DEMANDS OF AGENTS IN A PROBLEM OF DEGRESSIVELY PROPORTIONAL ALLOCATION OF GOODS

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Abstract

The Treaty of Lisbon validated the degressive proportionality principle as a rule of allocating seats in the European Parliament. The principle can be comprehended twofold. First, the sequence of values resulting from an actual allocation is required to satisfy this condition, whereas second, it suffices that the sequence before rounding to integers satisfies the condition. Evidently, the latter understanding significantly increases the number of feasible solutions. The paper deals with one of the aspects of the degressive allocation rule perceived in this way. It is about adjusting the demands of agents in such a manner that the proportional allocation rule can be directly applied.

Key words: *degressive proportionality, fair division, European Parliament, voting*

1. INTRODUCTION

Proportionality originating from Aristotle is one of universally accepted standards of how benefits and obligations are divided. According to this principle, agents entitled to division participate in the allocation of a good proportionally to the level of specific characterizing quantities. Those quantities are typically expressed as numbers, but their interpretation can be significantly diverse. In case of dividing profits of a company, it is for example the number of owned shares, whereas in problems of voting for social representatives it is populations of respective electoral districts.

Generally, if a sequence of positive numbers $P = (p_1, p_2, \dots, p_n)$ is a sequence of such quantities assigned to n agents, then the allocation $S = (s_1, s_2, \dots, s_n)$ is proportional with respect to the sequence P if and only if $\frac{s_i}{p_i} = \frac{H}{V}$ holds for every i, where $H = s_1 + s_2 + \cdots + s_n$ and $V = p_1 + p_2 + \cdots + p_n$. In case of indivisible goods, the elements of the sequence S are also required to be natural numbers.

In certain cases there are also other possible schemes, which do not create proportional allocations. In particular, a situation can come about that the common interest of all agents will be to violate the rule of proportional allocation. It happens mostly when the quantities characterizing individual agents are at wide variance. Consider a case when residents of homes located on a dead-end street are to co-finance a made road. A proportional allocation rule based on the length of road used by respective residents can lead to the solution imposing an extremely high cost on the farthest resident who exploits the longest segment of the road, so that the investment would be unfeasible. As a result, all interested will suffer, because they will have to continue driving on a dirt road. Therefore they should agree to a different allocation of investment cost, so that upgrade works can be underway at all.

A similar case arises when allocating the European Parliament mandates among the member states. The interest of the community is to appropriately and venerably represent individual countries, and while the numbers of population are extremely different, the proportionality rule would exclude the least populated countries. For that reason the Treaty of Lisbon introduced the degressively proportional allocation of mandates.

2. DEGRESSIVE PROPORTIONALITY

Degressive proportionality of allocation formally implies that:

(1) $s_i \leq s_{i+1}$ holds for all $1 \leq i \leq n$, and

(2) $\frac{p_i}{s_i} \leq \frac{p_{i+1}}{s_{i+1}}$ holds for all $1 \leq i < n$.

It is easy to notice that (2) involves a weaker condition written as the inequality unlike proportional allocation involving the equality. In case of the allocation of mandates, more populated countries reduce their representation for the sake of less populated countries. In case of the allocation of investment cost, the degressive proportionality can be explained as incremental cost incurred by residents who less exploit the road and reduce the burden sustained by those, whose proportional shares would be greatest. Yet if characterized by unselfish care for the common good, even at the cost of some agents, however, the principle of degressive proportionality is basically flawed, i.e. a unique solution is not guaranteed. Even if the so-called boundary conditions of the allocation are given, i.e. the total amount of allocated good *H*, the smallest share $s_1 = m$ and the largest share $s_n = M$, the number of feasible solutions can be large. A sequence of quotas $Q = (q_1, q_2, \dots, q_n)$, where $q_1 = \frac{p_1 H}{V}$, is a uniquely determined allocation with respect to the proportional rule. The only problem in this case can be the integer representation of the sequence *Q* of indivisible goods. Thereby these two principles are applied in a different way. Degressive proportionality requires a precise specification of additional conditions so as to achieve a unique solution. There are several ideas put forward in the subject matter literature (Grimmett, G., Laslier, J. F., Pukelsheim, F., Ramirez-González, V., Rose, R., Słomczyński, W., Zachariasen, M., Życzkowski, K., 2011; González, V. R., Aroza, J. M., Garcia, A. M., 2012; Delgado-Márquez, B., Kaeding, M., Palomares, A., 2013; Łyko J., Rudek R., 2013) to deal with this issue. Below we propose adjustment of claims as a method that is nearest to the idea of seeking a so-called allocation function (Słomczyński W., Życzkowski K., 2012).

