

THE INEQUALITY IN SOCIETY AND A MULTIPOINT EXTENSION OF NASH BARGAINING PROBLEM

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Abstract

We use the microeconomic model of supply and demand of financial resources and investment opportunities, in which each contract can be interpreted as a specific Nash bargaining problem, where payoffs are expressed as income from the use of investment opportunities. We identify and describe the phenomenon of investing in a social position. A player investing in a social position can achieve a payoff distribution that is an alternative to the game associated with solving the Nash bargaining problem. Subsequently, we consider games in which players can react to the choice of alternative, providing as a multipoint extension of the Nash bargaining problem. These reactions lead to numerous non-trivial tasks that we describe and interpret in terms of real problems associated with the distribution of social wealth. We analyze games of positional investments with retaliation, games of positional investment with repetition, and games associated with reforms that allow players to expand the area of potential investment opportunities.

Keywords: capital market, cooperative game, human capital, investment, Nash bargaining problem, productive consumption economics, social inequality, social wealth

1. INTRODUCTION

In the economics of productive consumption is this consumption productive in the sense that it brings significant income effects. The phenomenon of positional investment plays an important role in answering the question why investment opportunities related to the acquisition, preservation and utilisation of human capital are not exploited in the competitive capital market environment based on their rate of return, regardless of who is the owner of the financial resources and investment opportunities (Černík, Valenčík, Wawrosz, 2015). The presented paper shows the concepts and models of multipoint extension of Nash bargaining problem in several directions. When choosing the directions of extension, the authors based their decisions on tasks that have practical relevance. One of the important results is the identification of the phenomenon of aversion to reforms that create a greater degree of equality of opportunity for the acquisition, preservation and utilisation of human capital, and the discovery of the causes of this phenomenon.

2. MATERIALS AND METHODS

The subject of the research is the possibility of multipoint extensions of Nash bargaining problem (Nash 1950a, 1950b, 1953) in order to analyse the phenomenon of positional investment and the practical social contexts associated with reforms aimed at promoting investment in the acquisition, preservation and utilisation of human capital.

To do this, we create the appropriate concepts and models:

– By concept we mean the expression of certain connections between selected variables describing the behaviour of players in a complex of interrelated games, which at a given level of knowledge cannot be sufficiently separated so that we could express them by the relevant model and solve them as such. In creating these concepts, we emphasise clarity. We start from the contextual nature of the games, i.e. from the fact that in reality each game is played in the context of other games that affect its parameters.

– By model we understand relations between quantities describing a certain game, which can be described analytically. At the same time, what is understood as a solution of the game is defined, and the tools by which the solution can be found are usually described (or if necessary, there is a limit put on the set in which the solution can be found or in which it can be proven that a solution does not exist).

The dual model of the capital market is an important support for the use of theoretical tools of the microeconomic model of the capital market and theoretical tools derived from Nash bargaining problem or from some approaches to solving the problem of water distribution. Furthermore, standard tools of non-cooperative and cooperative games are also used (Osborn, 2004) (Peleg, 2007), (Selten, 1999a), (Selten, 1999b).

Our contribution deals with the reforms for creating a greater degree of equality of opportunity for the acquisition, preservation and utilisation of human capital, in a specific historical social context. We understand the orientation of research as subordinated to the fact how priorities successively arise through partial knowledge in relation to social realities. Therefore we use a procedure based on the expansion of the basic (elementary) model, which in this case is the chosen variant of two-point extension of Nash bargaining problem. I.e. it is a conscious focus of attention on comparing the possibilities of expanding the elementary model with how real social problems in the given area (of the aforementioned type of reforms) appear through the prism of partial results (Binmore, 1994), (Binmore, 1998), (Binmore, 2007), (Binmore, 2009), (Carmichael, 2005).

One of the most demanding research achievements is precisely the fact that partial results change the prism through which we perceive social reality. It is necessary to choose the right sequence of steps and constantly perceive the real problems and their relevance, thus gradually deciphering reality. Not only in terms of progress of knowledge, but also in terms of clarity of interpretation of achieved results. Experience from research work shows that from time to time knowledge is shifted to a higher level, which is manifested by the fact that the problem, its solution and the interpretation of the problem and its solution becomes more comprehensible from a certain theoretical perspective. Just as when walking up a hill, a person gets to a place where he or she can see more clearly both the distant target and the path already behind, with all the dead ends and unnecessary twists made when climbing. It is an experience that we have gone through in the field of presented issues.

