

ONE FORGOTTEN DIRECTION OF BUSINESS ANALYSIS: THE CASE OF ESTONIA

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Abstract

In the 1960s and 1970s, Estonian Academician Uno Mereste developed a methodology for performance analysis, which could be called Integrated System of Performance Analysis (ISoPA). Several papers on ISoPA were published in the 1980s. Unfortunately, they were all written in Estonian or Russian. For that reason, the methodology of ISoPA is virtually unknown outside of Estonia. ISoPA allows an analyst to evaluate the business results of an individual entity or corporate group from many different perspectives, comparing them with set goals, historical data or data of other entities or groups. Based on a combination of a system approach with matrix modeling and index number theory, ISoPA became an alternative approach to analyze and evaluate entities' business activities, yet for various reasons the methodology is almost by present-day business managers and financial analysts. The aim of the paper is to introduce the use of ISoPA methodology and to discuss the selection and ranking of initial indicators for assessing the performance of a business entity. The authors of the current paper believe that the use of the ISoPA methodology to examine the interrelationships between business performance indicators provides a promising opportunity to improve the level of business analysis.

Keywords: Estonia, performance measurement, Integrated System of Performance Analysis, index numbers

1. INTRODUCTION

In the 1960s and 1970s, Estonian Academician Uno Mereste, whose main field of research was the index number theory, developed basic principles of a new performance analysis methodology, called *Integrated System of Performance Analysis (ISoPA)*. In his doctoral dissertation *The Use of Index Number Theory in Economic Analysis (Theoretical and Methodological Issues)* Mereste developed further the theory of analytical index numbers (he called them super indexes) (Mereste, 1970). This approach was originally started in the 1871 by the famous German economist Etienne Laspeyres. It must be noticed that from 1869 to 1873 Laspeyres served as Professor of Geography, Ethnography, Statistics and Political Economy at Dorpat, now Tartu, Estonia. In the then current practice of business analysis, index numbers were used either in the form of stand-alone, or as a systemic factor indexes. Mereste noticed that there is a third option that differs qualitatively from the options mentioned, namely in the form of index matrices. He also proposed a version of classification of index numbers and developed a method that allows studying the interaction and mutual influence of any number of interrelated economic phenomena using analytical factor indexes. In 1984 was published Mereste's monograph *The Integrated Analysis and Efficiency* (alternatively *The Integrated Analysis and Effectiveness*), summarizing his research and development work.

In the 1980s, *ISoPA* was used to some extent in Estonia and some Estonian researchers further developed the methodology of *ISoPA*. Unfortunately, all their publications were in Estonian or Russian.

In 1991, after the collapse of the USSR Estonia regained independence. The transition from command economy to market economy had a huge impact on a great number of aspects in all society. Former researchers became politicians, businesspersons, auditors ... The school of Estonian business analysts headed by Mereste collapsed and using of *ISoPA* stopped.

After a long break, a handbook *Bank Performance Analysis and Financial Management* (Vensel, 2001) and article *Bank Performance Analysis Methodology and Empirical Evidence (Estonian Banking System, 1994–2002)* (Vensel et al., 2004) partly devoted to *ISoPA* were published. Currently some researchers and PhD students of the Department of Business Administration at Tallinn University of

Technology have begun to use and develop *ISoPA* in their research (Alver and Startseva, 2013; Alver and Startseva, 2011; Startseva and Alver, 2011; Startseva *et al.*, 2013; Siimann 2011).

2. THE FACTOR INDEXES AND TIERED INDEXES AND THEIR ECONOMIC CONTENT

The methodology used by *ISoPA* is based on combining systems analysis with matrix modeling and index number theory. To explain *ISoPA* the following key terms are used:

Quantitative phenomenon – an economic phenomenon that is measured in simple units (euros, tons, kilograms, root or cubic meters, liters, hours, pieces, pairs etc.) or their multiples (million euros, thousand tons, dekaliters, man-hours, kilowatt-hours, thousand pieces, thousand pairs). Sometimes the term *extensive phenomenon* is used instead.

Quantitative indicator – an indicator that represents a quantitative phenomenon.

Qualitative phenomenon – an economic phenomenon that is measured indirectly by fractional units of measure (price, productivity, profitability etc.). Sometimes the term *intensive phenomenon* is used instead.

Qualitative indicator – an indicator that represents a qualitative phenomenon.

Factor – a part or element that influences something or contributes to a particular result. If the factors have numerical values then multiplied together they give a resultant aggregate amount.

