

MILK COLLECTION NETWORK DESIGN IN A FUZZY ENVIRONMENT

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Abstract

Milk and dairy products have been consumed around the world for thousands of years. There are many factors that determine the quality of raw milk. Animal health, seasonal changes, and other circumstances are among the factors that influence the amount and quality of the milk produced. One of the important processes in the milk supply chain is the transportation of raw milk from farms to dairies by tankers under uncertain weather, road, and safety conditions. The problem of milk collection involves the obtaining of raw milk from producers at different locations by multi-compartment tankers. In this paper, we present a novel fuzzy mathematical model for a multi-compartment milk collection problem with split deliveries under uncertainty. The uncertainty in this model is regarding the amount of changeable volume of milk production and service duration time. The model aims to minimize the total duration time by assigning milk to tankers. Results show that uncertainty has a very critical effect on the design of collection networks.

Keywords: milk collection problem, vehicle routing problem, fuzzy optimization

1. INTRODUCTION

It is estimated that humans began consuming milk sometime between 7000 and 9000 BC. Milk has a nutritional value that can be described as a miracle of life, and its place in the human diet is as old as history. It is a nutrient-rich fluid which contains calcium and various vitamins, therefore food vital for humanity. Approximately 150 million households around the world are engaged in milk production. Developing countries account for a large proportion of production worldwide. In developing countries, milk is produced by small-scale producers who are dispersed in rural areas. According to the Food and Agriculture Organization of the United Nations, an estimated 80 to 90 percent of milk in developing countries is produced by small-scale farms.

Milk should be cooled to 4 °C within the first three to four hours after milking to combat bacteria growth. The cooling process is done by refrigerators or cooling tanks. Milk collection centers have been established to cool the milk for many reasons, such as lack of power in rural areas, technical problems, and the high cost of cooling systems. The milk which is separated and cooled by quality type in each collection center or large-scale farm is collected by tankers and delivering to a milk processing center. The milk must be delivered to the dairy factory within a specific time limit or it will be considered spoiled, so the total duration of each tanker's route must be well planned. There are many factors that can affect this duration, for instance, road conditions, the weather, and the farm service time to the tanker. Farm service time to the tanker can be extended by changing the various reason, for example, loading personnel skills, cleaning of loading tool, etc. The amount of milk production can be uncertain for many reasons, such as milk being highly perishable food, seasonal changes, animal problems, and technical problems. Technical problems occur frequently in rural areas, for example power outages due to the weather etc.

The Milk Collection Problem (MCP) has existed in the literature since the 1950s. The MCP is basically concerned with the collection of raw milk at dairy factories via tankers under problem-specific constraints. In the literature, this problem is considered to be in the class of location routing problems (LRP) (Prodhon and Prins, 2014), rich vehicle routing problems (RVRP) (Lahyani et al., 2015), and truck and trailer routing problems (TTRP) (Cuda et al., 2015).

Jones (1953) identified regional milk collection cost items in his study and used these costs to establish the order of visits to farms assigned to tankers. Basnet et al. (1999) proposed an integer mathematical programming model for scheduling milk collection tankers. Frye (2013) analyzed

conditions and rules that should be considered during milk collection. In addition, Thomson et al. (2005) and Endrizzi et al. (2012) examined the effects of milk quality, mixing of different kinds of milk, the waiting time of milk in collection centers, and delivery time to the dairy factory on the quality of final milk products. In most of the studies in MCP literature, the problem is generally solved by separating into subproblems (assignment and routing) or sequentially (first assignment, then routing).