In this paper we present the main result based on the multipoint extension of Nash bargaining problem. William Thomson and Terje Lensberg (Thomson, Lensberg 1989) formulated axioms to specify how solutions should respond to change, and provided another characterizations of all the major solutions. We don't rewrite their theory of bargaining and we identify with their published results.

3. DUAL MODEL OF THE CAPITAL MARKET

We start from the following specific case of Nash (S, d) bargaining problem:

We let $1, 2, \dots, n$ represent n fixed players. We let R^n denote the n - dimensional Euclidean space, and R^+_n is the non-negative orthant of R^n . A bargaining game of these n players is a set S , subset of R^+_n , which satisfies three conditions.

1. S is convex and compact.
2. S is comprehensive, i.e., if $x \in S$, $y \in R^+_n$, and $y \leq x$, then $y \in S$.
3. There is an $x \in S$ such that $x_i > 0$ for $i=1, 2, \dots, n$.

The points in S represent the feasible von-Neumann Morgenstern utility levels that the n players can reach simultaneously in a particular bargaining situation, i.e., $x = (x_1, x_2, \dots, x_n) \in S$ if and only if there is a possible outcome in this bargaining situation which would assign player 1 a utility level x_1 , would assign 2 a utility level x_2 , etc. If the players fail to agree on an outcome in S then they each

The figure consists of two side-by-side graphs. The left graph shows a coordinate system with axes labeled y and x . It features a straight budget line labeled S , a convex indifference curve labeled I^0 , and another convex curve labeled C . A point E is marked at the intersection of S and I^0 . A shaded circular region around E contains a small angle labeled α . Dashed lines from E project onto the axes at x_d and y_d . Points x_0 and y_0 are also indicated on the axes. Three numbered labels (1), (2), and (3) are placed near the top of the graph. The right graph has a vertical axis labeled Y' and a horizontal axis labeled Y . It shows two downward-sloping curves, MY^{-1} and MY^{-2} , intersecting at point E . A shaded circular region around E contains a small angle labeled β . Dashed lines from E project onto the axes at Y_1 and $Y_1 + Y_2$. A label y_d points to the horizontal axis.

Source: Own creation

Shaded area between points d and E: The area of Pareto improvements of given opportunities to use one's own financial resources to realise the other player's investment opportunities; an area meeting the prerequisite of individual rationality and achievability in the (S, d) Nash bargaining problem given by the opportunity to exploit investment opportunities no matter who owns them, the only point here

that also meets the prerequisite of collective rationality (Pareto optimality) is point E; its shape (of the boundary) is determined by the course of marginal yield curves from investment opportunities (in the left part of the figure MY¹ a MY²)

Dashed vertical line: Threshold of the budget constraint of the poorer player from the right and the richer player from the left

Darker grey triangle: The area of Pareto improvements of given possibilities to use own financial resources of one of the players to realise the investment opportunities of the other player

Lighter grey bevelled quadrangle: Payoff (future income) of the richer player at the point of disagreement or in a situation where Pareto improvements of given possibilities are not utilised to utilise one's own financial resources to realise the other player's investment opportunities

Some connections between the left and right sides of the model (shown by dotted lines with arrows):

- (1) Point E on the left side of the figure corresponds to point E on the right side of the figure
- (2) The triangle highlighted in dark grey on the left side of the figure corresponds to the shaded closed area between points d and E on the right side of the figure
- (3) The bevelled quadrangle highlighted by the lighter colour on the left side of the figure corresponds to value y_d (the coordinate of point d of the richer player) on the right side of the figure

(we do not provide more context in order not to reduce the clarity of the figure)

Nash bargaining problem means that we are looking for a function $f: \{(S, d)\} \rightarrow R$ which meets the specified requirements. In doing so

f is the corresponding function

S is a set that meets the prerequisite of achievability, i.e. the solution must be a part of it

d is the default point of disagreement

R is the space in which we are looking for solutions

In our case:

For S it is true that: x, y belongs to $S \leftrightarrow (x+y) \leq s$ and at the same time ≥ 0

