly vital due to creation of the new technologies of metalworking. These problems come into sharp focus in the field of nanotechnologies which the roughness topology is considered for not as a secondary property being a "response" of the surface layer structure to the influence of a certain physical process (for example, like in the cutting work) but as a property of the structure itself all the more dimensions of such layers are comparable with the electrons mean free path. In [19], at microrelief level of such processed surfaces, we demonstrated existence of fractal clusters with irregularities distributed by the power laws with heavy tails (Fig. 11).

Presence of fractality in such different media can be controlled in particular on a change of the skin effect and impedance. Exactly the spatial/time evolution of the current allows the electromagnetic field "feel" fractal properties (fractal signatures) of the physical medium under investigation.

Scaling models of the rough layer of solid's surface can be represented as an electric circuits analogy which has the form of the Cantor dust for example and so on [3-5].

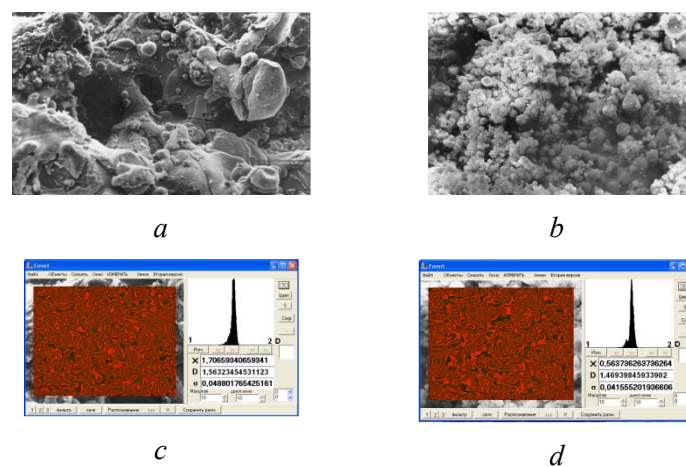


Figure 11. Fractal analysis of the samples surfaces with a plasma-sprayed covering of hydroxyapatite: (a, b) 2D – image of samples surface; (c, d) the field and histogram of local fractal dimensions (note the distributions' heavy tails).

9. FRACTAL IMPEDANCES AND SIMULATION OF FRACTIONAL OPERATORS

In practice, as it's pointed out above, random variables sum converges not to the Gaussian distributions but to the stable ones or “Levi-Pareto” distributions with the heavy tails pretty often. The distribution function of such distributions is “wide”. It results in the fact that some moments of such a distribution will be infinite formally. Simulation of random variables distributed by Levi-Pareto results in processes of abnormal diffusion described by fractional derivatives on space or/and time variables [4, 8, 9]. As a matter of fact equations with fractional derivatives describe non-Markovian processes with memory.

Physical simulation of fractional integral and differential operators allows creating radio elements on passive elements, modelling fractal impedances $Z(j\omega)$ with frequency scaling on the basis of nanotechnologies [20-22]

$$Z(j\omega) \equiv A(j\omega)^{-\eta}, \quad (2)$$

where $0 \leq \eta \leq 1$, A – const, ω - angular frequency.

10. THE FIRST FRACTAL CONDENSER

Even before proposing an idea of fractal radio systems we considered [108] the problem of simulation of elementary fractal impedance in the form of a hypothetical "fractal condenser" like an interesting task to be done for the first time in the world. As it is well known, in the case of the final stage of building an electrical circuit analogy for RC -circuits (Fig. 12,*a,b*) one can write the following expression after the Laplace transformation:

$$\frac{u(p)}{R_2 i(p)} = 1 + \frac{\frac{1}{C_1 R_2}}{p + \frac{i_1(p)}{C_1 u_1(p)}} \quad (3)$$

For the right part of Fig. 12,*b* we can write:

$$\frac{u_1(p)}{R_1 i_1(p)} = 1 + \frac{\frac{1}{C_0 R_1}}{p + \frac{1}{C_0 R_0}} \quad (4)$$