In studies conducted by Hoff and Løkketangen (2007), Caramia and Guerriero (2010), Pasha et al. (2014), and Amiama et al. (2015) some vehicle routing constraints and location constraints were also taken into consideration. Caramia and Guerriero (2010) took into account the collection of the different milk types without mixing. However, in the solution phase, the problem was solved sequentially. This reduced the quality of the solution, although it enables more realistic solutions to be obtained in practice by taking into account problem-specific constraints. In the literature, the stochastic nature of the MCP problem has only been considered by Huang et al. (2018). They proposed a mathematical programming model for the problem by considering the stochastic travel time between farmers and the dairy factory. In order to solve the problem, authors developed a set covering problem (SCP) and several heuristics-based solution approaches. Even as the authors considered the stochastic nature of travel times, they excluded the uncertainty of raw milk production by farms and collection centers and stochastic service time of trucks in the facilities.

In this study, a fuzzy mathematical model has been developed which considers the uncertainty possibilities above and provides a concurrent solution with split demand for transporting milk via tankers with different capacities. The uncertainty in this model is the amount of milk production, speed of the trucks, and service time spent in the facilities. The remaining parts of this manuscript are structured as follows. Section 2 describes the nature of the problem and the model of the problem with fuzzy parameters. Section 3 presents a real-life hypothetical case study to evaluate this model's performance. Finally, the concluding remarks and some directions for future research are given in Section 0.

2. MATHEMATICAL MODEL

MCP is concerned with decisions of selecting farms/milk collection centers to be visited, type of tanker to be used; tanks to be stored; and finally, the visiting sequence to be optimized. For this reason, MCP has two kinds of subproblems: The Vehicle Assignment Problem (VAP) and the Vehicle Routing Problem (VRP). Fig. 1 represents the milk collection problem.

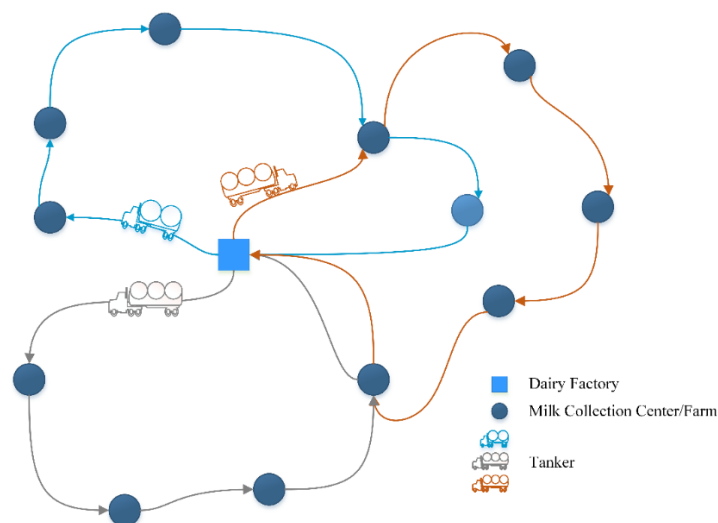


Fig. 1. Milk Collection Problem

The amount of milk produced depends on many factors. For instance, failure of the cooler tank will cause milk to spoil and be discarded, decreasing the total amount. Another example of variability is that less milk is produced in the summertime than in spring and autumn. In the summertime, besides the amount of producing milk decrease, air temperature is getting more and more increased. Because of this temperature increase, raw milk could be spoiled rapidly. Milk is a perishable food; therefore, all these factors should be considered when planning the milk collection process. Therefore, expected milk production amounts are considered to be fuzzy.

The change in the amount of milk produced causes a change in the total service time. In addition, service times may vary for various reasons. For example, the skill of the personnel delivering the milk is one of the important factors affecting its duration time. For this reason, the service time of the farm/milk collection center is considered fuzzy regardless of demand divisibility.

During the collection of raw milk, tanker speed can vary over time because of weather conditions, location problem, driving safety, etc. Due to this uncertainty, tanker speed is also considered as fuzzy numbers.