3. ADJUSTMENT OF CLAIMS

All previous approaches to deal with the uniqueness problem of degressively proportional allocation presented in the literature focused on a direct specification of the desired sequence S. Such allocation can be also obtained indirectly, firstly adjusting the claims of agents. This approach, even if mathematically similar to seeking the allocation function, is actually a better sign of the idea of degressive proportionality. The agents participating either in the division of the good or in collective obligations willingly adjust either their claims to possess the good or their shares in common burden, in a way that goes best with their shared goals. This can be explained so that an originally defined sequence of claims *P* is not appropriate to be applied in the particular case of proportional division, therefore the actual quantities assigned to individual agents should be modified in such a way that the application is possible. As a result, another sequence $P' = (p'_1, p'_2, \dots, p'_n)$ is obtained that is degressively proportional with respect to the sequence *P*, and satisfies $p'_1 + p'_2 + \dots + p'_n = V$ but the proportional solution does not contradict the general interest. Thus all known methods of proportional allocation can be applied. Such an approach is also justified by the idea of the so-called unrounded degressive proportionality where the condition of degressive proportionality must be satisfied only before rounding to integers. In this way, the interpretation is unrestrained and similar to the integer representation of the sequence *Q* of quotas in a proportional division. The boundary conditions demonstrate how the adjustment is carried out in case of the smallest and the greatest agent. The proportional allocation will assign the amount $s_1 = m$ of a good to the smallest agent if the respective value in the adjusted sequence of claims equals $p'_1 = \frac{mV}{H}$. Similarly, in case of the greatest agent and the boundary condition $s_n = M$, the relevant value is $p'_n = \frac{MV}{H}$.

Because no other indications are given, the remaining values of the sequence P' must be arbitrary. One of the relevant proposals is to find the adjustment function, i.e. the function $f: [p_1, p_n] \rightarrow [p'_1, p'_n]$ such that the sequence $f(P) = (f(p_1), f(p_2), \dots, f(p_n))$ is degressively proportional with respect to the

sequence *P*, with $f(p_1) = \frac{mV}{H}$, $f(p_n) = \frac{MV}{H}$ and $f(p_1) + f(p_2) + \cdots + f(p_n) = V$. In such a case we may assume that $p_i' = f(p_i)$.

Another proposal, more natural in view of accepting the Aristotelian principle, is the adjustment by means of the rule of proportion. To this end, the equation of the linear function $f(x)$ suffices whose graph, the straight line, passes through the points (p_1, p_1') and (p_n, p_n'), and the values of the function in points $p_1, p_2, \dots, p_n$ can be calculated. This function is given by the formula $f(x) = \frac{p_n' - p_1'}{p_n - p_1}x + \frac{p_n p_1' - p_n' p_1}{p_n - p_1}$. Certainly, the sequence $f(P) = \big(f(p_1), f(p_2), \dots, f(p_n)\big)$ is degressively proportional with respect to the sequence *P*. Moreover, we have $f(p_1) = \frac{mV}{H}$ and $f(p_n) = \frac{MV}{H}$, but generally $f(p_1) + f(p_2) + \cdots + f(p_n) = V_f$ is not equal *V*, i.e. it is not a simple correction or reallocation of claims. Accordingly, in order to obtain a desirable sequence *P'* we have to distribute the amount $|V - V_f|$ of the good, i.e. we give it or take away, depending on the sign of the difference $|V - V_f|$ One approach is again proportionality, this time with respect to the sequence $f(P)$, thus it can be referred to as the method of Double Proportion. Hence, considering the boundary conditions, we should exclude the first and the last term, assuming that $p_1' = f(p_1)$ and $p_n' = f(p_n)$, with other terms defined by the formula $p_i' = f(p_i) + f(p_i)\frac{(V - V_f)}{\big(V_f - f(p_1) - f(p_n)\big)}$. The sequence P'obtained in this way satisfies the boundary conditions, but it is not always degressively proportional with respect to the sequence *P*. As shown in (Łyko J., Maciuk A., 2016), if $V > V_f$, then the condition of degressive proportionality is satisfied for all $1 < i < n$. In other cases we should also monitor the allocation in this stage so that the constraints resulting from the conditions (1) and (2) of degressive proportionality are not violated.