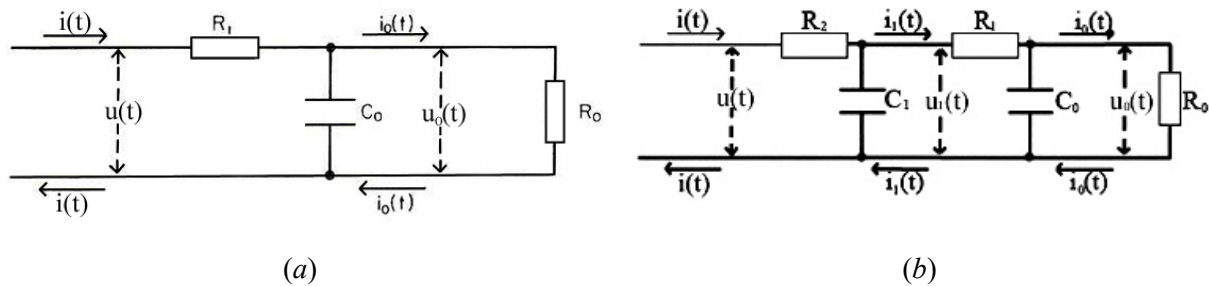


Figure 12. A process of realization of fractional operators: *a*) - the initial electrical circuit, *b*) - adding a capacitor integrator R_1, C_1 into the initial circuit.

Let us define the following frequencies:

$$\omega_0 \equiv \frac{1}{C_0 R_0}, \omega_1 \equiv \frac{1}{C_0 R_1}, \omega_2 \equiv \frac{1}{C_1 R_1}, \omega_3 \equiv \frac{1}{C_1 R_2}, \quad (5)$$

which are typical for couples of elements under consideration. By using the introduced designations we combine equations (3) and (4):

$$\frac{u(p)}{R_2 i(p)} = 1 + \frac{\omega_3}{p + \frac{\omega_2}{1 + \frac{\omega_1}{p + \omega_0}}} \quad (6)$$

Expression (6) can be rewritten in the form:

$$\frac{u(p)}{R_2 i(p)} = 1 + \frac{\omega_3}{p + 1} \frac{\omega_2}{1 + \frac{\omega_1}{p + 1}} \frac{\omega_0}{1}, \quad (7)$$

that is in the form of the continued fraction.

Let us generalize the circuit presented on Fig. 12,b by adding n integrating circuits R_{n+1}, C_n . We come to the circuit (Fig. 13) which we can write by induction for:

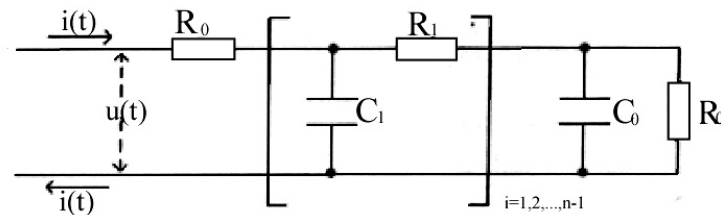


Figure 13. The fractional operator $d^{-1/2}/dt^{-1/2}$ or “semi-integrator” on the basis of $(2n + 1)$ elemental components.

$$\frac{u(p)}{R_n i(p)} = 1 + \frac{\omega_{2n-1}}{p +} \frac{\omega_{2n-2}}{1 +} \frac{\omega_{2n-3}}{p +} \dots \frac{\omega_2}{1 +} \frac{\omega_1}{p +} \frac{\omega_0}{1}, \quad (8)$$

where

$$\omega_{2j} = \frac{1}{C_j R_j}, \quad \omega_{2j+1} = \frac{1}{C_j R_{j+1}}. \quad (9)$$

Let us further simplify equation (8) by dividing each of frequencies ω_j by p . So if $\omega_j / p = v_j$ then equation (8) can be written as an expression of continued fraction:

$$\frac{u(p)}{R_n i(p)} = 1 + \frac{v_{2n-1}}{1 +} \frac{v_{2n-2}}{1 +} \frac{v_{2n-3}}{1 +} \dots \frac{v_2}{1 +} \frac{v_1}{1 +} \frac{v_0}{1}. \quad (10)$$

