SPECTRAL METHOD OF ARTIFICIAL BIPOLAR NEURON SYNTHESIS

Alexander N. Sychev

Embassy of the Russian Federation in Latvia, Riga, LV-1010, Latvia

Abstract

The structure of an artificial bipolar neuron with local linear filters at the inputs is proposed. A spectral algorithm for the synthesis of the structure is described. An example of synthesis for a randomly assigned logical function is demonstrated. The effectiveness of the application of the structure is estimated.

Key words: *Walsh spectrum, bipolar neuron, notch filter*

1. INTRODUCTION

In pattern recognition, image pixel signals can be represented by binary variables. Because of this, the classes of images are quite comparable with certain logical functions (LF). The number of pixels used often determines the large dimension of these functions. It is clear that the processes of forming, transmitting, receiving and analyzing images are accompanied by various kinds of interference and errors, with are reflected in the nature of the signals of pixels and functions. As a result, pattern recognition becomes the task of analyzing big data containing noise and minor details.

The neural network performing the recognition must obviously carry out the appropriate filtering, which ensures the elimination of interference and the identification of the useful image signal. In engineering practice and communication theory the characteristics of the transmitted signal must be matched with the characteristics of the communication channel for a greater reliability of data transmission in the conditions of interference. The signal in demand should be noise-like. For this, communication devices have special structural elements that provide the introduction and removal of noise. Similarly, it is advisable to have elements with appropriate noise characteristics as part of a neural network in order to transform the incoming information efficiently.

Traditionally used artificial neurons appear to be quite monotonous logical functions, the properties of which are quite different from the characteristics of noise-like ones. At the same time, in biological prototypes, there are neural structures that provide signal isolation against the background of disturbances. For example, "on"("off")-structure of the retina. Analysis [1,2] of the composition of these structures indicates a bipolar neuron, the function of which has the desired noise-like properties. It will be useful to introduce an artificial analogue of a bipolar neuron into the element base of neural networks that carry out pattern recognition.

The aim this research papers is to develop a method for the synthesis of an artificial bipolar neuron using Walsh spectral representation in order to solve pattern recognition problems.

2. THE LOGICAL STRUCTURE OF THE ARTIFICIAL BIPOLAR NEURON

Figure 1 show the logical circuit of the "on"("off")-model – the structure of the retina, which was previously considered in [1]. The function of a bipolar neuron here is performed by a two-threshold element (TTE) with positive and negative values of the input weights. The OR-NOT element, combining the outputs of the inverters, represents the composition of the cones with the inputs of the horizontal cell of the retina. In essence, this connection forms a local filter that provides an acceptable approximation of a certain fragment of the domain of a function representing a class of images. The connection of a horizontal cell with a bipolar neuron is characterized by an inhibitory effect. Therefore, the output of the element OR-NOT is connected to the input of the TTE having a negative weight value.

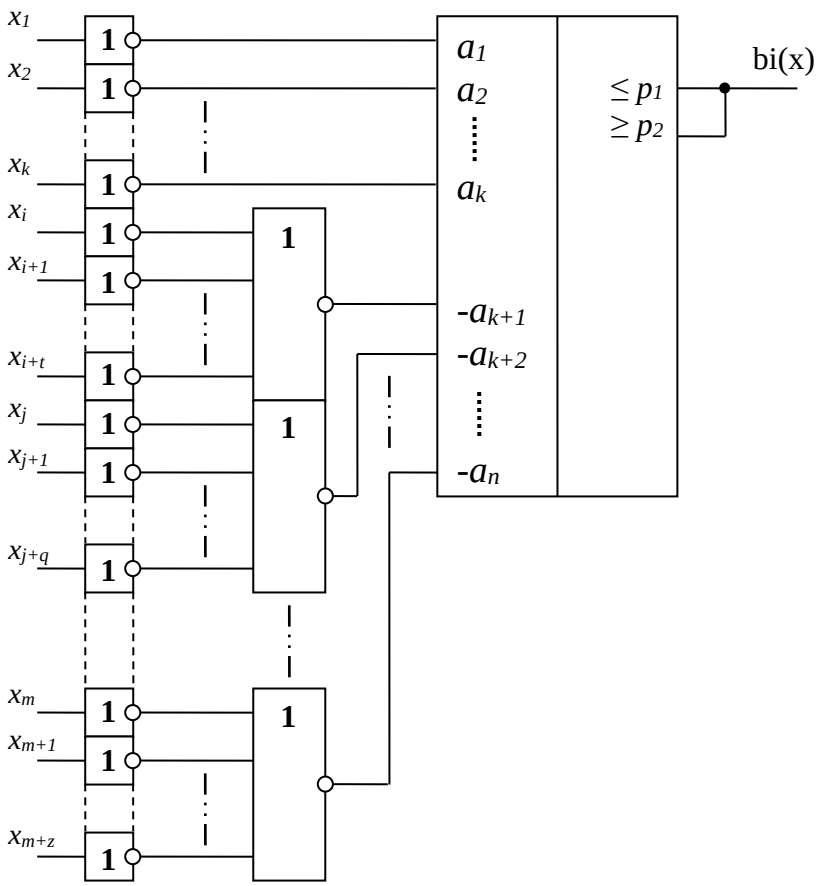


Fig. 1. Logical scheme of the excitement function of bipolar cell with 'off'- center

To increase the efficiency of using a bipolar neuron, local filters should be placed both at the inputs with negative values of the weight and at the inputs with positive weights of the neuron. Secondly, the functionality of neural networks will increase significantly with the use of linearization,- the implementation of linear transformations at the inputs of network elements.

Mathematically, the linearization of the inputs is described by a linear transformation of $\sigma \in GL_n(2)$ of the type $\sigma \otimes x$, *mod(2)*, where the binary representation σ and x

$$\sigma = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} \\ \dots & \dots & \dots & \dots \\ \sigma_{m1} & \sigma_{m2} & \dots & \sigma_{mn} \end{vmatrix}, \; x = \begin{vmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{vmatrix}, \; \sigma \otimes x = \begin{vmatrix} \lambda_1 \\ \lambda_2 \\ \dots \\ \lambda_m \end{vmatrix}, \; \lambda_i = \lambda_i(x) = \sigma_{i1}x_1 \oplus \dots \oplus \sigma_{in}x_n, \; i = 1 \div m.$$

As a final result, the local filters $\varphi_q(x)$ of multiplicity k have the form $\varphi_q(x) = \prod_{j=1 \div k} \lambda_j(x)$. The use of these filters makes it possible to abandon the calculation of input weights a_q, in the synthesis of a neuron, limiting itself to their values $a_q = \pm 1$.

Along with this, negative values of the weights can also be abandoned. It is easy to check that the inputs of a neuron with negative weights $a_q = -1$ can be replaced with inverted inputs, which allows switching to a completely logical representation of the "on"("off")-structure model.

3. ALGORITHM FOR SYNTHESIS OF A BIPOLAR NEURON

So, the functional efficiency of the bipolar neuron is determined by two factors. Firstly, it is based on a two-threshold element with positive and negative input weights. In this case, the value of the sum of positive weights and the absolute value of the sum of negative weights is sufficiently balanced. Secondly, the functional significance of the TTE inputs is enhanced by the preliminary placement of various local filters $\varphi_q(x)$ on them.