In this study, we propose a fuzzy mathematical model to solve the milk collection problem by integrating VAP and VRP. This problem is modeled as a multi-compartment vehicle routing problem with split deliveries (MC-VRP-SD). The mathematical model's assumptions for this problem are:

- The milk produced by each farm/milk collection center must be collected and transported to the dairy factory according to supplier agreement.
- Raw milk is ready for collection under ideal temperature conditions.
- The amount of produced raw milk is considered fuzzy.
- The amount of raw milk collected cannot exceed the tankers' capacities.
- Each tanker may visit each farms/collection center just once.
- Each farm/collection center can be visited by different tankers if required.
- Each tanker must start and end its route at the dairy factory.
- Raw milk must be collected within the specified time limit.
- The travel speed of the tankers is assumed to be fuzzy.
- Service times are considered fuzzy.
- The division of raw milk does not affect the service time.

The objective is to minimize the total distance traveled by the tankers in the network. The notation used for the MCP is:

Indices

$i, j \in N$	set of nodes (0 = dairy factory, 1..N = farms/collection centers)
$k \in K$	set of tankers
$l \in L$	set of tanks on a tanker

Parameters

C_{ij}	distance between nodes i and j
Q_{kl}	capacity of tank l on tanker k
$\tilde{D}_i$	amount of raw milk to be collected from farm/collection center i
$\tilde{S}_i$	service time at farm/collection center i

$\tilde{V}$ average speed of tankers

T maximum route duration for delivering raw milk to the dairy factory

P a sufficiently big number

Decision variables

x_{ijk} 1: if the arc between the node i and j is served by tanker k , 0 otherwise

w_{kl} 1: if tank l on tanker k is assigned the milk, 0 otherwise

z_{ikl} 1: if the raw milk is produced at node i and it is loaded into tank l on tanker k , 0 otherwise

y_{ik} 1: if tanker k visited node i , 0 otherwise

b_i 1: if the raw milk stored at node i assigned, 0 otherwise

f_{iklm} Fulfillment ratio for tank l on tanker k , which is loaded with raw milk from node i

Multi tanker with split deliveries under the uncertainty model is;

$$\text{minimize } Z = \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} C_{ij} x_{ijk} / \tilde{V} + \sum_{i \in N} \sum_{k \in K} \tilde{S}_i Y_{ik} \quad (1)$$

s.to

$$z_{iklm} \leq y_{ik} \quad \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (2)$$

$$z_{ikl} \leq w_{kl} \quad \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (3)$$

$$w_{kl} \leq \sum_{i \in N \setminus \{0\}} z_{ikl} \quad k \in K, l \in L \quad (4)$$

$$\sum_{j \in N} x_{ijk} + \sum_{j \in N} x_{jik} = 2y_{ik} \quad \forall i \in N, k \in K \quad (5)$$

$$\sum_{i \in Z} \sum_{j \in Z, i < j} x_{ijk} \leq \sum_{i \in Z} y_{ik} - y_{zk} \quad Z \subseteq N \setminus \{0\}, z \in Z, k \in K \quad (6)$$

$$\sum_{j \in N \setminus \{0\}} x_{jik} \leq y_{ik} \quad \forall i \in N \setminus \{0\}, k \in K \quad (7)$$

$$x_{i0k} \leq 2y_{ik} \quad \forall i \in N \setminus \{0\}, k \in K \quad (8)$$

$$\sum_{i \in N} \sum_{k \in K} x_{iik} = 0 \quad (9)$$

$$\sum_{k \in K} \sum_{l \in L} f_{iklm} = b_i \quad \forall i \in N \setminus \{0\} \quad (10)$$

$$b_i P \geq \tilde{D}_i \quad \forall i \in N \quad (11)$$

$$b_i \leq \tilde{D}_i \quad \forall i \in N \quad (12)$$

$$\sum_{i \in N \setminus \{0\}} \tilde{D}_i f_{ikl} \leq Q_{kl} \quad k \in K, l \in L \quad (13)$$