At certain relationships between values v the continued fractions in equation (10) can be expressed in a more preferred way. It is possible when all condensers have one capacity value which equals $C_0=C_1=C_2=\dots=C_{n-1}=C$, and all resistors except for one R_n have one resistance value. Value of resistance R_n , equals to a half of every other resistor's resistance. Thus,

$$R_0=R_1=R_2=\dots=R_{n-1}=R, \quad R_n=R/2. \quad (11)$$

Let us define v as:

$$v_0 = v_1 = v_2 = \dots = v_{2n-3} = v_{2n-2} = \frac{1}{CRp} \equiv v, \quad (12)$$

then

$$v_{2n-1} = \frac{2}{CRp} = 2v. \quad (13)$$

Similarly with equation (10) we write

$$\frac{2u(p)}{Ri(p)} = 1 + 2 \frac{v}{1 +} \frac{v}{1 +} \dots \frac{v}{1 +} \frac{v}{1}, \quad (14)$$

where the continued fraction has $2n$ factors v in the numerator. It can be shown [8] that:

$$\frac{v}{1+} \frac{v}{1+} \dots \frac{v}{1+} \frac{v}{1+} = \frac{\sqrt{4v+1}}{1 + \left[\frac{\sqrt{4v+1}-1}{\sqrt{4v+1}+1} \right]^{2n+1}} - \frac{\sqrt{4v+1}}{2} - \frac{1}{2} \quad (15)$$

For the model introduced above we get the main expression by combining equations (14) and (15) and dividing by coefficient $2\sqrt{v}$:

$$\frac{u(p)}{i(p)} \sqrt{\frac{Cp}{R}} = A(v) = \sqrt{\frac{4v+1}{4v}} \left[\frac{\left[\frac{\sqrt{4v+1}+1}{\sqrt{4v+1}+1} \right]^{2n+1} - \left[\frac{\sqrt{4v+1}-1}{\sqrt{4v+1}-1} \right]^{2n+1}}{\left[\frac{\sqrt{4v+1}+1}{\sqrt{4v+1}+1} \right]^{2n+1} + \left[\frac{\sqrt{4v+1}-1}{\sqrt{4v+1}-1} \right]^{2n+1}} \right] \quad (16)$$

In case of the final stage of building an equivalent electrical circuit for RC -circuits we can regulate frequency bands which the necessary power dependence of impedance of the form $\omega^{-\alpha}$ will be observed in when we use n -th matching fraction for the given continued fraction. An elementary algorithm of realization of a fractional operator of the form $d^{-1/2}/dt^{-1/2}$ or a “semi-integrator” is presented above on Fig. 13.

Thus independently of [8] the authors designed an impedance model $Z(j\omega)$ in the form of an infinite continued fraction. In case of the final stage of building an equivalent electrical circuit for RC -circuits we can regulate frequency bands which the necessary power dependence of impedance of the form $\omega^{-\alpha}$ will be observed in when we use n -th matching fraction for the given continued fraction. In this case we will for the first time in practice realize a nonlinear “*fractal condenser*” in the analogous and numerical form [20].

One should also refer an entire array of problems of simulation of microelectronic fractal impedances for fractal radio elements of low frequency and high frequency wave bands to this direction.

11. FRACTAL MEMRISTOR

In 1971, L.O. Chua [23] proposed a new passive element: a memristor. Until recently, the memristor was considered to be a theoretical element, which can be implemented only using fairly cumbersome transistor or operational amplifier electronic circuits; therefore, memristors found little application. In recent years, however, this viewpoint has been changed [24]; therefore, of interest is thorough investigation of the properties of the memristor.

Let us consider the concept of the memristor. Fig. 14 shows a square whose corners correspond to such general quantities of the circuit theory as charge Q , magnetic flux Φ , electric current I , and electric voltage U .

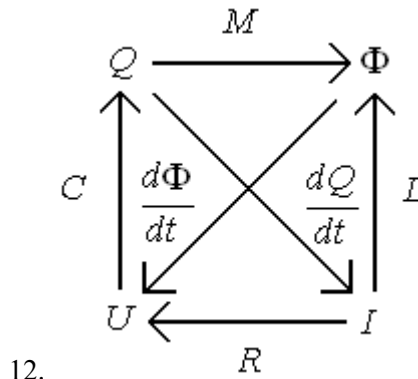


Figure 14. Relations between four basic characteristics of the electric circuits.

The square corners are connected by the lines with arrows that point out the relations between two individual characteristics. As follows from the symmetry considerations, there should be six such relations [23, 24] specified by the equations

$$Q = CU, \quad \Phi = LI, \quad U = RI, \quad \Phi = MQ, \quad I = \frac{dQ}{dt}, \quad U = \frac{d\Phi}{dt}, \quad (17)$$

where C is the capacitance, L is the inductance, R is the resistance, and M is the memristivity (the parameter absent in the traditional theory).