The synthesis of a bipolar neuron, taking into consideration all these factors, is carried out similarly to the synthesis of a threshold element described in the research [3]. For m inputs of a bipolar neuron sequentially, starting with the first local filters $\varphi_q, q = 1 \div m$ are determined, and positive and negative input weights a_q are assigned alternately, for example, $a_1 = +1,\ a_2 = -1,\ a_3 = +1,\ a_4 = -1, \ldots,\ a_q = +1,\ a_{q+1} = -1, \ldots$

The criterion of the filter $\varphi_q = \varphi_q(x) = \prod_{j=1 \div k} \lambda_j^q(x)$ efficiency is its approximation index δ^q relatively to the selected fragment $X^q \subseteq X$ in the domain X of the given function $f(x)$. The index here is characterized by the expression $\delta^q = k_1^q - k_0^q$, where

$k_1^q = \left| X^{q1} \bigcap X^{\varphi} \right|, \quad X^{q1} = \left\{ x : (x \in X^q)\ \&\ (f(x) = 1) \right\}, \quad X^{\varphi} = \left\{ x : \varphi_q(x) = 1 \right\},$

$k_0^q = \left| X^{q0} \bigcap X^{\varphi} \right|, \quad X^{q0} = \left\{ x : (x \in X^q)\ \&\ (f(x) = 0) \right\}, \quad X^{\varphi} = \left\{ x : \varphi_q(x) = 1 \right\}.$

When forming the filter φ_q, the fragments X^q are set in accordance with the levels of neuron excitation $w(x) = \sum_{1}^{q-1} a_i \varphi_i(x)$. The range of levels $w(x) \in [-1, 0, 1]$ is considered at that. So, when calculating an input filter φ_q with a positive weight where $a_q = +1$ the fragment $X^q = X_w^{-1} \cup X_w^0$ is analyzed, in case the input weight is negative (corresponds to an inhibitory synapse), $a_q = -1$, then fragment $X^q = X_w^{+1} \cup X_w^0$ is analyzed, where for arguments $x \in X$ $X_w^{-1} = \left\{ x : w(x) = -1 \right\}, \quad X_w^0 = \left\{ x : w(x) = 0 \right\}, \quad X_w^{+1} = \left\{ x : w(x) = 1 \right\}.$

Every filter φ_q is formed by sequential computation of its simple linear components $\lambda_j^q,\ j = 1 \div k$. If the input weight is $a_q = +1$, then the linear component $\lambda_j^q = \omega_1^m x_1 \oplus \omega_2^m x_2 \oplus \ldots \omega_n^m x_n$ is determined by the nonzero argument $\omega^m = (\omega_1^m, \omega_2^m, \ldots, \omega_n^m)$ of the maximum (in absolute value) coefficient $s_{j-1}^{qm}(\omega^m) = \max \left\{ \left| S_{j-1}^q \right| \right\}$ in the Walsh spectrum $S_{j-1}^q = \left\{ s_{j-1}^q(\omega), \omega = 0 \div (2^n - 1) \right\}$ of the two ternary functions difference $h_w^0(x) - h_w^{-1}(x)$. For the input from $a_q = -1$, the component λ_j^q is determined by the nonzero argument of the dominant in the spectrum S_{j-1}^q in the ternary functions difference $h_w^0(x) - h_w^{+1}(x)$. A pair $h_w^{-1}(x), h_w^0(x)$ represents LF $f(x)$ in the corresponding fragments X_w^{-1}, X_w^0, and a pair $h_w^{+1}(x), h_w^0(x)$ corresponds to the fragments X_w^{+1}, X_w^0.

In case when, for example, the first components $\lambda_1^q, \lambda_2^q, ..., \lambda_{j-1}^q$ have been determined, then the following component λ_j^q is determined with the help of spectrum S_{j-1}^q on the fragments $X_{j-1}^q = \left\{ x : (x \in X^q) \,\&\, (\prod_{i=1}^{j-1} \lambda_i^q(x) = 1) \right\} = \left\{ x : (x \in X_w^{-1} \cup X_w^0) \,\&\, (\prod_{i=1}^{j-1} \lambda_i^q(x) = 1) \right\}$. Or

$X_{j-1}^q = \left\{ x : (x \in X^q) \,\&\, (\prod_{i=1}^{j-1} \lambda_i^q(x) = 1) \right\} = \left\{ x : (x \in X_w^{+1} \cup X_w^0) \,\&\, (\prod_{i=1}^{j-1} \lambda_i^q(x) = 1) \right\}$.

Following the determination of the component λ_j^q, it is necessary to calculate the approximation index δ_j^q, corresponding to the new composition of the filter $\varphi_q = \lambda_1^q \cdot \lambda_2^q \cdot \ldots \cdot \lambda_j^q$, and to compare its value with the index δ_{j-1}^q of the previous filter composition $\varphi_q = \lambda_1^q \cdot \lambda_2^q \cdot \ldots \cdot \lambda_{j-1}^q$. The end of the calculations is fixed in the position $\delta_{j-1}^q \geq \delta_j^q$. In this case, the resulting multiplicity k of the local filter φ_q value $k = j - 1$ is designed.

It is necessary to note that spectral arguments ω^m, corresponding to components $\lambda_i^q, i = 1 \ldots k$ of the filter φ^q must be linearly independent.

In order to maintain technical efficiency, it is advisable to limit the number of m inputs of the bipolar neuron $m \leq 2n$ as the approximation index δ^q for the subsequent filters diminishes.

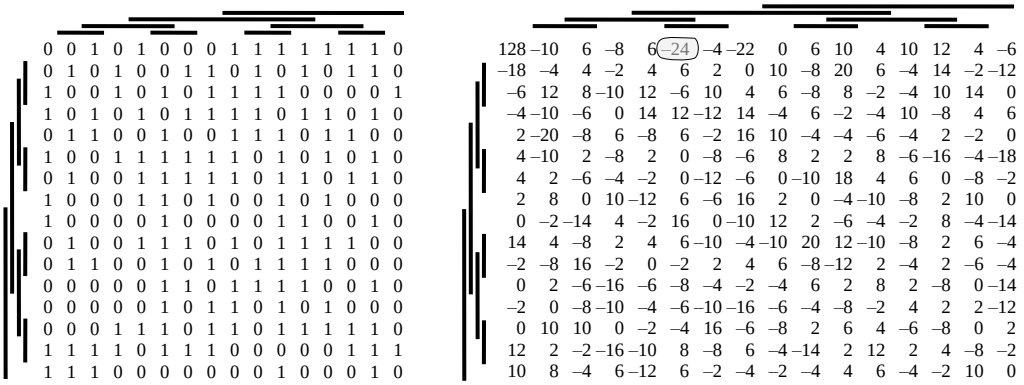
It is also necessary to make a separate clarification regarding the calculation of the initial local filters φ_1 and φ_2. At the first step of the algorithm, when determining the first input, the weight $a_1 = +1$ is assigned. For filter φ_1, when X is selected as an analyzed fragment,- it is the entire domain of the initial function $f(x)$ determination. Therefore, at first, the spectrum S of the function f is calculated the usual way over the entire region X. Among coefficients $s(\omega)$, constituting spectrum S, maximal coefficient in absolute value $s^m(\omega^m) = \max\{|S(\omega)|\}, \omega \neq 0$ is selected. The argument $\omega^m = (\omega_1^m, \omega_2^m, ..., \omega_n^m)$ determines the first component $\lambda_1^1(x)$ of the first filter φ_1: $\lambda_1^1(x) = \omega_1^m x_1 \oplus \omega_2^m x_2 \oplus ... \oplus \omega_n^m x_n$. In order to determine the second component $\lambda_2^1(x)$ in accordance with [3] ternary function $h(x)$ is determined in the fragment $X_1^1 = \{x : x \in X \,\&\, \lambda_1^1(x) = 1\}$. Then, it is necessary to calculate spectrum S_1^1 of the function h for the fragment X_1^1, and pick out the argument ω^m of its dominant $s_1^{1m}(\omega^m)$. As a result, $\lambda_2^1(x) = \omega_1^m x_1 \oplus \omega_2^m x_2 \oplus \ldots \oplus \omega_n^m x_n$. If its necessary, then the determination of the following component $\lambda_j^1(x), j = 3, 4, \ldots$ of the filter φ_1 is executed according to spectrum S_{j-1}^1 of the ternary function $h(x)$ for the fragment $X_{j-1}^1 = \left\{ x : x \in X \,\&\, (\prod_{i=1}^{j-1} \lambda_i^1(x) = 1) \right\}$.