R^2 , i.e. in the case of two players we consider two-dimensional space

d belongs to S

players' payoffs, x, y , are interpreted as future returns of players from the combination of financial resources and investment opportunities

for every x satisfies $\int_0^{Y_1} x(Y) dY$

for every y satisfies: $\int_{Y_1}^{Y_1+Y_2} y(Y_1+Y_2 - Y) dY$

for s satisfies: $s = \max \left\{ \int_0^{Y_1} x(Y) dY + \int_{Y_1}^{Y_1+Y_2} y(Y_1+Y_2 - Y) dY \right\}$

4. ELEMENTARY MODEL OF POSITIONAL INVESTMENT

Let us summarise the most important facts of the previous part. The way of interpreting the payoff of players is very important in analysing games of this type. It is usually understood as a benefit in the sense of a subjective experience that is non-transferable and inter-subjectively incomparable. Let us now return to the games we are dealing with. In our case we are solving a specific kind of cooperative game of the (S, d) Nash bargaining problem type:

– Payoffs may be expressed in comparable and transferable financial units.

- The point of disagreement d is at the boundary of the convex set, the curvature of which is given by the declining rate of return on investment opportunities available to one and the other player.
- The position of point d is determined by the budget constraints of players or more precisely by the amount of financial resources of each player.
- Cooperative improvement is determined by the “creditor – debtor” relationship (one player can use the financial resources owned by the other player to realise their investment opportunities), so that the investment opportunities are used based on their rate of return, regardless of who is their owner.
- Set S (the improvement over point of disagreement d) is specific, bounded by a 45° slope line and represents the points determined by the different options for distributing the maximum return on investment opportunities for a given sum of financial resources and given investment opportunities.

In the Valenčík et al. (2020), it has been shown that what prevents the exploitation of investment opportunities associated with the acquisition, utilisation and preservation of human capital based on their rate of return, irrespective of who owns the financial resources and investment opportunities, is the investment in social position or the positional investment. The simplest (elementary) case of positional investment can be expressed in the form of a two-point extension of Nash bargaining problem.

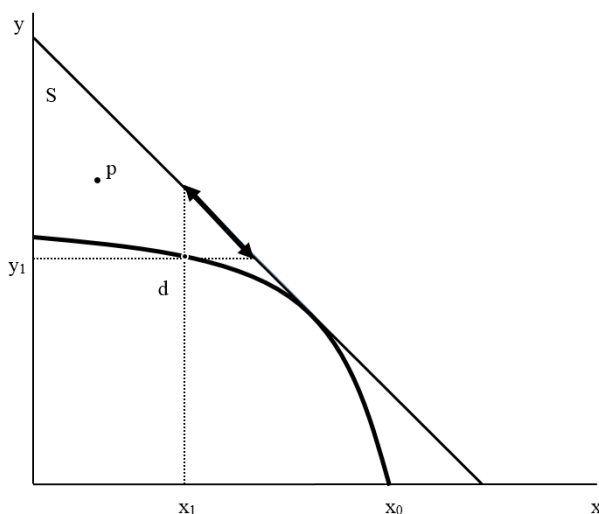


Fig. 2. Elementary model of positional investment

Source: Own creation

If the richer player has the opportunity to use his or her material advantage to achieve the division at p (and the poorer player will not be able to respond), he or she will not have an interest (in terms of individual rationality) to participate in solving Nash bargaining problem.

Three situations can arise:

- | | |
|-----------------------------|---|
| $y_p \geq y_{\max}$ | the richer player will choose a positional investment strategy |
| $y_{\max} > y_p > y_{\min}$ | the solution of Nash bargaining problem influenced by the position of point p |
| $y_{\min} \geq y_p$ | the opportunity of positional investment does not affect the game |

One of the interpretations of positional investment is that the profitability of the richer player's investment opportunities increases and that the poorer player's investment opportunity profitability decreases.

In the paper of Valenčík et al. (2019) it has been shown that in most cases an explanation based on the shift of the MX' and MY' curves is the most appropriate.