As seen from Eq. (17), memristivity relates the charge and the magnetic flux and has the dimensionality of resistance. Based on (1), we can write two expressions for memristivity:

$$M = \frac{L}{RC}, \quad M = \frac{\Phi(0) + \int_0^t U(\tau) d\tau}{Q(0) + \int_0^t I(\tau) d\tau}. \quad (18)$$

The first expression describes a series or a parallel RLC circuit. Here, memristivity characterizes the circuit Q factor, which is equal to $\sqrt{M/R}$ or $\sqrt{R/M}$. The second expression allows us to conclude that the simplest memristor is a conductor with current and memristivity is the resistance of this conductor. In the general case, however, memristivity is a nonlocal function of time, i.e., it may depend on the signal duration.

In this case, the element itself should no longer be just a simple connection of resistors, capacitors, and inductors. The equation describing the relation between voltage and current that follows from expressions (18) is

$$U(t) = M(t) \left[I(t) + \left(Q(0) + \int_0^t I(\tau) d\tau \right) \frac{d \ln M(t)}{dt} \right]. \quad (19)$$

Similarly to the classical circuit elements, memristors, being actually implemented elements, can exhibit nonlinear and inertial properties (memory, hysteresis of the I – V characteristic, etc). In view of this fact, the attempts were made to develop the memristor on the basis of analog circuits by applying the mathematical tool of fractional integro-differentiation, which is widely used in synergetics and fractal physics [4, 8, 9].

In our opinion, it is reasonable to introduce operators that contain fractional time derivatives of the electric charge and magnetic flux directly in definition (17). We write the fractional operators in the form [25]

$$z(x) = D_{0x}^{m-1} \frac{dy(x)}{dx} = \partial_{0x}^m y(x), \quad 0 \leq m \leq 1, \quad (20)$$

where

$$D_{sx}^m y(x) = \begin{cases} \frac{\text{sign}(x-s)}{\Gamma(-m)} \int_s^x \frac{y(x') dx'}{|x-x'|^{m+1}}, & m < 0, \\ y(x), & m = 0, \\ \text{sign}^n(x-s) \frac{d^n}{dx^n} D_{sx}^{m-n} y(x), & n-1 < m \leq n, n \in N, \end{cases}$$

$$\partial_{sx}^m y(x) = \text{sign}^n(x-s) D_{sx}^{m-n} \frac{d^n y(x)}{dx^n}, \quad n-1 < m \leq n, n \in N,$$

$\Gamma(-m)$ is the Euler gamma function, and D_{sx}^m and ∂_{sx}^m are the fractional integro-differential Riemann - Liouville and Caputo operators of order m .

Finally we get [25]

$$z(x) \approx \Gamma(1+m) \frac{\Delta y}{(\Delta x)^m}. \quad (21)$$

The ratio $\Delta y/(\Delta x)^m$ in (21) determines the so-called Holder derivative introduced for fractal functions instead of an ordinary derivative and expresses the nonlinear increment law. This derivative and, consequently, expression (20) can be used quite reasonably in definition (17) to take into account possible fractional dynamics of the charge and the magnetic flux during operation of the memristor.

Thus, replacing the first derivatives in (17) by operators (20) and performing some transformations, we find

$$U(x) = D_{0x}^\alpha \left[M(x) \left(\frac{Q(0)}{t_0} + D_{0x}^{-\beta} I(x) \right) \right] - \frac{\Phi(0)}{t_0} \frac{1}{\Gamma(1-\alpha)x^\alpha}, \quad (22)$$

where exponents α and β characterize randomness and fractality of the time changes in the magnetic flux and electric charge, $x = t/t_0$ is the dimensionless time, and t_0 is some characteristic time. In the special case of $M = \text{const}$, Eq. (22) is simplified to

$$U(x) = M \left(\frac{Q(0)}{t_0} \frac{1}{\Gamma(1-\beta)x^\beta} + D_{0x}^{\alpha-\beta} I(x) \right) - \frac{\Phi(0)}{t_0} \frac{1}{\Gamma(1-\alpha)x^\alpha}. \quad (23)$$

Obtained Eqs. (3) and (22) and (23) describe the processes with perfect and partial memory, respectively. The presence of memory means that the voltage across the memristor will depend on the features of current flow from switching on to time point t . At $\alpha = \beta = 1$, Eq. (22) transforms to Eq. (19) and Eq. (23) transforms to the Ohm's law. It is shown that the integral quantum Hall effect can be the physical basis of memristor's operational principle [25].