The formation of filter φ_1 at the first input with a weight $a_1 = 1$ initiates the division of the X determination region into two fragments $X_w^0 = \{x : \varphi_1(x) = 0\}$ and $X_w^1 = \{x : \varphi_1(x) = 1\}$ in accordance with the formed levels of excitation $w \in [0,1]$.

Further, the second input of TTE is assigned the weight $a_2 = -1$, and to the local filter φ_2 the fragment $X_1^2 = X_w^0 \cup X_w^1$ is assigned for spectral analysis. Subsequent calculations are performed in accordance with the synthesis algorithm described above.

4. AN EXAMPLE OF SYNTHESIS OF BIPOLAR NEURON FOR LOGICAL FUNCTION OF 8 VARIABLES

Let LF $f(x)$ of 8 variables be given randomly with a uniform probability density. Its table Karnaugh is shown in Fig. 2a. The function contain k_1 and k_0 respectively of unit and zero values: $k_1 = |X_1|$, $X_1 = \{x : f(x) = 1\}$, $k_0 = |X_0|$, $X_0 = \{x : f(x) = 0\}$. To complicate the task function with $k_1 = k_0 = 128$ is chosen, its difference is $\theta = \log_2(k_1 / k_0) = 0$.



a) b)

Fig. 2. Karnaugh tables: a) function $f(x)$, b) spectrum $S(\omega)$.

To determine the first filter φ_1, spectrum S of f function is calculated, and it is represented by table Karnaugh in Fig.2b. The codes of the variables x_i of the argument $x = (x_1, x_2, \ldots, x_8)$ of the function f and variables ω_i of the argument $\omega = (\omega_1, \omega_2, \ldots, \omega_8)$ of S spectrum are represented on the map by lines. Horizontal lines represent $x_1 \div x_4$, $\omega_1 \div \omega_4$, beginning from the top. Vertical lines, beginning from the left, represent the codes of the variables $x_5 \div x_8$ and $\omega_5 \div \omega_8$. In spectrum S dominant $s^m(\omega^m) = -24$, $\omega^m = 01110000$ is highlighted. According to the definition of Walsh functions, the first component of the first filter, in this case, represents a linear function $\lambda_1^1(x) = x_2 \oplus x_3 \oplus x_4$. Fig.3a shows, the allocated by it, fragment X_1^1 of the area of X determination with the indication of function f value. Dashes indicate the values that are not included in X_1^1. According Fig.3a index δ_1^1 of the first variant of approximation for filter $\varphi_1 = \lambda_1^1$ will be $\delta_1^1 = k_1 - k_0 = 76 - 52 = 24$. For the determination of the second component λ_2^1 in the fragment $X_1^1 = \{x : x \in X \ \& \ \lambda_1^1(x) = 1\}$ in accordance with [3] ternary function $h(x)$ is allocated. Its spectrum S_1^1 is represented in Fig.3c. Here, the dominant $s_1^{1m}(\omega^m) = 4.10$, $\omega^m = 11110001$ is highlighted, that determines $\lambda_2^1(x) = \overline{x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_8}$. The second variant of filter

$\varphi_1(x) = \lambda_1^1(x) \cdot \lambda_2^1(x)$ forms fragment $X_2^1 = \{x : x \in X \ \& (\varphi_1(x) = 1)\}$. Fragment X_2^1 represented in Fig.3b. The approximation index $\delta_2^1 = k_1 - k_0 = 46 - 18 = 28$ corresponds to it. Thus, $\delta_1^1 < \delta_2^1$.