The classification of index numbers used in *ISoPA* distinguishes the following types: *the composite index, the index of quantitative factor, the index of qualitative factor, the index of variable composition, the index of constant composition and the index of structural change*. Figure 1 summarizes formulas of index numbers of each type.

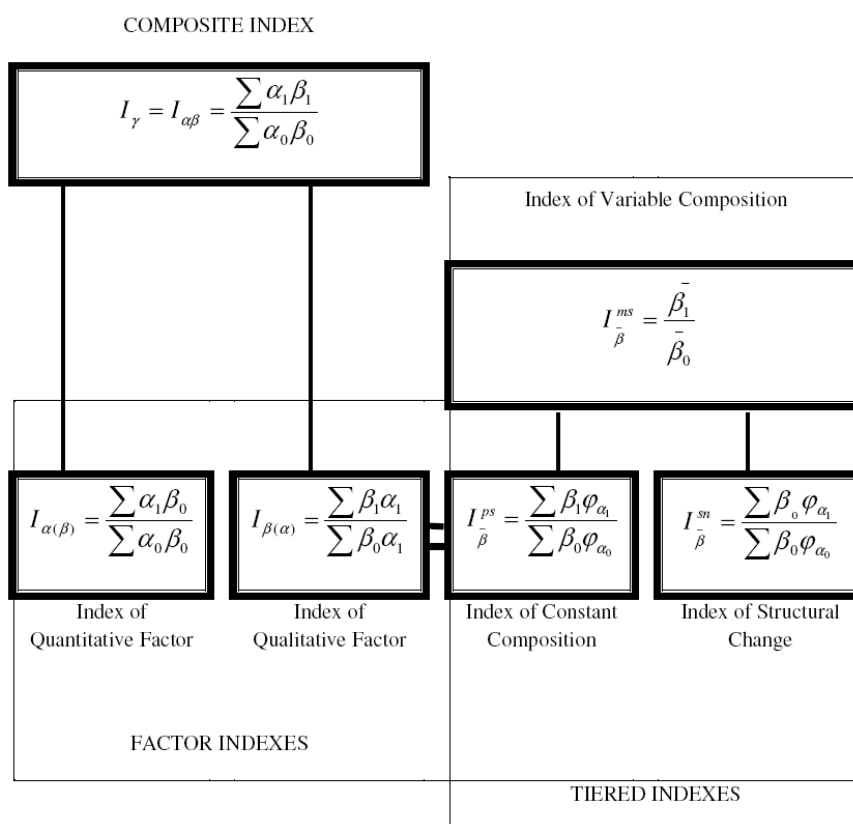


Fig. 1. The classification of index numbers used in *ISoPA*

Source: Mereste (1984, p. 84).

The composite index $I_\gamma (= I_{\alpha\beta})$ expresses the effect of changes in both of two factors (variables), factor α (quantitative factor) and factor β (qualitative factor), i.e. the change of aggregate amount under the influence of both influencing factors involved. It is not an analytical index, but a synthetic one, as it reflects the relative change in one quantity affected by changes in two factors.

$$I_\gamma = I_{\alpha\beta} = \frac{\sum \alpha_1 \beta_1}{\sum \alpha_0 \beta_0} \quad (1),$$

where

α_1 - (value of) the quantitative indicator α in the period '1', for which comparison is made;

α_0 - (value of) the quantitative indicator α in the base or reference period (period '0');

β_1 - (value of) the qualitative indicator β in the period '1', for which comparison is made;

β_0 - (value of) the qualitative indicator β in the base or reference period (period '0').

The importance of the composite index in the aggregated form is that it is the basis for the construction of analytical factor indexes, and along with them, it belongs to the same system of index numbers.

The composite index can be converted into an index reflecting the effect of only one factor (variable) with the help of the so-called *logical isolation mode*. If the index formula (1) is to be converted so that only one variable factor remains in it, and the second factor in the numerator and the denominator will have the same value, then it can be assumed that the effect of changing the item with the same value in the numerator and the denominator will be eliminated from the value of index number.

The index numbers of the quantitative factors are calculated according to the well-known Laspeyres formula:

$$I_\alpha = \frac{\sum \alpha_1 \beta_0}{\sum \alpha_0 \beta_0} \quad (2)$$

The index numbers of the qualitative factors are calculated according to the well-known Paasche formula:

$$I_\beta = \frac{\sum \alpha_1 \beta_1}{\sum \alpha_1 \beta_0} \quad (3)$$

The index I_α (2) shows the impact of the change in the quantitative factor on the aggregated amount $\sum \alpha\beta$. The index I_β (3) shows the impact of the change in the qualitative factor on the aggregated amount $\sum \alpha\beta$.