12. AUTHOR'S CONCEPTION OF FRACTAL ELEMENTS AND RADIO SYSTEMS

The elaboration of the first standard dictionary of fractal features of target classes and permanent improvement of the algorithmic software are the main stages of the development and prototyping of a fractal nonparametric detector of radar signals designed as a dedicated processor. On the basis of these results, we can speak about the design of not only fractal units (devices) but also the entire fractal

radio system [3-5, 13-15, 20-22, 26-43]. Such fractal radio systems (Fig. 15) contain (starting from the input) fractal antennas and digital fractal detectors and use fractal data-processing methods; future devices will use fractal methods for modulation and demodulation of radio signals.

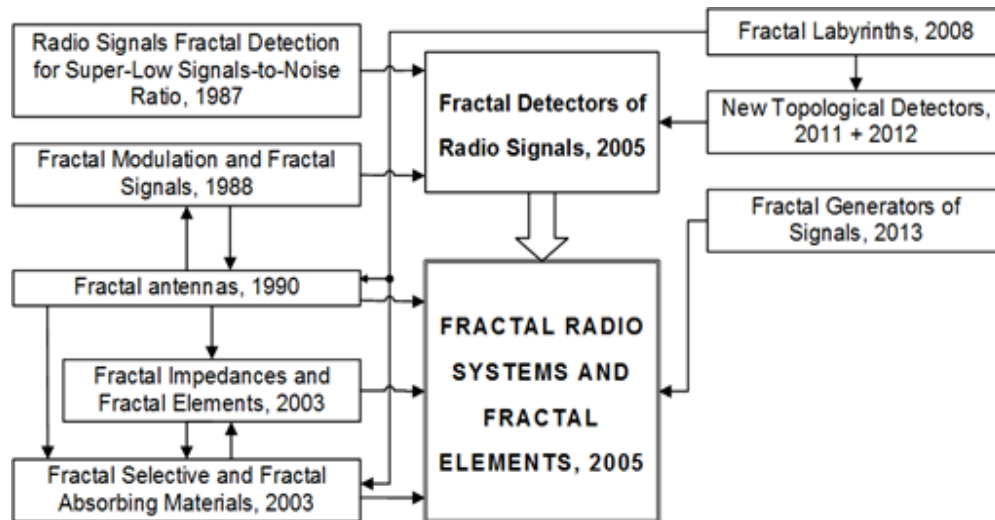


Figure 15. The author's conception of fractal radio systems, devices and radio elements.

Application of a recursive procedure allows, in principle, formation of a self-similar hierarchical structure, up to formation of individual conducting strips in a microcircuit.

13. CONCLUSION

Fractal theory formation - striking example of new science investigation line that is based equally on both progress in abstract mathematics areas and on a new view on a long known empiric material which could not be interpreted and described scientifically before the valid models appear. Fractal theory application allows us to discover a great deal of previously unused reserves and to apply it in different technical applications field.

Effectiveness of radio location and telecommunication systems can be significantly enhanced by taking into account fractality of wave phenomena which develop at every stage of waves radiation, scattering and propagation in different media. Representation of signals obtained by a system in the fractional measure space and application of the scaling ratios during its processing allow to bring absolutely new ideas and methods into conventional areas of classical radio physics and radio electronics and to get results rather unexpected for practice which are however physically valid. A fundamentally new approach to the informational components of a signal and the field as a whole forms the main distinction of fractal radio physics and radio electronics from the classical one. Application of the fractal theory allows revealing huge resources which have been unexplainable earlier and use them in the area of different technical applications.

For the purpose of technical implementation of fractal radio electronics methods it is necessary to have the new elements base which allows processing signals in the fractional measure space and simulating fractal objects and processes which have the dynamics following the differential equations of fractional order.

Promising elements of fractal radio electronics are also functional elements whose fractal impedances are realized basing on the conductors fractal geometry on a surface (fractal nanostructures) and in space (fractal antennas), fractal geometry of substrates surface microrelief or fractal structure of polymer composites and so on.

New challenging information technologies development and introduction on the basis of fractal radio physics and fractal radio electronics principles relates with training of specialists for these scientific fields. First of all, in a number of universities there is a need to organize courses on the simultaneous study of fractal theory foundation, determined chaos theory, fractional operators theory and its physical-technical application.