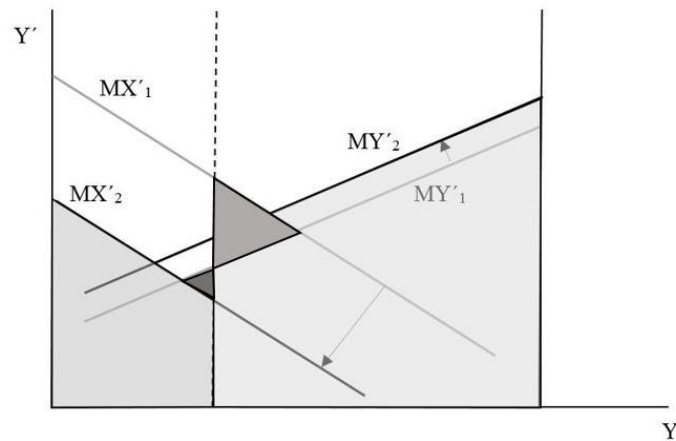


Fig. 3. One of the interpretations of positional investment

Source: Own creation

Figure 3 corresponds to the following figure 4 in our dual model.

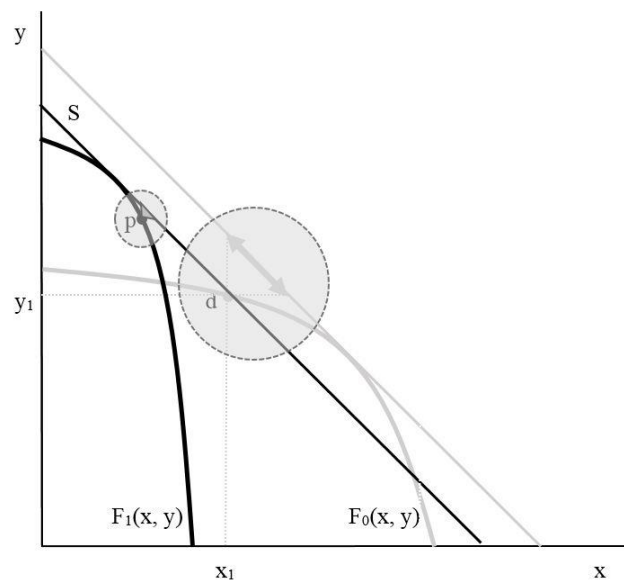


Fig. 4. One of the interpretations of positional investment

Source: Own creation

Note that positional investment, which affects the rate of return on investment opportunities available to players, can “reverse” the relationship between the creditor and the debtor.

A larger lighter triangle indicates the original area of Pareto improvements resulting from the use of the capital market.

A smaller, darker triangle indicates the area of Pareto improvements resulting from the use of the capital market after a change in the profitability of investment opportunities as a result of positional investment.

We know this situation well. Poorer people save not only for the worse, but also because they do not have sufficient investment opportunities (they do not hold the right position in the society). Large

projects are realised from their savings by those who in the right position (in terms of image, social contacts, inside information, etc.).

5. GAMES WITH OPPOSITION ACTION BY THE PLAYERS

So far, we have only considered the case where one (the richer) player can invest in his or her social position to increase his or her payoff. Either by affecting the cooperative game in question, or by achieving higher payoffs than in the cooperative game. The second (poorer) player had no opportunity to respond to the decision of the richer player. However, in reality, the poorer player very often does not have to be just a mere by-stander, but he or she may react or show resistance in order to:

- enforce an increase in his or her payoff;
- reduce the payoff of the richer player (either by punishing him or her, but mainly to discourage him or her from investing in a social position, even at the cost of reducing his or her own payoff).

This is described in the following figure:

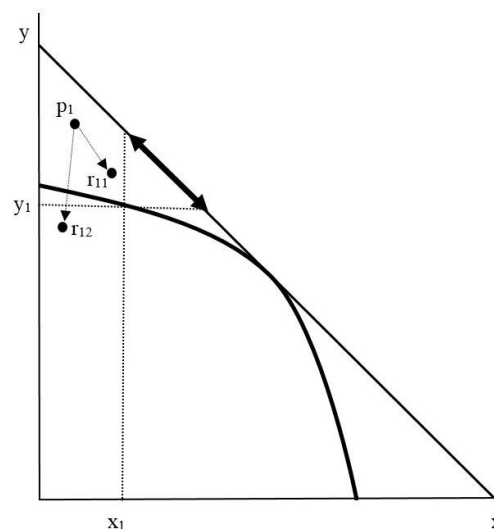


Fig. 5. Poor player reaction to positional investment

Source: Own creation

Here, points r_{11} and r_{12} represent the poorer player's reaction to the first player's positional investment (realised or intended), where:

- If point r_{11} is reached, the effect of increasing the income of the poorer player in the form of his or her resistance prevails, and the reduction of the payoff of the richer player (punishing the richer player) is an accompanying effect.
- If point r_{12} is reached, the effect of punishing the richer player prevails, even at the cost of the poorer player reducing his or her own payoff. This can have two causes. Either the poorer player is trying to discourage the richer player from the positional investment strategy, or the poorer player is preparing for a follow-up game involving the use of positional investment.