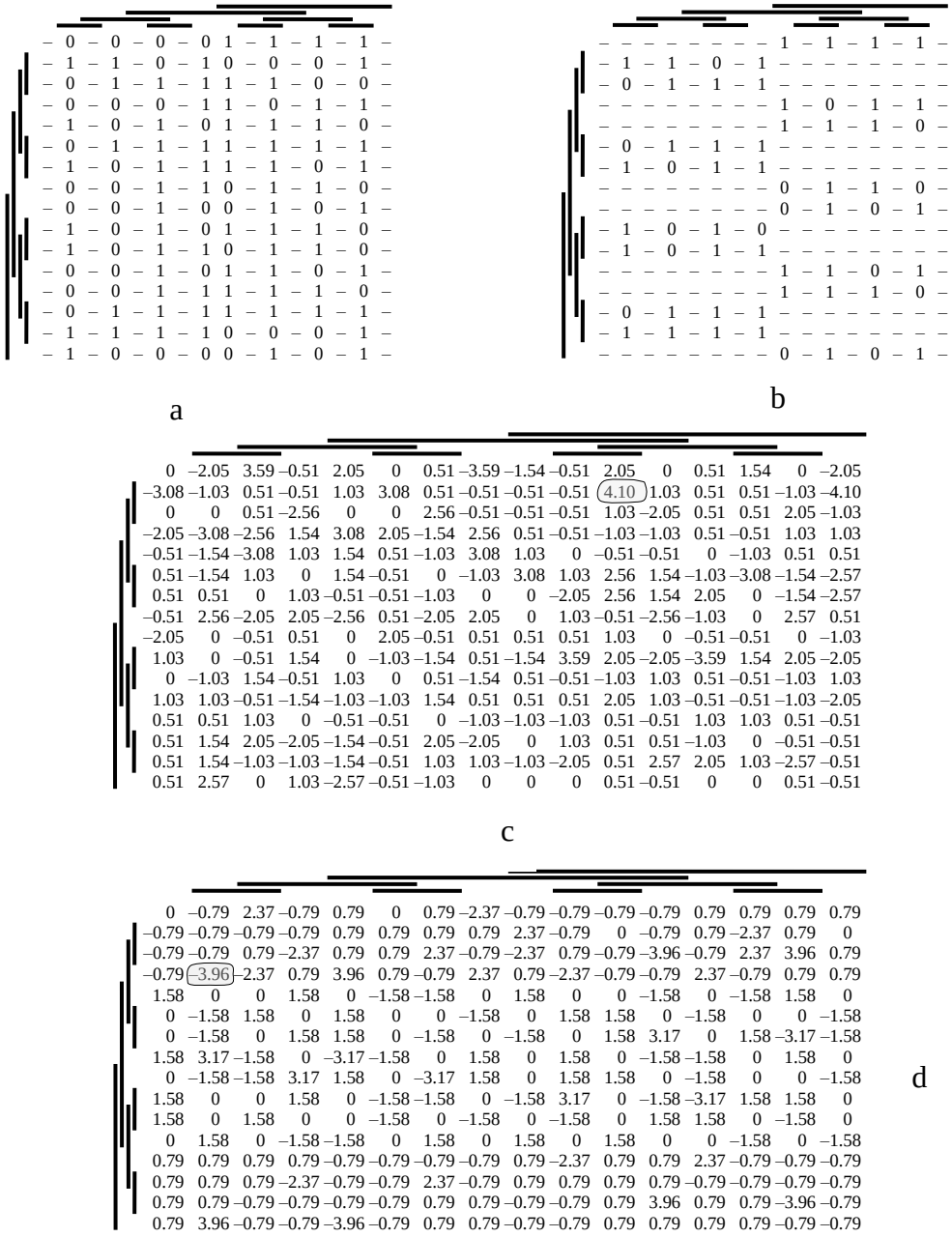


Fig. 3. Karnaugh tables: a) function $f(x)$ for fragment X_1^1,

b) function $f(x)$ for fragment X_2^1,

c) spectrum $S_1^1(\omega)$, d) spectrum $S_2^1(\omega)$.

Continuing the formation of φ_1, we determine h for X_2^1. Its spectrum S_2^1 is represented in Fig.3d. Here dominant $s_2^{1m}(\omega^m) = -3.96, \quad \omega^m = 00010010$. And $\lambda_3^1(x) = x_4 \oplus x_7$ accordingly. The next

variant of filter is $\varphi_1 = \lambda_1^1 \cdot \lambda_2^1 \cdot \lambda_3^1$, If we consider fragment X_3^1 determined by it, then its index of approximation will be $\delta_3^1 = k_1 - k_0 = 28 - 4 = 24$. Consequently $\delta_1^1 < \delta_2^1 > \delta_3^1$ and it is necessary for filter φ_1 to determine the multiplicity $k = 2$, $\varphi_1(x) = \lambda_1^1(x) \cdot \lambda_2^1(x)$ and the final value of approximation $\delta^1 = \delta_2^1 = 28$.

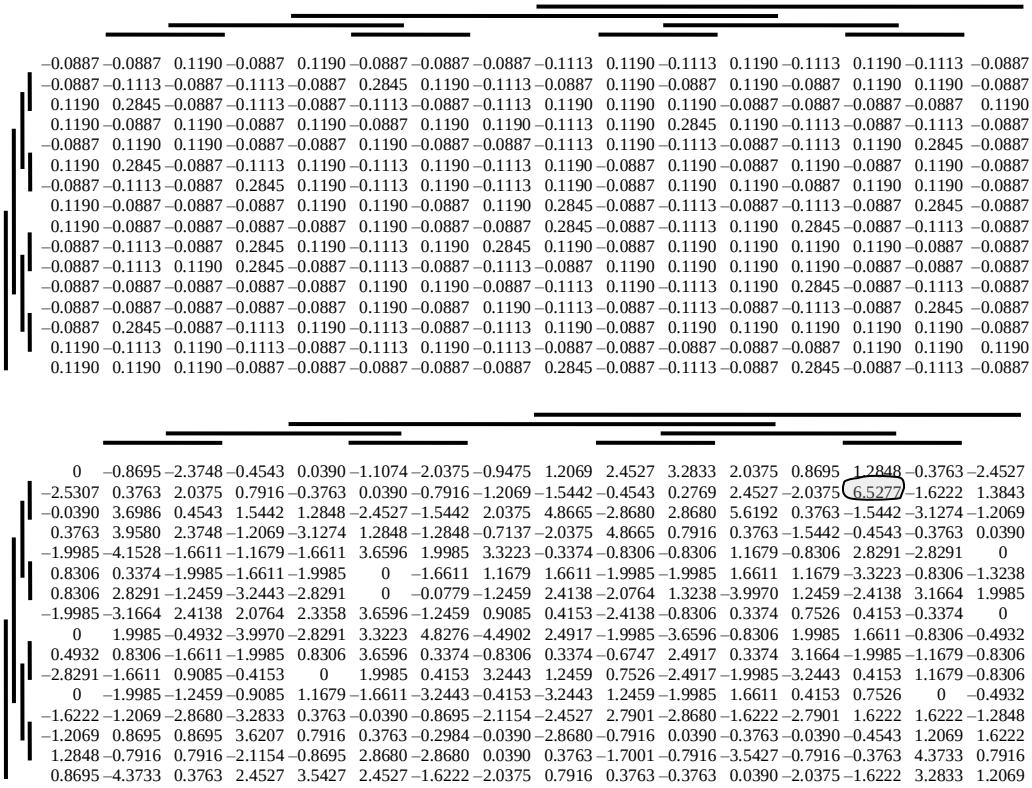
The distribution of function f value for $x \in X$ according to the levels $\boldsymbol{w}$ of neuron excitation after filter φ_1 has been completed is presented in Table 1.

Table 1

w	0	1
k_1	82	46
k_0	110	18

The determination of the second inhibitory input of the neuron with a weight $a_2 = -1$ and its filter φ_2 is carried out by analysis of two fragments X_w^{+1}, X_w^0. A bipolar neuron generates an output signal, being both in the excitation state and when it comes out of the inhibited state. Therefore, the filter φ_2 is to highlight the value of the function $f(x) = 1$ for $x \in X_w^0$, out of the set k_1 in the column "0", with the aim to transfer them into the breaking sate $w(x) = -1$. Simultaneously, it is necessary to provide the transfer of the values $f(x) = 0$ from the multitude k_0 from the fragment of X_w^{+1} excited state, i.e. column "1", into the fragment X_w^0 neutral state. They are highlighted in grey in Table 1.

The initial value of the approximation index δ_0^2 of the second input of the neuron is determined by the indices of approximation ${}^0\delta_0^2$ of the fragment X_w^0 and the index ${}^{+1}\delta_0^2$ of the fragment X_w^{+1}: $\delta_0^2 = {}^0\delta_0^2 + {}^{+1}\delta_0^2 = (82 - 110) + (18 - 46) = -56$. Further, for each of the fragments X_w^{+1}, X_w^0 ternary functions $h_w^{+1}(x), h_w^0(x)$ and their difference $h_w^0(x) - h_w^{+1}(x)$ according to [3] are formed.



a

b

Fig. 4. Karnaugh tables: a) difference $h_w^{\vee}(x) - h_w^{\wedge}(x)$,

b) spectrum S_0^2 for difference $h_w^0(x) - h_w^{+1}(x)$

$$h_w^{+1}(x) = \begin{cases} 0.1113, & if\ x \in X_w^{+1},\ f(x) = 1, \\ -0.2845, & if\ x \in X_w^{+1},\ f(x) = 0, \\ 0, & if\ x \notin X_w^{+1}. \end{cases} \qquad h_w^0(x) = \begin{cases} 0.1190, & if\ x \in X_w^0,\ f(x) = 1, \\ -0.0887, & if\ x \in X_w^0,\ f(x) = 0, \\ 0, & if\ x \notin X_w^0. \end{cases}$$