As result of lecturing on the fractal technologies developed by author in IREE RAS and reports on ISTC project in USA (Washington, New-York, Huntsville, Atlanta, Franklin) in 2000 and 2005 American specialists wrote in official letter for Director of IREE RAS academician Yu.V. Gulyaev in December of 2005 "...Dr. A. Potapov has successfully presented several seminars in the Center for Space Plasma and Aeronomic Research (CSPAR) at the University of Alabama in Huntsville. The seminars were of essential interests and confirmed high scientific credentials of Dr. A. Potapov. RADAR technologies presented by Dr. Potapov are novel and based on the fractal theory. Their importance for the international community of specialists and scientists is undeniable". At the same time (December of 2005) scientific meeting of author and fractal geometry founder B. Mandelbrot.

ACKNOWLEDGMENT

This work was supported in part by the project of International Science and Technology Center No. 0847.2 (2000 – 2005, USA), Russian Foundation for Basic Research (projects №№ 05-07-90349, 07-07-07005, 07-07-12054, 07-08-00637, 11-07-00203), and also was supported in part by the project "Leading Talents of Guangdong Province", № 00201502 (2016 – 2020) in the JiNan University (China, Guangzhou).

REFERENCES

1. Volakis, J.L. (ed.) 2007, *The Antenna Engineering Handbook*, 4th Edition, McGraw Hill, New York.
2. Munk, B.A. 2000, *Frequency Selective Surfaces: Theory and Design*, Wiley, New York.
3. Potapov, A.A. 2002, *Fractals in Radio Physics and Radar*, Logos, Moscow.
4. Potapov, A.A. 2005, *Fractals in Radio Physics and Radar: Topology of Sample*, 2-th issue ed. and correct, University Library, Moscow.
5. Potapov, A.A. 2006, "Fractals and Chaos as Basis of New Challenging Technologies in Modern Radio Systems", in R. M. Crownover, *Introduction to Fractals and Chaos*, Tekhnosfera, Moscow, pp. 374–479.
6. Jaggard, D.L. 1990, "On Fractal Electrodynamics", in *Recent Advances in Electromagnetic Theory*, ed. H.N. Kritikos & D. L. Jaggard, Springer, New York, pp. 183–224.
7. Gulyaev, Yu.V., Nikitov, S.A., Potapov, A.A. & Davydov, A.G. 2005, "Design of Fractal Radio Systems: Numerical Analysis of Electromagnetic Properties of the Sierpinski Fractal Antenna", *Journal of Communications Technology and Electronics*, vol. 50, no. 9, pp. 988–993.
8. Oldham, K.B. & Spanier, J. 1974, *The Fractional Calculus*, Academic Press, New York.
9. Metzler, R. & Klafter, J. 2000, "The Random Walk's Guide to Anomalous Diffusion: A Fractional Dynamics Approach", *Phys. Rep.*, vol. 339, pp. 1–77.
10. Cristea L.L. & Steinsky, B. 2009, "Curves of infinite length in 4×4 -labyrinth fractals", *Geom Dedicata*, vol. 141, pp. 1–17.
11. Potapov, A.A., Slezkin, D.V. & Potapov, V.A. 2013, "Fractal Labyrinths as Geometry Base of New Kinds of Fractal Antennas and Fractal Antenna Arrays", *Radiotekhnika* (Moscow), no 8, pp. 31–36.