The connection between index numbers is identical to the connection between corresponding indicators:

$$I_\alpha \times I_\beta = I_{\alpha\beta} = I_\gamma \quad (4)$$

Tiered indexes or level indexes are index numbers whose value expresses the change in the average value of a feature altogether or under the influence of some factor. For the purpose of this paper, structural changes are changes in the way different indicators of structure relate to each other. Structural change is expressed as an increase in the share of one or more components of a specified phenomenon and the simultaneous decrease in the share of other components.

The tiered index makes it possible to study the effects of changes in the structure of the phenomenon. The importance of examining structural changes is determined by the following:

- structural changes have a profound impact on many results of operations of the entity;
- structural changes take place in a more or less hidden form.

The index of variable composition expresses the change in the aggregate (average) amount, affected by all factors, including structure:

$$I_{\beta} = \frac{\bar{\beta}_1}{\bar{\beta}_0} \quad (5)$$

In the aggregated form the formula (5) looks like

$$I_{\beta} = \frac{\sum (\beta_1 \frac{\alpha_1}{\Sigma \alpha_1})}{\sum (\beta_0 \frac{\alpha_0}{\Sigma \alpha_0})} = \frac{\sum \beta_1 \varphi_{\alpha_1}}{\sum \beta_0 \varphi_{\alpha_0}} \quad (6)$$

where $\varphi_{\alpha} = \frac{\alpha}{\Sigma \alpha}$.

In the index of constant composition I_{β}^{ps} , the effect of elements' structure is eliminated:

$$I_{\beta}^{ps} = \frac{\sum \beta_1 \varphi_{\alpha_1}}{\sum \beta_0 \varphi_{\alpha_1}} \quad \text{or} \quad I_{\frac{ps}{\beta}} = I_{\beta} \quad (7)$$

The index of structural change makes it possible to measure the effect of changes in the structure of an aggregate on the average value of a qualitative indicator:

$$I_{\beta}^{sn} = \frac{\sum (\beta_0 \frac{\alpha_1}{\Sigma \alpha_1})}{\sum (\beta_0 \frac{\alpha_0}{\Sigma \alpha_0})} = \frac{\sum \beta_0 \varphi_{\alpha_1}}{\sum \beta_0 \varphi_{\alpha_0}} \quad (8)$$

3. DESIGN OF PERFORMANCE MEASUREMENT SYSTEM

3.1. Efficiency versus effectiveness

Efficiency expresses a relationship between inputs and outputs. In general, efficiency is a measurable concept, quantitatively determined by the output to input ratio.

Effectiveness is the relationship between the output of a company and the goals set up. It is a relatively vague, non-quantitative concept that is related to the achievement of goals. Effectiveness continuously measures whether the actual output corresponds to the desired output. When company meets its goals, this is generally referred to as effectiveness. Being *effective* means achieving the goals of the company.

Efficiency	Effectiveness
Efficiency, per se, is not related to goals. Efficiency is process focused.	Effectiveness is always related to goals.
An efficient company is one, which does whatever it does with the lowest consumption of resources; but if what it does (i.e., its output) is an inadequate contribution to the accomplishment of the company's goals, it is ineffective.	Effectiveness is focused on achieving / not achieving the final goals, taking into consideration any variables that were to change in the future.
Efficiency refers to doing things in a right manner.	Effectiveness refers to doing the right things.
If we compare the data of the just ended period with the data of the previous period, we can find out whether the company was efficient.	If we compare the data of the just ended period with the budget data of that period, we can find out whether the company was effective.

Table 1. Relationship of efficiency and effectiveness

Source: Developed by the authors.

Example. In May, *One Company* produced 1,000 units of product (its output) at a cost of 4,000 € (its input). The relationship of the 4,000 € cost to the 1,000 units measures the efficiency of *One Company*. In this case, we can express the ratio as a cost per unit: $4,000 \text{ €} / 1,000 \text{ units} = 4 \text{ € per unit}$.

If in June, *One Company* produced 1,000 units of product at a cost of 3,900 €, its efficiency is measured as 3.90 € per unit, and the indication is that *One Company* was more efficient in June than in May. But if according to the budget *One Company* was obliged to produce 1,000 units of product at a cost of 3,850 € (i.e. 3.85 € per unit), the indication is that *One Company* was ineffective in June.