The richer player may answer the reaction of the poorer player. The richer player can also react to it, while considering two different effects – either wanting to reduce the payoff of the poorer player (to punish him or her or to create better conditions for positional investment in the follow-up game), or wanting to increase his or her payoffs despite the resistance of the poorer player. This can be illustrated by the following figure.

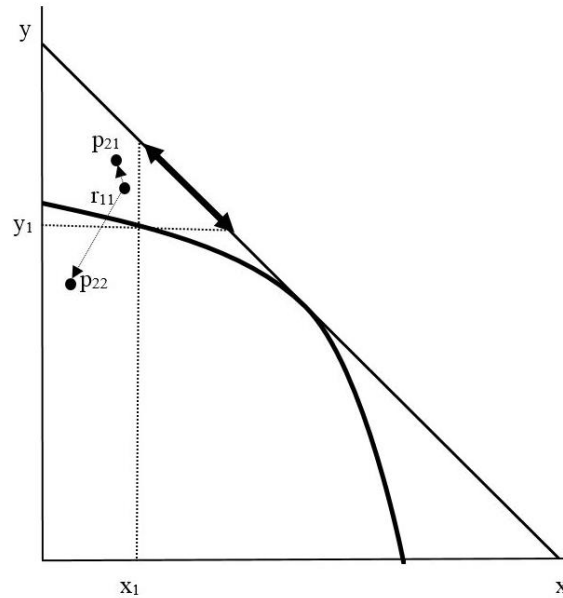


Fig. 6. Reaction of the richer player to the reaction of the poorer player

Source: Own creation

Figure 6 shows a case where the richer player responds to point r_{11} (the poorer player attempted to increase his or her payoffs by opposing positional investment) while:

- If point p_{21} is reached, the richer player is mainly concerned with increasing his or her own payoff.
- If point p_{22} is reached, the effect of punishing the poorer player for opposing positional investment prevails, even at the cost of the richer player reducing his or her own payoff, which in this case may have two causes similar to the case of the poorer player.

The poorer player can respond to the reaction of the richer player, the richer player can respond with another reaction, etc. There are two ways to analyse such a chain of reactions:

- Either we have the opportunity to express the whole chain of reactions through a certain model (for example a game in an explicit form) and investigate it by achievable methods. This is only possible in exceptional cases.
- Or we will assume that both players are in some way able to appreciate the endpoints of the chain of reactions depending on the initial positional investment. This reduces the very complicated, multi-round game to a one-round game, and by analysing it we can gain valuable insights.

Whether we analyse a single-round game or have a game that was created by reducing the chain of events to a single-round game, we get a situation like of this type.

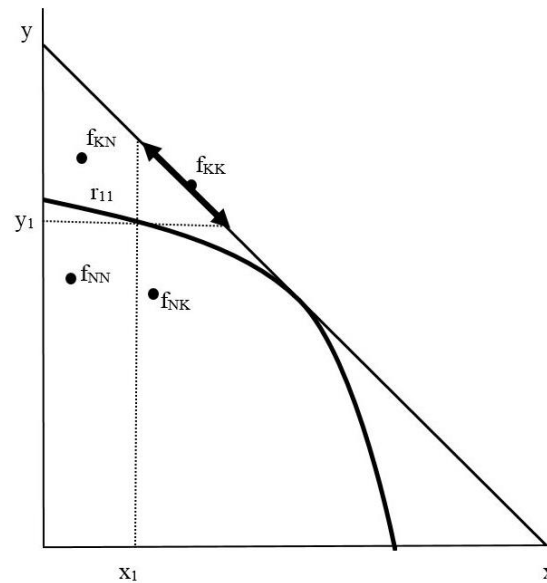


Fig. 7. Reaction of the richer player to the reaction of the poorer player

Source: Own creation

Here:

f_{KK} is the final situation where both players cooperate, in which case they are solving the relevant cooperative game

f_{KN} is the final situation where the poorer player cooperates and the richer does not

f_{NK} is the final situation where the richer player cooperates and the poorer does not

f_{NN} is the final situation where none of the players cooperate

The general rule here is that if one player does not cooperate, the other player has a lower payoff if he or she cooperates than if he or she does not cooperate.