12. Potapov, A.A. & Slezkin, D.V. 2014, "Fractal Labyrinths as Small Antennas Development Base", *Zbornik radova Konferencije MIT - Matematicke i informacione tehnologije* (Vrnjackoj Banji od 5. do 9. septembra i u Becicima od 10. do 14. septembra 2013. godine), Kosovska Mitrovica: Prirodno-matematički fakultet Ulverziteta u Pristini (Serbia), pp. 553–559.
13. Potapov, A.A. 2006, "On the Fractal Radio Physics and Fractal Radio Electronics", *Proc. Jubilee Sci.-Tech. Conf. Dedicated to the 60th Birthday of OAO Radiotekhnicheskii Inst. im. A.L. Mints and Faculty of Aircraft Electronics, Moscow Aviation Inst. "Innovation in Radio Engineering Information - Telecommunication Technologies"*, Moscow 24 – 26 October, pt. 1, pp. 66–84.
14. Potapov, A.A. 2005, "New Information Radiophysical Technologies Fractal in Radiolocation: Fractal and Synergetic", *Proc. of 28th ESA Antenna Workshop on pace Antenna Systems and Technologies* (Noordwijk, The Netherlands, 31 May–03 June 2005), ESTEC, Noordwijk, pt. 2, pp. 1047–1050.
15. Potapov, A.A., Gulyaev, Yu.V., Nikitov, S.A., Pakhomov, A.A. & German, V.A. 2008, *Newest Images Processing Methods*, ed. Potapov, A.A., FIZMATLIT, Moscow.
16. Joannopoulos, J. D., Johnson, S.G., Win, J. N. & Meade, R. D. 2008, *Photonic crystals: molding the flow of light*, Princeton, Princeton Univ. Press.
17. Hou B., Liao X. Q. & Poon J. K. S. 2010, "Resonant infrared transmission and effective medium response of subwavelength H-fractal apertures", *Optics Express*, vol. 18, no. 4, pp. 3946–3951.
18. Potapov A.A. 2011, "Fractal Antennas, Nano-technologies, Resonances and Plasmons", *Achievements of Modern Radioelectronics*, no. 5, pp. 5-12.
19. Potapov, A.A., Bulavkin, V.V., German, V.A. & Vyacheslavova O.F. 2005, "Fractal Signature Methods for Profiling of Processed Surfaces", *Tech. Phys.*, vol. 50, no. 5, pp. 560–575.
20. A.A. Potapov, A.A. Potapov (Jr.), and V.A. Potapov, 2006, "The Fractal Capacitor, Fractal Operators, and Fractal Impedances," *Nonlinear World*, vol. 4, no. 4-5, pp. 172–187.
21. Potapov, A.A. 2009, "Can We Build an Adaptive Fractal Radio System?", *Proc. PIERS 2009 in Moscow "Progress in Electromagnetics Research Symp.*, Cambridge, MA, pp. 1798–1802.
22. Potapov A.A. 2009, "The Textures, Fractal, Scaling Effects and Fractional Operators as a Basis of New Methods of Information Processing and Fractal Radio Systems Designing", *Proc. SPIE*, vol. 7374, pp. 73740E-1–73740E-14.
23. Chua L.O. 1971, "Memristor - The Missing Circuit Element", *IEEE Transactions on Circuit Theory*, vol. CT-18, no. 5, pp. 507-519.
24. James M. Tour & Tao He. 2008, "The fourth element", *Nature*, vol. 453, no. 1, pp. 42-43.
25. Rekhviashvili S.Sh. & Potapov A.A. 2012, "Memristor and the Integral Quantum Hall Effect", *Journal of Communications Technology and Electronics*, vol. 57, no. 2, pp. 189–191.
26. Gulyaev, Yu.V., Nikitov, S.A., Potapov, A.A. & German, V.A. 2006, "Concepts of Scaling and Fractal Dimension in the Design of a Fractal Detector of Radio Signals", *Journal of Communications Technology and Electronics*, vol. 51, no. 8, pp. 909–916.
27. Potapov, A.A. 2003, "New Information Technology in Radar Detection of Low-Contrast Targets Based on Probabilistic Texture and Fractal Features", *Journal of Communications Technology and Electronics*, vol. 48, no. 9, pp. 1012–1029.
28. Potapov, A.A. 2007, "The Theory of Functionals of Stochastic Backscattered Fields", *Journal of Communications Technology and Electronics*, vol. 52, no.3, pp. 245–292.
29. Potapov, A.A. 2007, "On the Conception of Fractal Radio Systems and Fractal Radio Elements," *Nonlinear world*, vol. 5, no. 7-8, pp. 415–444.