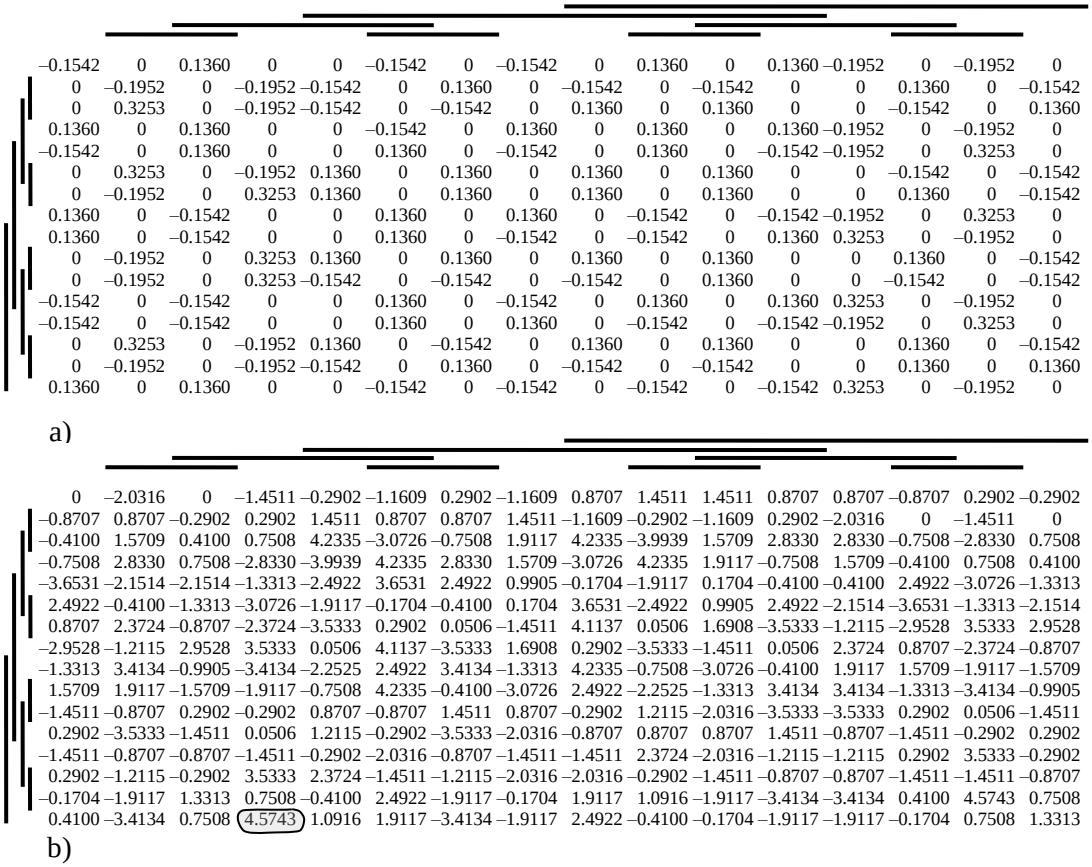
The difference $h_w^0(x) - h_w^{+1}(x)$ and its spectrum S_0^2 are shown in Fig.4a and Fig.4b respectively. In Fig.4b a spectral dominant $s_0^{2m}(\omega^m) = 6.5277$, $\omega^m = 10110001$ is highlighted, so the first variant of the filter is $\varphi_2 = \lambda_1^2(x) = \overline{x_1 \oplus x_3 \oplus x_4 \oplus x_8}$.

The distribution of the constituent according to the levels of the neuron excitation on the fragments limited by local filter φ_2 is shown in Table 2.

Table 2

$\varphi \backslash w$	**0**	**1**
k_1	**51**	**20**
k_0	**45**	**12**

And now the first variant of the approximation index value of the second input $\delta_1^2 = {}^0\delta_1^2 + {}^{+1}\delta_1^2 = (51-45)+(12-20) = -2$. The comparison of indices $\delta_0^2 < \delta_1^2$.



a)

b)

Fig. 5. Karnaugh tables: a) difference $h_w^0(x) - h_w^{+1}(x)$ for $\varphi_2 = \lambda_1^2$,

b) spectrum S_1^2 for difference $h_w^0(x) - h_w^{+1}(x)$

While continuing the formation of filter φ_2 according to fragments $X_w^{+1}\,\&\,(\varphi_2(x)=1)$, $X_w^0\,\&\,(\varphi_2(x)=1)$ functions are determined

$$h_w^0(x) = \begin{cases} 0.1360, & if\ x \in X_w^0,\ f(x)=1,\ \varphi_2(x)=1, \\ -0{,}1542, & if\ x \in X_w^0,\ f(x)=0,\ \varphi_2(x)=1, \\ 0, & if\ x \notin X_w^0. \end{cases}$$

$$h_w^{+1}(x) = \begin{cases} 0.1952, & if\ x \in X_w^{+1},\ f(x)=1,\ \varphi_2(x)=1, \\ -0.3253, & if\ x \in X_w^{+1},\ f(x)=0,\ \varphi_2(x)=1, \\ 0, & if\ x \notin X_w^{+1}\ . \end{cases}$$

The difference $h_w^0(x) - h_w^{+1}(x)$ and its spectrum S_1^2 are represented in Fig.5a and Fig.5b. A spectral dominant in Fig.5b $s_1^{2m}(\omega^m) = 4.5743$, $\omega^m = 00101000$ is highlighted, therefore, the second component is $\lambda_2^2(x) = \overline{x_3 \oplus x_5}$ and the second variant of the filter is

$\varphi_2 = \lambda_1^2(x) \cdot \lambda_2^2(x) = \overline{x_1 \oplus x_3 \oplus x_4 \oplus x_8} \cdot \overline{x_3 \oplus x_5}$. The distribution of values $f(x)$ according to the levels of neuron excitation with a limitation in fragment $X_{\varphi 2}$ of filter φ_2 is represented in Table 3.

Table 3

φ^1 **w**	**0**	**1**
k_1	**28**	**7**
k_0	**20**	**9**

The value of the second variant of the second input approximation index is $\delta_2^2 = {}^0\delta_2^2 + {}^{+1}\delta_2^2 = (28-20)+(9-7) = 10$. Comparison of indices: $\delta_0^2 < \delta_1^2 < \delta_2^2$.

Continuing the formation of filter φ_2 according to fragments $X_w^{+1}\ \&\ (\varphi_2(x)=1)$, $X_w^0\ \&\ (\varphi_2(x)=1)$ the following functions are determined

$$h_w^0(x) = \begin{cases} 0.1737, & if\ x \in X_w^0,\ f(x)=1, \varphi_2(x)=1, \\ -0.2432 & if\ x \in X_w^0,\ f(x)=0, \varphi_2(x)=1, \\ 0, & if\ x \notin X_w^0. \end{cases}$$

$$h_w^{+1}(x) = \begin{cases} 0.4032, & if\ x \in X_w^{+1},\ f(x)=1, \varphi_2(x)=1, \\ -0.3136, & if\ x \in X_w^{+1},\ f(x)=0, \varphi_2(x)=1, \\ 0, & if\ x \notin X_w^{+1}\ . \end{cases}$$

The difference $h_w^0(x) - h_w^{+1}(x)$ and its spectrum S_2^2 are shown in Fig.6a and Fig.6b. A spectral dominant $s_2^{2m}(\omega^m) = 5.2516$, $\omega^m = 01110100$ is highlighted in Fig.6b, therefore, the third component $\lambda_3^2(x) = \overline{x_2 \oplus x_3 \oplus x_4 \oplus x_6}$ and the third variant of filter is $\varphi_2 = \lambda_1^2(x) \cdot \lambda_2^2(x) \cdot \lambda_3^2(x) = \overline{x_1 \oplus x_3 \oplus x_4 \oplus x_8} \cdot \overline{x_3 \oplus x_5} \cdot \overline{x_2 \oplus x_3 \oplus x_4 \oplus x_6}$. The distribution of values $f(x)$ according to the levels of neuron excitation with the limitation on the third variant of filter φ_2 is represented in Table 4.