In the above example, efficiency and effectiveness were expressed in relative terms; that is, *One Company* was said to be more efficient in June than in May and at the same time ineffective in June.

The relationships between efficiency / inefficiency and effectiveness / ineffectiveness are shown in Table 2.

		Efficient / inefficient use of resources (Doing things right)	
		Efficient	Inefficient
Achieving / not achieving goals (Doing the right things)	Effective	Effective (achieving goals) and efficient (high ROI, cost-efficient)	Effective (achieving goals), but inefficient (costs are high)
	Ineffective	Ineffective (not achieving goals), but efficient (not producing enough but low-cost)	Ineffective (not achieving goals) and inefficient (not producing enough and are expensive)

Table 2. Relationships between efficiency / inefficiency and effectiveness / ineffectiveness

Source: Developed by the authors.

3.2. Efficiency field and its matrix form

Suppose that one would like to conduct analysis of financial statements in order to assess economic efficiency or effectiveness based on the following five initial indicators:

- Profit (P)
- Sales (S)
- Cost of Goods Sold (C)
- Assets (A)
- Equity (E).

It is possible to develop $n^2 - n$ qualitative relationships between initial *quantitative* indicators, where n is the number of initial indicators. In our case, based on 5 initial indicators 20 *qualitative* indicators (ratios) can be formed (see Fig. 1). It is necessary to emphasize that all these relations (qualitative indicators) have a fixed economic content. The network, which consists of 5 initial quantitative indicators and 20 qualitative indicators, is called the *field of effectiveness* or the *efficiency field* (it depends from comparison base).

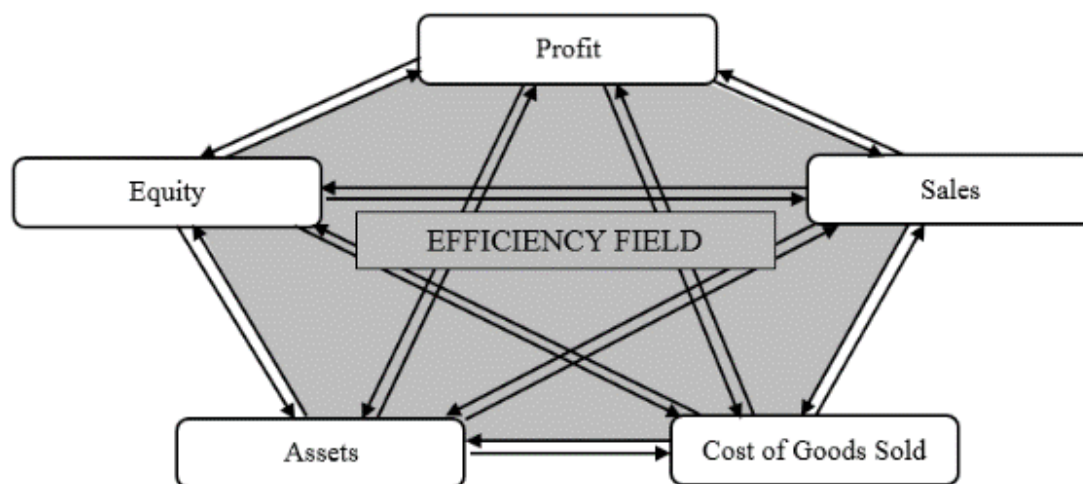


Fig. 1. An example of the efficiency field including 20 *qualitative* indicators (the direction of arrows is from financial ratio numerator to denominator) surrounded by 5 initial *quantitative* indicators

Source: Developed by the authors.

Graphical representation of the relations is very clear, but it is technically inconvenient for large sets of indicators. It is much more reasonable to present the same relationships in the form of an organized matrix model (Fig. 2).