$$z_{ikl} \geq f_{ikl} \quad \forall i \in N, k \in K, l \in L \quad (14)$$

$$\sum_{i \in N} \sum_{\substack{j \in N \\ i \neq j}} \left(\frac{C_{ij} x_{ijk}}{\tilde{V}} \right) + \sum_{i \in N} \tilde{S}_i y_{ik} \leq T \quad k \in K \quad (15)$$

$$f_{ikl} \geq 0 \quad \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (16)$$

$$x_{ijk} \in \{0,1\} \quad \forall i, j \in N \setminus \{0\}, k \in K \quad (17)$$

$$w_{kl}, z_{ikl}, y_{ik}, b_i \in \{0,1\} \quad \forall i, j \in N, k \in K \quad (18)$$

Objective function (1) seeks to minimize the total duration time in the network. The total duration time is the sum of the total service time and total transport time. Constraints (2) and (3) ensure that if the raw milk which is produced at node i is assigned to tank l on tanker k , then tanker k must visit node i and tank l on tanker k will be loaded with milk. Constraint (4) means that if tank l on tanker k is assigned to raw milk, then the milk produced by at least one node is assigned to tank l on tanker k . Constraint (5) specifies the degree of each node while constraint (6) prohibits illegal subtours. Constraints (7) and (8) are flow constraints, a way to ensure that if at least one j producer is visited, i.e. (7), or tanker k returns from node i to node 0 (the dairy factory), i.e. (8), so node i must have been visited by tanker k . Constraint (9) aims to improve model performance. Constraints (10), (11), and (12) ensure the division of raw milk that is ready to collect. Constraint (13) guarantees that the amount of raw milk assigned to tank l on tanker k cannot exceed the tank's capacity. Constraint (14) ensures the connection between a node assignment and the ratio of the demand related to this node. Constraint (15) provides that the duration of time milk is stored in a tank on the tanker does not exceed the maximum time limit. Constraints (16), (17), and (18) ensure the binary and non-negativity restrictions on decision variables.

The parameters are denoted by fuzzy numbers, which reflect the data more efficiently and realistically when the amount of data is insufficient. In the fuzzy model, fuzzy numbers may be transformed into their crisp equivalents by using defuzzification pursuant to satisfying membership functions at a defined degree (Carlsson and Korhonen, 1986). There are several models developed in the literature to transform fuzzy models with indefinite parameters into equivalent crisp models. In this study, the ranking fuzzy number through the comparison of its expected intervals approach was used for defuzzification. This method was developed by Jiménez (1996) and enhanced by Jiménez et al. (2007) and Parra et al. (2005).

3. HYPOTHETICAL CASE STUDY

For the purpose of testing the given mathematical model, a real lifelike hypothetical case study was created. In this case study, there are nine farms/collection centers and one dairy factory. Fig. 2 shows the network for the case study. Tankers collect the milk within the specified time limit. In this hypothetical case study, tanker speed is considered a triangular fuzzy number which is [60,45,30] km/hour.

As mentioned earlier, because milk is a highly perishable, the total duration time is important for milk quality. For this case study, the total duration time limit is set at 3.5 hours (210 minutes). Table 1 displays the produced amount of milk ($\tilde{D}_i$), the service time ($\tilde{S}_i$), and the location of the nodes along the X and Y axes. The amount of milk produced, and the service time are presented as triangular fuzzy numbers. Table 2 shows the capacity of each tanker and the number of tanks each has.