–0.2432	0	0	0	0	0	0	–0.2432	0	0.1737	0	0	0	0	–0.4032	0
0	–0.4032	0	0	0	0	0.1737	0	–0.2432	0	0	0	0	0	0	–0.2432
0	0.3136	0	0	0	0	–0.2432	0	0.1737	0	0	0	0	0	0	0.1737
0.1737	0	0	0	0	0	0	0.1737	0	0.1737	0	0	0	0	–0.4032	0
–0.2432	0	0	0	0	0	0	–0.2432	0	0.1737	0	0	0	0	0.3136	0
0	0.3136	0	0	0	0	0.1737	0	0.1737	0	0	0	0	0	0	–0.2432
0	–0.4032	0	0	0	0	0.1737	0	0.1737	0	0	0	0	0	0	–0.2432
0.1737	0	0	0	0	0	0	0.1737	0	–0.2432	0	0	0	0	0.3136	0
0	0	–0.2432	0	0	0.1737	0	0	0	0	0	0.1737	0.3136	0	0	0
0	0	0	0.3136	0.1737	0	0	0	0	0	0.1737	0	0	0.1737	0	0
0	0	0	0.3136	–0.2432	0	0	0	0	0	0.1737	0	0	–0.2432	0	0
0	0	–0.2432	0	0	0.1737	0	0	0	0	0	0.1737	0.3136	0	0	0
0	0	–0.2432	0	0	0.1737	0	0	0	0	0	–0.2432	–0.4032	0	0	0
0	0	0	–0.4032	0.1737	0	0	0	0	0	0.1737	0	0	0.1737	0	0
0	0	0	–0.4032	–0.2432	0	0	0	0	0	–0.2432	0	0	0.1737	0	0
0	0	0.1737	0	0	–0.2432	0	0	0	0	0	–0.2432	0.3136	0	0	0

a)

0	–0.9507	–0.5558	–0.7168	–1.5506	–1.3896	1.5506	–1.9454	–0.7168	0.9507	0.7168	2.3843	1.5506	–0.1169	0.1169	–1.5506
–0.1169	1.5506	–1.5506	0.1169	0.9507	–0.7168	2.3843	0.7168	–1.3896	–1.5506	–1.9454	1.5506	–0.9507	0	–0.7168	–0.5558
2.1504	0.9507	–0.4829	0.7168	1.5506	–2.9842	–1.5506	2.9842	2.3843	–3.8179	0.9507	0.4829	1.3167	–1.5506	–2.9842	–0.1169
–1.5506	1.3167	–0.1169	–2.9842	–3.8179	2.3843	0.4829	0.9507	–2.9842	1.5506	2.9842	–1.5506	0.9507	2.1504	0.7168	–0.4829
–2.6182	–2.6182	–4.0518	–0.7168	–3.2181	0.1169	1.5506	1.5506	0.9507	–0.7168	0.7168	–0.9507	–0.1169	1.5506	–3.2181	–1.5506
1.5506	–0.1169	–1.5506	–3.2181	–0.7168	0.9507	–0.9507	0.7168	0.1169	–3.2181	1.5506	1.5506	–2.6182	–2.6182	–0.7168	–4.0518
–0.7168	0.4829	0.7168	–0.4829	–4.6517	–0.1169	2.9842	–1.5506	5.2516	–0.9507	–0.2490	–0.7168	–1.5506	–4.4178	1.5506	4.4178
–4.4178	–1.5506	4.4178	1.5506	–0.9507	5.2516	–0.7168	–0.2490	–0.1169	–4.6517	–1.5506	2.9842	0.4829	–0.7168	–0.4829	0.7168
1.5506	4.4178	–1.5506	–4.4178	–0.2490	–0.7168	5.2516	–0.9507	2.9842	–1.5506	–4.6517	–0.1169	0.7168	–0.4829	–0.7168	0.4829
–0.4829	0.7168	0.4829	–0.7168	–1.5506	2.9842	–0.1169	–4.6517	–0.7168	–0.2490	–0.9507	5.2516	4.4178	1.5506	–4.4178	–1.5506
–3.2181	–1.5506	–0.1169	–1.5506	0.7168	–0.9507	0.9507	–0.7168	1.5506	1.5506	–3.2181	0.1169	–4.0518	–0.7168	–2.6182	–2.6182
–0.7168	–4.0518	–2.6182	–2.6182	1.5506	1.5506	0.1169	–3.2181	–0.9507	0.7168	–0.7168	0.9507	–1.5506	–3.2181	1.5506	–0.1169
–2.9842	–0.1169	1.3167	–1.5506	0.9507	0.4829	2.3843	–3.8179	–1.5506	2.9842	1.5506	–2.9842	–0.4829	0.7168	2.1504	0.9507
0.7168	–0.4829	0.9507	2.1504	2.9842	–1.5506	–2.9842	1.5506	0.4829	0.9507	–3.8179	2.3843	–0.1169	–2.9842	–1.5506	1.3167
0.1169	–1.5506	1.5506	–0.1169	0.7168	2.3843	–0.7168	0.9507	1.5506	–1.9454	–1.5506	–1.3896	–0.5558	–0.7168	0	–0.9507
–0.7168	–0.5558	–0.9507	0	–1.9454	1.5506	–1.3896	–1.5506	2.3843	0.7168	0.9507	–0.7168	–1.5506	0.1169	–0.1169	1.5506

b)

Fig. 6. Karnaugh tables: a) difference $h_w^{\nu}(x) - h_w^{\nu+1}(x)$ for $\varphi_2 = \lambda_1^2 \cdot \lambda_2^2$,

b) spectrum S_2^2 for difference $h_w^0(x) - h_w^{+1}(x)$

Table 4

$\varphi^{\uparrow} w$	**0**	**1**
k_1	**16**	**1**
k_0	**8**	**7**

The value of the third variant of the second input approximation index is $\delta_3^2 = {}^0\delta_3^2 + {}^{+1}\delta_3^2 = (16-8)+(7-1) = 14$. The comparison of indices shows: $\delta_0^2 < \delta_1^2 < \delta_2^2 < \delta_3^2$.

While doing similar calculations of the fourth variant of filter φ_2 option in spectrum S_3^2 the dominant $s_3^{2m}(\omega^m) = -5.2103$, $\omega^m = 00010110$ is highlighted. The fourth component of filter is $\lambda_4^2(x) = x_4 \oplus x_6 \oplus x_7$. Table 5 shows general distribution of function values for the filter variant where $\varphi_2 = \lambda_1^2(x) \cdot \lambda_2^2(x) \cdot \lambda_3^2(x) \cdot \lambda_4^2(x)$.