4. CASE STUDY

The presented adjustment method of claims can be applied to the allocation of mandates in the European Parliament. The numerical illustration is based on the sequence P of populations in the member states given in the Council Decision (Council Decision 2016/2353, 2016). The first allocation was carried out assuming the membership of the United Kingdom, and the results are presented in table 1. The boundary conditions in this case were not modified and assumed as *H*=751, *m*=6, and *M*=96.

Table 2 shows the results obtained in case of 27 member states, exclusive of the United Kingdom, i.e. after the actual Brexit. In these cases, the three different values of *H* were assumed, *H*=751, *H*=701, *H*=678. The two latter numbers follow from the debate in the Committee on Constitutional Affairs where the restriction of the assembly size after Brexit was discussed. The second number is simply the current total size minus the number of mandates held by the British representatives in the present term. The first number is greater by 23 mandates. This size is necessary to satisfy the principle of degressive proportionality of allocation declared in the Treaty of Lisbon that used to be repeatedly violated due to political reasons ever since the principle was adopted. The additional 23 mandates can ensure the degressive proportionality of allocation with no harm to political status quo.

In each case the allocation based on the adjusted sequence of populations *P'* was indicated. The adjustment of claims was carried out using the method of Double Proportion. The integer representation was obtained using the conventional divisor method with rounding to the nearest integer.

Table 1. Allocation of mandates using the method of Double Proportion (DP) and the Cambridge Compromise (CC) for 28 member states of the European Union

Lp.	Country	P	P'	DP	CC
1	Malta	434403	4081443,7	6	6
2	Luxembourg	576249	4218178,9	6	6
3	Cyprus	848319	4436558,9	7	7
4	Estonia	1315944	4811903,3	7	7
5	Latvia	1968957	5336051,4	8	8
6	Slovenia	2064188	5412489,6	8	8
7	Lithuania	2888558	6074179,4	9	9
8	Croatia	4190669	7119333,2	10	10
9	Ireland	4664156	7499382,8	11	11
10	Slovakia	5407910	8096365,2	12	12
11	Finland	5465408	8142516,6	12	12
12	Denmark	5700917	8331550,5	12	12
13	Bulgaria	7153784	9497710,3	14	14
14	Austria	8711500	10748028	16	16
15	Hungary	9830485	11646194	17	17
16	Sweden	9998000	11780652	17	17
17	Portugal	10341330	12056229	18	18
18	Czech Rep.	10445783	12140070	18	18
19	Greece	10793526	12419190	18	18
20	Belgium	11289853	12817572	19	19
21	Netherlands	17235349	17589790	26	26
22	Romania	19759968	19616204	29	29
23	Poland	37967209	34230448	50	50
24	Spain	46438422	41029961	60	60
25	Italy	61302519	52960793	78	78
26	UK	65341184	56202471	83	83
27	France	66661621	57262335	84	84
28	Germany	82064489	65303099	96	96
Total		510860699	510860699	751	751

Source: own works.