30. Potapov, A.A., Gil'mutdinov, A.Kh. & Ushakov P.A., 2009, *Fractal Elements and Radio Systems: Physical Aspects*, ed. A.A. Potapov, Radiotekhnika, Moscow.
31. Potapov, A.A. 2014, "Oscillations, waves, structures and systems illustrated by examples of the global fractal-scaling method (measure zero sets, singularity, scaling, sample topology, sprites, jets, elves, memristors, oscillators, fractal mazes, robust antenna arrays and fractal detectors), *Non-Linear World* (Moscow), vol. 12, no. 4, pp. 3–38.
32. Potapov, A.A. 2015, "Fractal scaling or scale-invariant radiolocation principles and their application in radars with synthetic aperture", *III All-Russian Sci. Tech. Conf. "Development prospects of long-range detection radar, integrated systems, VKO infoware complexes and control and information processing complexes: "RTI VKO systems - 2015"*, Radiotekhnicheskii Inst. im. A. L. Mintsy, MGTU im. Bauman, Moscow, pp. 573–590.
33. Potapov, A.A. 2015, "Fractals and Scaling in the Radar: A Look from 2015", *Book of Abstracts 8th Int. Conf. (CHAOS' 2015) on Chaotic Modeling, Simulation and Applications* (26 - 29 May 2015, Paris, France), Henri Poincaré Institute, Paris, p. 102.
34. Potapov A.A. 2016, "Strategic Directions in Synthesis of New Topological Radar Detectors of Low-Contrast Targets against the Background of High-Intensity Noise from the Ground, Sea and Precipitations", *Proc. 2016 CIE Int. Conf. on Radar «Radar 2016»*, ed. Wu Shunjun (China, Guangzhou, Oct. 10 - 12, 2016), Chinese Institute of Electronics (CIE), Beijing, pp. 692–696.
35. Potapov A.A. 2016, "Fractality and scaling problems in radio location and radio physics with new methods of detection of low-contrast targets against a background of high intensity noise", *Proc. of the XV Int. Academic Congress "Fundamental and Applied Studies in the Modern World"* (United Kingdom, Oxford, 06 - 08 September 2016), Oxford University Press, Oxford, pp. 314-322.
36. Potapov A.A. 2016, "Analysis and Synthesis of Topological Radar Detectors of Low-Contrast Targets Against the Background of High Intensity Noise as a New Branch of Radiolocation and the Theory of Statistical Solutions", *Eurasian Physical Technical Journal*, vol. 13, no. 2(26), pp. 13–24.
37. Potapov A.A. 2016, "New Conception of Fractal Radio Device with Fractal Antennas and Fractal Detectors in the MIMO Systems", *Book of Abstracts 9th Int. Conf. (CHAOS' 2016) on Chaotic Modeling, Simulation and Applications* (23 - 26 May 2016, Senate House, University of London, London, UK), University of London, London, p. 85.
38. Potapov A.A. & Potapov V.A. 2016, "Scaling of the Fractals Antennas", *Book of Abstracts 9th Int. Conf. (CHAOS' 2016) on Chaotic Modeling, Simulation and Applications* (23 - 26 May 2016, Senate House, University of London, London, UK), University of London, London, pp. 86-87.
39. Potapov A.A. & Rassadin A.E. 2017, "The latest achievements in the development of the concept of radio- and optical systems quasiparticle based on Feynman integrals", *Proc. SPIE*, vol. 10342. doi: 10.1117/12.2270592.
40. Kostromina O.S., Potapov A.A., Rakut I.V., Rassadin A.E. & Tronov A. A. 2017, "On nonstationary regimes in electric circuits with ferroelectric negative capacitance", *Proc. of the Int. Siberian Conf. Control and Communications SIBCON-2017* (Kazakhstan, Astana, June 29 – 30, 2017), Tomsk IEEE Chapter & Student Branch of the Institute of Electrical and Electronics Engineers, Astana, Tomsk.
41. Potapov A.A. 2017, "Diffraction at frequency 36 GHz which are observed at radar scattering of an electromagnetic wave by a fractal surface and wave catastrophes in fractal randomly inhomogeneous media", *Proc. XIII Int. Conf. "Zababakhin Scientific Talks" Dedicated to 100th anniversary of academician E.I. Zababakhin* (Snezhinsk, March 20 – 24, 2017), RFNC–VNIITF, Snezhinsk, pp. 137-138.

42. Potapov A.A. 2016, "Chaos Theory, Fractals and Scaling in the Radar: A Look from 2015", in *The Foundations of Chaos Revisited: From Poincaré to Recent Advancements*, ed. C. Skiadas, Springer Int. Publ., Switzerland, Basel, pp. 195-218.
43. Potapov A.A. 2017, "Fractal Scaling or Scale-invariant Radar: A Breakthrough into the Future", *Universal Journal of Physics and Application* (USA), vol. 11, no. 1, pp. 13–32.