Table 5

$\varphi^{\uparrow} w$	**0**	**1**
k_1	**11**	**0**
k_0	**1**	**4**

The value of the fourth variant of the approximation index for the second input is $\delta_4^2 = {}^0\delta_4^2 + {}^{+1}\delta_4^2 = (11-1)+(4-0) = 14$. The comparison of indices shows $\delta_0^2 < \delta_1^2 < \delta_2^2 < \delta_3^2 = \delta_4^2$. Thus, for filter φ_2 it is necessary to determine the multiplicity $k = 3$, $\varphi_2(x) = \lambda_1^2(x) \cdot \lambda_2^2(x) \cdot \lambda_3^2(x)$ and the final approximation index $\delta^2 = \delta_3^2 = 14$.

The distribution of function $f(x)$ according to the levels of neuron excitation after filters φ_1 and φ_2 at the inputs with the weights $a_1 = 1$ and $a_2 = -1$ is represented in Table 6. The neuron approximation index is $\delta = \delta^1 + \delta^2 = 42$.

Table 6

w	**–1**	**0**	**1**
k_1	**16**	**67**	**45**
k_0	**8**	**109**	**11**

The determination of the third excitatory input of the neuron with the weight $a_3 = 1$ and the corresponding filter φ_3 is carried out by the analysis of two fragments X_w^{-1}, X_w^0. The filter shall transfer values $f(x)$ from the multitude k_0 of the breaking state, i.e. from fragment X_w^{-1} to the fragment neutral state X_w^0, and shall also transfer the values $f(x) = 1$ from the fragment of the neutral state X_w^0 into the fragment of the excitatory state X_w^{+1}.They are highlighted in grey in Table 6.

Further on, similar to the previous steps, for each variant of calculating the component λ_j^3, $j = 1,2,...$ of filter φ_3 from the fragments X_w^{+1}, X_w^0 in accordance with [3] pairs of ternary functions $h_w^{+1}(x), h_w^0(x)$ are formed, their differences are $h_w^0(x) - h_w^{-1}(x)$ and their spectra $S_j^3(\omega)$. As a result , for φ_3 the multiplicity $k = 3$ and $\varphi_3 = \overline{x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 \oplus x_7 \oplus x_8} \cdot \overline{x_2 \oplus x_3 \oplus x_4 \oplus x_7} \cdot \overline{x_4 \oplus x_7}$ will be determined. The final distribution of values $f(x)$ (Table 7) in accordance with the levels of neuron excitation in the area of $X_{\varphi 3}$ action of the local filter φ_3

Table 7

$\varphi^{\uparrow} w$	**–1**	**0**	**1**
k_1	**0**	**13**	**6**
k_0	**4**	**7**	**2**

And the general distribution of values $f(x)$ in the whole area of X determination according to the levels of neuron excitation is represented in Table 8.

Table 8

w	–1	0	1	2
k_1	16	54	52	6
k_0	4	106	16	2

Now the neuron approximation level reaches the value $\delta = \delta^1 + \delta^2 + \delta^3 = 52$. In the table, the number of single value k_1 in the fragment X_w^0 and the number of values k_0 in the fragment X_w^{+1}, which should be taken into account when calculating filter φ_4, are highlighted in grey.

Continuing the alternate formation of inhibitory and excitatory inputs, we shall obtain the following results:

$$a_4 = -1, \quad k = 4, \quad \delta = 63,$$

$$\varphi_4 = (\overline{x_2 \oplus x_3 \oplus x_4 \oplus x_6 \oplus x_7}) \cdot (\overline{x_3 \oplus x_4 \oplus x_8}) \cdot (x_4 \oplus x_5) \cdot (\overline{x_5});$$

$$a_5 = +1, \quad k = 3, \quad \delta = 72,$$

$$\varphi_5 = (x_3 \oplus x_5 \oplus x_7) \cdot (x_1 \oplus x_3 \oplus x_4 \oplus x_7) \cdot (x_5 \oplus x_6);$$

$$a_6 = -1, \quad k = 4, \quad \delta = 81,$$

$$\varphi_6 = (\overline{x_1 \oplus x_3 \oplus x_6 \oplus x_8}) \cdot (x_2 \oplus x_4 \oplus x_5 \oplus x_8) \cdot (x_3 \oplus x_5 \oplus x_6 \oplus x_7) \cdot (\overline{x_4 \oplus x_5 \oplus x_7 \oplus x_8});$$

$$a_7 = +1, \quad k = 4, \quad \delta = 88,$$

$$\varphi_7 = (x_2 \oplus x_3 \oplus x_4 \oplus x_5 \oplus x_6 \oplus x_7) \cdot (x_7) \cdot (\overline{x_3 \oplus x_6 \oplus x_7 \oplus x_8}) \cdot (x_1 \oplus x_2 \oplus x_4 \oplus x_8);$$

$$a_8 = -1, \quad k = 4, \quad \delta = 94,$$

$$\varphi_8 = (\overline{x_1 \oplus x_2 \oplus x_3 \oplus x_7 \oplus x_8}) \cdot (x_2 \oplus x_5 \oplus x_6) \cdot (x_4 \oplus x_5 \oplus x_8) \cdot (x_5 \oplus x_6 \oplus x_7);$$

The final distribution of constituents according to the levels of neuron excitation is represented in Table 9.

Table 9

w	–4	–3	–2	–1	0	1	2	3
k_1	—	—	4	32	16	64	10	2
k_0	—	—	—	4	110	10	4	—

Distribution of fragment X_w^0 constituents is highlighted in colour in the table. If we continue the synthesis further, then, at a certain step we shall obtain the value $k_1 = 0$ for fragment X_w^0. But for the functions of large dimension such a result will require an excessive number of filters φ_q and TTE inputs due to a decrease in the values of δ^q approximation indices in the sum $\delta = \sum_{q=1}^{m} \delta^q$ for $q \geq 2n$.

Further solution of the pattern recognition problem is achievable on the basis of logical switching networks [4,5] using bipolar neuron.

At this step of the algorithm, upon reaching an acceptable approximation of the original function, further synthesis of the bipolar neuron stops. The total value of the number m inputs and input filters is determined. m =8.

The biological prototype, the bipolar neuron, has a certain range of levels of excitation and inhibition, in which no output pulse is generated. In this synthesis method, a bipolar neuron with a narrow "silence" range containing only one neutral level $w(x) = 0$ is formed. Taking into account that the input local filters select rather narrow fragments of the domain of definition of the original function, the general construction can be referred to as a notch filter. At the end of the example, the synthesis result is shown in Fig.7.

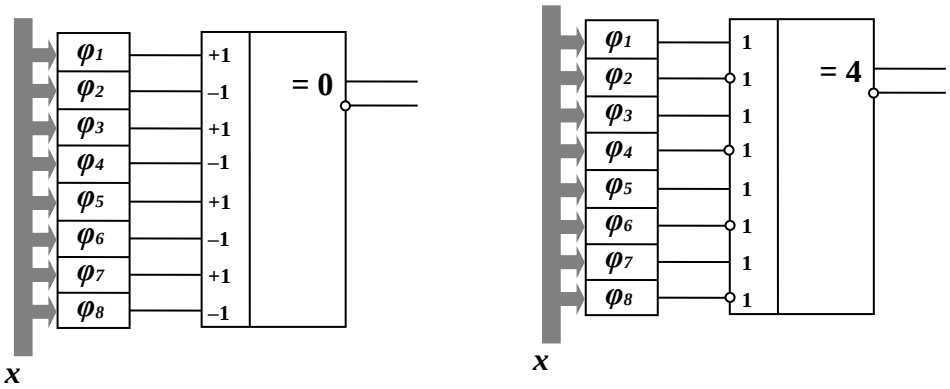


Fig. 7. Variants of notch filter construction: a) with an arithmetic model of a rejection neuron; b) with a logical model of a rejection neuron.