Table 2. Allocation of mandates using the method of Double Proportion (DP) and the Cambridge Compromise (CC), exclusive of the United Kingdom (*H*=751, *H*=701, *H*=678)

Lp.	Country	P	P'	DP	CC	P'	DP	CC	P'	DP	CC
1	Malta	434403	3559410,2	6	6	3813291,1	6	6	3942650,6	6	6
2	Luxembourg	576249	4403500,8	7	6	4354137,4	7	6	4328985,3	7	6
3	Cyprus	848319	4618058,9	8	7	4566290,3	7	7	4539912,7	7	7
4	Estonia	1315944	4986834,3	8	7	4930931,7	8	7	4902447,7	7	7
5	Latvia	1968957	5501809,2	9	8	5440133,6	9	8	5408708,2	8	8
6	Slovenia	2064188	5576909,6	9	8	5514392,2	9	8	5482537,8	8	8
7	Lithuania	2888558	6227018,9	11	10	6157213,8	10	9	6121646,1	9	9
8	Croatia	4190669	7253881,2	12	11	7172564,9	11	11	7131131,9	11	11
9	Ireland	4664156	7627279,5	13	12	7541777,3	12	11	7498211,6	11	11
10	Slovakia	5407910	8213813,9	14	13	8121736,7	13	12	8074820,7	12	12
11	Finland	5465408	8259157,6	14	13	8166572,1	13	12	8119397,1	12	12
12	Denmark	5700917	8444883,2	14	14	8350215,7	13	13	8301979,9	13	12
13	Bulgaria	7153784	9590633,7	16	16	9483122,3	15	15	9428342,2	14	14
14	Austria	8711500	10819070	18	18	10697787	17	17	10635991	16	16
15	Hungary	9830485	11701516	20	19	11570342	18	18	11503505	18	17
16	Sweden	9998000	11833621	20	20	11700965	18	18	11633374	18	18
17	Portugal	10341330	12104376	20	20	11968685	19	19	11899547	18	18
18	Czech Rep.	10445783	12186749	21	20	12050135	19	19	11980526	18	18
19	Greece	10793526	12460983	21	21	12321295	19	19	12250120	19	19
20	Belgium	11289853	12852394	22	22	12708318	20	20	12634907	19	19
21	Netherlands	17235349	17541092	30	30	17344455	27	27	17244264	26	26
22	Romania	19759968	19532041	33	34	19313085	30	31	19201522	29	29
23	Poland	37967209	33890517	57	59	33510602	53	54	33317025	51	51
24	Spain	46438422	40571030	69	72	40116226	63	65	39884491	61	62
25	Italy	61302519	52293057	88	93	51706849	81	83	51408160	79	80
26	France	66661621	56519317	95	96	55885733	88	90	55562904	85	86
27	Germany	82064489	56950564	96	96	61012658	96	96	63082409	96	96
Total		445519516	445519516	751	751	445519516	701	701	445519516	678	678

Source: own works.

5. CONCLUSIONS

In all examined cases, the sequence *P'* obtained by means of Double Proportion is degressively proportional with respect to the sequence *P*. Therefore, it can be used to generate the integer allocation that is consistent with the principle of unrounded degressive proportionality. This principle is also satisfied by the allocation proposed in 2011, known as the Cambridge Compromise. This allocation was recommended as suitable for application in the subsequent terms of the European Parliament.

When comparing the results obtained, one can find that the Cambridge Compromise is more favorable for larger countries. Generally they are allocated more mandates than in case of adjusting the claims by means of Double Proportion, and then allocating mandates by use of the divisor method with rounding to the nearest integer. In the latter case, the smaller countries are favored because the division of the surplus $V - V_f$ after the first proportional adjustment of claims is carried out proportionally to the values $f(p_i)$, not to the values of sequence terms pi. Thus, the greater values of claims of smaller agents are taken. The greater surplus $V - V_f$ with respect to the total value *V*, the greater this difference apparently. It is demonstrated in tables with the results of allocations after Brexit.

The division of the difference $V - V_f$ proportionally to the values of the sequence *P* will evidently move the solution towards the result obtained in the allocation proposed by the Cambridge Compromise, because it better represents the original sequence *P* of claims. However, assuming that its adjustment is necessary due to the desirable correct representation of all member states, one has to admit in these circumstances that the division of the surplus $V - V_f$ is more natural if carried out proportionally to the preliminary adjustment of $f(p_i)$.

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