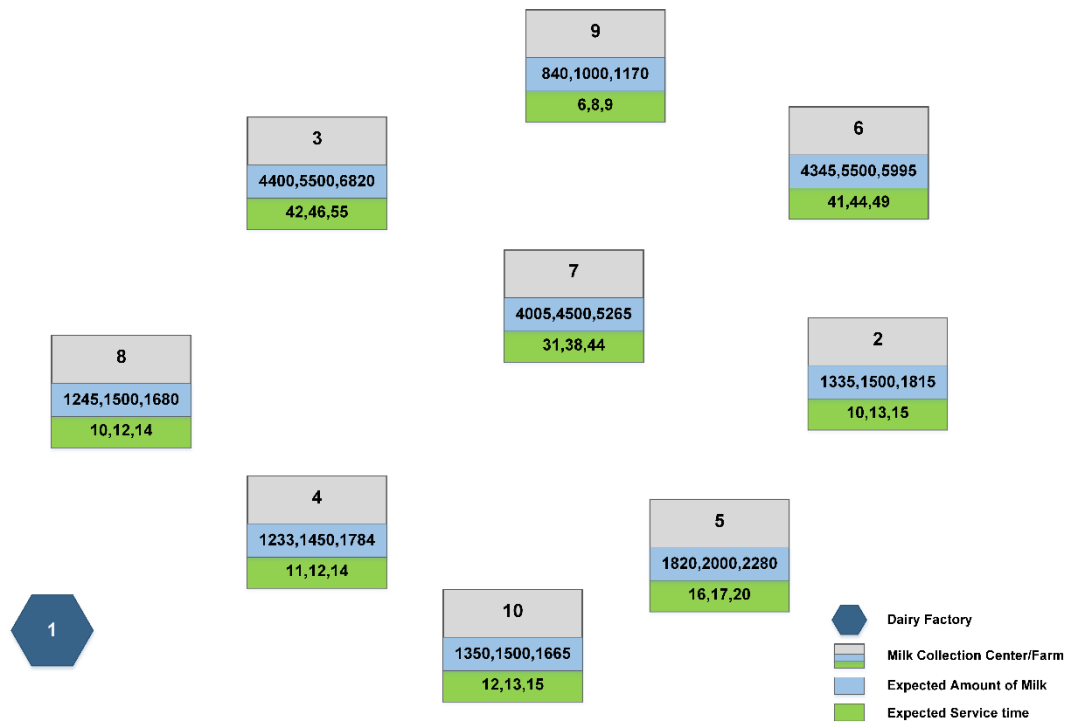


Fig. 2. Hypothetical Case Study Network

Node	$\tilde{D}_i$	$\tilde{S}_i$	X	Y
1	[0,0,0]	[0,0,0]	0	0
2	[1335,1500,1815]	[10,13,15]	975.690	411.985
3	[4400,5500,6820]	[42,46,55]	457.692	977.378
4	[1233,1450,1784]	[11,12,14]	336.499	148.107
5	[1820,2000,2280]	[16,17,20]	867.212	241.269
6	[4345,5500,5995]	[41,44,49]	919.882	547.194
7	[4005,4500,5265]	[31,38,44]	766.775	360.531
8	[1245,1500,1680]	[10,12,14]	376.221	264.250
9	[840,1000,1170]	[6,8,9]	998.429	979.084
10	[1350,1500,1665]	[12,13,15]	574.582	240.938

Table 1. Case Study Data Set

	Tank 1	Tank 2	Tank 3
Tanker 1	2000		
Tanker 2	5000	5000	5000
Tanker 3	3000	3000	3000
Tanker 4	2000	2000	

Table 2. Case Study Tank & Tanker Capacity

The proposed model was solved using GAMS 23.4.3 with CPLEX 12.1.0 solver. All computational results were produced on a workstation with an Intel Xeon 2.5 GHz processor and 8 GB of RAM. In this study, different α values were considered to show how the model output changes and accordingly demonstrates changes in the route and the tankers used. Therefore, the mathematical model was solved by changing α values between 0 and 1. The fuzzy constraints and objective function values were recalculated each time the α value was changed. The case study results for each of eleven α values ranging between 0 and 1 are summarized in Table 3.

With an α value of 0, the results obtained were unrealistic, while for an α value of 1, they were completely realistic. When the results of the model were examined, it was observed that the model results were stable within specific α value intervals. These occur at $\alpha=0.1$ to 0.2 and $\alpha=0.4$ to 0.5.