The signal from the inverse output 0f the bipolar (rejection) neuron module represents the given function $f(x)$ with an approximation index $\delta = 94$. Figure 7b shows the logical interpretation of this construction. The replacement of the number r of the negative weights "–1" at the inputs of the neuron by inversion transformations leads to a change in the "silence" level from $w(x) = 0$ to $w(x) = r$. In this example $w(x) = 4$. The reaction of the rejection neuron on the excitation level $w(x) = 4$ is realized here by the combinatorial contact circuit "4 out of 8" (Fig.8).

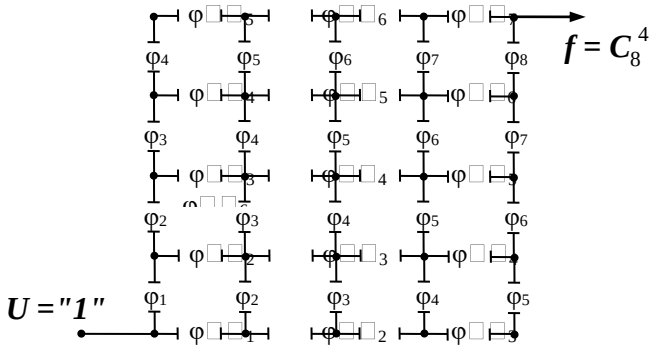


Fig. 8. Combinatorial contact circuit, controlled by input signals of the local filters φ_q, where $\tilde{\varphi}_q$ corresponds to the inverse value of the filter output signal.

5. THE EFFICIENCY OF AN ARTIFICIAL NEURON

Let us compare the obtained solution with the traditional one-threshold or monopolar model of the neuron, comparing the corresponding approximation indices. For the function chosen in the example, the best index δ_p corresponds, in the monopolar model, to the integer version of the structure vector $A = (p;\ a_1,\ a_2,\ldots,\ a_8) = (4;\ 1,\ 3,\ 2,\ 2,-2,-1,\ 1,\ 3)$. In this case, out of the distribution of constituents in Table 10, it follows that $\delta_p = 37$.

Table 10

w	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
k_1	*1*	*1*	*1*	*2*	*4*	*9*	*11*	*19*	*21*	*20*	*15*	*12*	*5*	*4*	*2*	*1*
k_0	—	*2*	*5*	*10*	*14*	*15*	*20*	*14*	*12*	*11*	*9*	*6*	*7*	*2*	*1*	—

$$\lambda = \sigma \otimes x = \begin{vmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \omega_6\\ \omega_7\\ \omega_8\end{vmatrix} \otimes \begin{vmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{vmatrix} = \begin{vmatrix}01110000\\ 01000000\\ 00010110\\ 11110001\\ 11011101\\ 00000001\\ 10000111\\ 11110101\end{vmatrix} \otimes \begin{vmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{vmatrix} = \begin{vmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\\ \lambda_5\\ \lambda_6\\ \lambda_7\\ \lambda_8\end{vmatrix} \pmod 2$$

A variant of a monopolar neuron with a simple linear transformation of the input signal allows you to operate functionally with more significant variables $\lambda_1 \div \lambda_8$, for which the corresponding spectrum coefficients $s(\omega_1) \div s(\omega_8)$ form a dominant tuple $[-24,-22,-20,20,20,-18,-18,18]$. The structural vector A^σ of the linearized monopolar neuron $A^\sigma = (3;\ 2,1,1,-1,-2,1,1,-1)$ is

determined by a simple search. In accordance with the distribution of constituents to the levels of neuron excitation, shown in Table 11, the approximation index is $\delta_\sigma = 50$.

Table 11

w	-4	-3	-2	-1	0	1	2	3	4	5	6
k_1	—	1	2	12	16	21	30	24	15	6	1
k_0	1	5	15	20	30	31	16	8	2	—	—

Thus, the ratio of the approximation indices for the considered variants of the example is

$$\delta : \delta_\sigma : \delta_p = 94 : 50 : 37 .$$

We should also consider the option of using a monopolar model of a neuron with local filters φ at the inputs, by analogy with the model of a bipolar structure. Let $\delta_{p\varphi}$ be the approximation index, which is provided a monopolar construction by n inputs and filters for a certain logical function of n size. In Table 12 one may find approximation results of separate LF values with a uniform density of probability. Randomly chosen functions, at that, contain single constituents with the number $k_1 = 2^{n-1}$. In case of functions of small dimension index $\delta_{p\varphi}$ will be only slightly inferior to the index of a bipolar approximation δ. But with the increase in the dimension n, the advantage of a bipolar neuron is becoming significant and it increases along with an increase of n.

Table 12

n	8	10	12	14	16	18	20
δ	96	247	687	1,632	3,804	8,585	19,948
$\delta_{p\varphi}$	90	245	626	1,498	3,391	8,052	18,048

The following shall be noted about the efficiency of using linear transformations. Having made an invariant transformation [3], one can simplify linear components λ_i^q of the local filters φ_q, $q = 1 \div 8$ with an aim in view to diminish the number of summation operations according to *mod 2*.

For example, for

$\varphi_6 = \overline{(x_1 \oplus x_3 \oplus x_6 \oplus x_8)} \circ (x_2 \oplus x_4 \oplus x_5 \oplus x_8) \circ (x_3 \oplus x_5 \oplus x_6 \oplus x_7) \circ \overline{(x_4 \oplus x_5 \oplus x_7 \oplus x_8)}$

the invariant transformation leads to the expression

$\varphi_6 = (x_1 \oplus x_4) \circ (x_2 \oplus x_7) \circ (x_3 \oplus x_4 \oplus x_6 \oplus x_8) \circ (x_1 \oplus x_5 \oplus x_7 \oplus x_8)$, where, in order to make the perception easier, sign "$\circ$" represents the conjunction operation “&”.

After such transformations, the whole block of local at the input rejector neuron will contain $U \le m \times n$ of two-input adders to *mod 2*, where m represents the number of neuron’s inputs. A set of filters, like the example of [3], can be made in the form of matrix from $m \times n$ of two-input adders.

This is a positive result in case of a hardware implementation. In case of software implementation, all simple linear components λ_i^q of the transformed filters φ_q are to juxtapose with a common matrix σ.

Parallel computation algorithms will provide efficient computation of matrix multiplication $\sigma \otimes x \pmod 2$ and final conjunction operations $\varphi_q = \prod_{i=1}^{k} \lambda_i^q$.

6. CONCLUSIONS

The proposed method for choosing linear filters effectively adapts the standard design of a bipolar neuron with respect to an arbitrary logical function. Thus, it provides the functional advantages of the used model of a bipolar neuron in relation to the traditional model of a monopolar, or threshold, neuron. The result obtained determines a significant reduction in the number of neurons used in artificial neural networks.

Matrix structures become constructive elements, which greatly simplifies the analysis of neural networks. Thanks to the use of Walsh spectral representations, the design of neural networks is carried out on the basis of matrix methods.

The proposed design of a bipolar neuron, containing the simplest input value weights, provides a significant increase in the speed of neural networks, both in hardware and software implementation.

The combination of a neuron-notch with local filters at the inputs, or a notch filter, surpasses the adaptive capabilities of a threshold element with adjustable input weights.

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