As for the relationship between payoffs on a case-by-case basis, there may be a variety of options, which we will discuss below.

It is relatively easy to see that the payoff values of the players in each case can be converted into a table of a two-matrix game, where the points have the following values:

$$f_{KK}(x_{KK}, y_{KK})$$

$$f_{KN}(x_{KN}, y_{KN})$$

$$f_{NK}(x_{NK}, y_{NK})$$

$$f_{NN}(x_{NN}, y_{NN})$$

		richer player B	
		cooperates	non-cooperates
poorer player A	cooperates	$x_{KK} : y_{KK}$	$x_{KN} : y_{KN}$
	non-cooperates	$x_{NK} : y_{NK}$	$x_{NN} : y_{NN}$

Table 1. Table of a two-matrix game

Source: Own creation

Depending on the inequalities involved, different types of two-matrix non-cooperative games can take place, and very often a game of the prisoner's dilemma type arises. This is also the case in our figure.

The possibility of acquiring a two-matrix game based on the concept of a cooperative game associated with positional investment and retaliation is quite a significant non-trivial result of our approach, which has significant practical applications.

6. DISCUSSION SUMMARY AND DISCUSSION

We have shown that a relatively elementary positional investment model (which has a trivial solution) can be expanded in several direction into models or, in some cases, at least into concepts that have a non-trivial solution and the actual solution or expression of the situation by a concept has non-trivial and valuable application concerning the identification of important phenomena in the field of social reality and related practical recommendations.

We considered extension in the following areas:

- Possibility of retaliation against these reactions by both the players with and without the possibility of positional investment. We have shown that in the case of two players, some situations can be expressed as two-matrix games.
- We presented a concept that allows to express the consequences of increasing the profitability of players' investment opportunities through reforms and showed that there can be a phenomenon of aversion to reforms, and even that in certain situations there may be alliances of players who expect the greatest benefit from positional investment in a derivative two-matrix game (and who opt for a non-cooperative strategy in that game) with the players who expect the greatest benefit from the possibility of retaliation in a derivative two-matrix game (and who also choose a non-cooperative strategy in this game). This creates a cross-alliance of players choosing a cooperative strategy and an alliance of players choosing a non-cooperative strategy.
- We have opened the issue of games containing the option of positional investment with repetition.
- We have not dealt with the previously analysed problem of a hitch that arises in the game without positional investment and which also complicates the possibility of using investment opportunities based on their rate of return, regardless of who owns the investment opportunities and who owns the financial resources. The analysis of this issue has been provided earlier and both in terms of the results of theoretical analysis and practical interpretations it follows that this is a much smaller obstacle than the positional investment (Černík, Valenčík, 2016).

We consider the biggest deviation of concepts and consequently also models from reality to be the fact that players estimate both the initial situation (their possibilities and those of other players) and possible changes (especially the consequences of possible reforms) significantly differently. This difference in valuation of real situations and their changes (also in terms of economics of productive consumption) manifests itself as a difference of preferences in relation to the available alternatives. To investigate these situations, we suggest using the apparatus (or possible modifications of this apparatus) of the solution to the theoretical problem of "fair cake-cutting" – dividing a divisible,

heterogeneous good Fair cake-cutting - Dividing a divisible, heterogeneous good. In this case, the negotiations would result in a consensus on the complex of reforms to be implemented, on how to sort the steps and how to set the parameters for these reforms.

7. CONCLUSIONS

The main message, which is to uncover the causes of the problems and, at the same time, the most serious consequences of dividing society into the rich and the poor, and then proposing reforms that lead to promising solution of the problems cause by such division of society into the rich and the poor require such wide coverage. In our opinion, an attempt to solve partial problems without their generally theoretical coverage is not realistic. We consider the context, in which we have tried to address the issue of wealth and poverty in terms of reforms that create the conditions for exploiting investment opportunities related to the acquisition, preservation and utilisation of human capital based on their rate of return, regardless of who owns these investment opportunities and who owns the financial resources, to be minimal. Minimal in terms of determination of the research area in which the problem of wealth and poverty (or a complex of problems related to the division of society into the rich and the poor) can be solved.

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