In cases where the amount of milk to be collected from a farm/milk collection center is high, the model allows for splitting the amount of milk to be collected. This milk is collected by different tankers or in different tanks on the tankers. Because of this qualification, the model is able to tolerate the change of α value in the specified intervals.

Scenario No	α	Total Duration Time (min)	Total Transport Time (min)	Total Service Time (min)	Total Distance (km)	Processing time (sec)	Number of Vehicle
1	0	914.774	709.775	205	532.331	47.92	2
2	0.1	924.686	719.687	205	539.765	108.52	2
3	0.2	924.686	719.687	205	539.765	71.72	2
4	0.3	936.956	719.707	217.25	539.78	114.75	2
5	0.4	1018.642	813.643	205	610.232	324.3	3
6	0.5	1018.642	813.643	205	610.232	235.33	3
7	0.6	1102.411	885.161	217.25	663.871	1192.53	3
8	0.7	1204.664	999.664	205	749.748	1490.67	3
9	0.8	1212.875	995.625	217.25	746.719	401.95	3
10	0.9	1216.934	999.684	217.25	749.763	715.36	3
11	1.0	1424.822	1219.823	205	914.857	551.3	4

Table 3. Model results for different α values

Table 3 indicates that total duration time varied between 914.774 and 1424.822 minutes, the total transport time between 709.775 and 1219.823 minutes, the total service time between 205 and 217.25 minutes, and finally the total travelled distance for the tankers, between 532.331 and 914.857 km. While the total transport time was highly sensitive to changes in α value, the effect on total service time was weaker than the total transport time. Because the division of the amount of milk does not affect the service time, the total service time was less affected by the changes in α value.

Total transport time is highly sensitive to the changes in the specified α values. When the model could not tolerate the changes in the α value, it reached the optimum value by changing the route and the tanker. In this situation, the total transport time, total service time, and of course the total duration time increased.

When all the results were analyzed, it became clear that it was unrealistic to choose the optimum model solution when the α value was 0. In this value, the model uses 2 tankers for optimal solution, and the amount of milk production will be less than usual. For α values between 0.2 and 0.9, the

model decided to use 3 tankers for optimum solutions and finally when α value is taken 1.0, the model decided to use 4 tankers to for an optimum solution.

In this hypothetical case study, the changes in the amount of raw milk produced are the most effective parameter on the total duration time, followed by the service time, and the speed of the tanker. In the data used in this study, the lowest milk production was expected to be 22,511 liters and the highest 26,462 liters, a difference of 3950.5 liters. In scenario 8, where α is 0.7, and scenario 11, where α is 1.0, the difference between the expected amounts of milk to be produced is approximately 1.186 liters. Considering the lowest and the highest expected amount of milk production and the varying tanker utilization rates depending on these values, scenario 8 which α value is taken as 0.7 would be appropriate for this case study.

4. CONCLUSION

Milk is a very important supportive food for human health. Since it is rapidly perishable food, its transportation to dairy factories should be well planned. The milk collection problem (MCP) is fundamentally concerned with the collection of raw milk from farms/milk collection centers via tankers under problem-specific constraints. The collected milk must be delivered to the milk production factory without spoiling. Therefore, it is very important for the whole supply chain to ensure that the raw milk is collected within the specified time limit.

In this study, we proposed a fuzzy mathematical model to solve the MCP by integrating VAP and VRP. The proposed model was solved using the ranking methods of fuzzy numbers through the comparison of their expected intervals method. A real-life hypothetical case study was constituted and solved by using different membership degrees in order to illustrate the importance of the fuzzy parameters and their effects on the results. The most important fuzzy parameter affecting the results in this study was the milk production supply.

The proposed linear programming model can be applied to not only one type of quality milk collection but also to milk collection problems with different quality types, or a study where service time is affected by the division of demand. Heuristic solution procedures must be applied to solve the instances with high number of nodes like the natural size of the problem.

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