Profit (P)	11 1	12 $\frac{S}{P}$ Sales to Profit	13 $\frac{C}{P}$ COGS to Profit	14 $\frac{A}{P}$ Assets to Profit	15 $\frac{E}{P}$ Equity to Profit
Sales (S)	21 $\frac{P}{S}$ Profit margin	22 1	23 $\frac{C}{S}$ COGS to Sales	24 $\frac{A}{S}$ Assets to Sales	25 $\frac{E}{S}$ Equity to Sales
Cost of Goods Sold (C)	31 $\frac{P}{C}$ Profit to COGS	32 $\frac{S}{C}$ Sales to COGS	33 1	34 $\frac{A}{C}$ Assets to COGS	35 $\frac{E}{C}$ Equity to COGS
Assets (A)	41 $\frac{P}{A}$ ROA	42 $\frac{S}{A}$ Asset Turnover	43 $\frac{C}{A}$ COGS to Assets	44 1	45 $\frac{E}{A}$ Equity to Assets
Equity (E)	51 $\frac{P}{E}$ ROE	52 $\frac{S}{E}$ Sales to Equity	53 $\frac{C}{E}$ COGS to Equity	54 $\frac{A}{E}$ Equity Multiplier	55 1

Fig. 2. Ordered matrix model for *ISoPA*

Source: Developed by the authors.

The matrix, presented in Figure 2, is a *Key Matrix*. It is the key to understanding analytical information presented in matrix format and used in *ISoPA*. The matrix consists of 20 qualitative indicators (ratios), surrounded by 5 initial quantitative indicators. Initial quantitative indicators are located in the header of table columns (γ_j) and the header of table rows (α_i). Consequently, the relationship between the elements of the matrix and the initial indicators can be expressed as

$$\beta_{ij} = \frac{\gamma_j}{\alpha_i} \quad (1)$$

where β_{ij} – element of row i and column j ;

i – row number of cell location ($i = 1, 2, 3, \dots, m$);

j – column number of cell location ($j = 1, 2, 3, \dots, n$).

The subscripts i and j determine the location of a particular phenomenon or component in the matrix. For example, β_{32} is a qualitative component in the intersection of row 3 and column 2, i.e. $\beta_{32} = \gamma_2 / \alpha_3$.

Formula (1) can be rewritten as follows

$$\alpha_i \beta_{ij} = \gamma_j \quad (2)$$

NB! γ_j is a resultant indicator affected by indicators α_i and β_{ij} .

A triangular matrix below the main diagonal of the matrix with the elements representing financial ratios that characterize different aspects of a company's business efficiency is called an *efficiency matrix*. Based on that assumption, the second triangular matrix may be called an *inverse efficiency matrix*, since its elements are reciprocal values of the efficiency indicators.

The entire set of data, organized in the form of a matrix, consists of 11 matrices and 2 vectors. Each matrix is a square matrix in the form of $m = n$, where m denotes the total number of rows and n is the total number of columns. The structures of all matrices are based on the *Key Matrix*.

3.3. The selection and the ranking of the initial indicators to build up a key matrix

The value of a company's *overall performance indicator (OPI)* and the findings of the analysis depend on the choice and order of placement of the initial indicators in the matrix model. The selection of the initial indicators depends on the objectives of the analysis. For example, if the purpose of the research is the measurement of overall performance, the set of initial indicators should include all the main indicators that characterize the resources, expenses and outcomes (results) of the entity's operations.

All initial indicators involved in the analysis can be divided into three groups: *resources*, *expenses* and *results*. Some theorists of this method have suggested grouping resources and expenses in the same group when classifying and ranking initial indicators. In our opinion, these should not be combined. The resources are always associated with a particular moment in time. By contrast, expenses are always associated with a certain period.

Expenses are a purposeful use of resources. A business entity is regarded as a mechanism that turns resources (inputs) through the expenses into desired outputs. Consequently, being located between resources and results, expenses are much closer to the output than resources are.

When sequencing the initial parameters, the following scheme should be used (Fig. 3). The arrows indicate the direction of desired increase of growth rates.

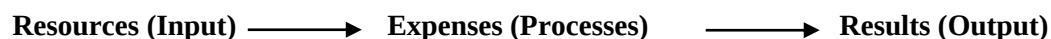


Fig. 3. Sequencing of initial indicators

Source: Developed by the authors.

Thus, the growth rate of results should be greater than the growth rate of expenses, and expenses should grow at a greater rate than resources. It should be mentioned that this scheme also reflects the cause and effect linkages between the given indicators (Alver, 1989; Alver and Startseva, 2013). The rule used in sequencing can be named the *RER-principle* (*RER = Resources, Expenses, Results*).

The matrix model can be divided vertically and horizontally into three parts, distinguishing the results, expenses and resources (Fig. 4).

	RESULTS	EXPENSES	RESOURCES
RESULTS	Indicator of Changes in Results' Structure J_{tt}		
EXPENSES	Indicator of Expenses' Efficiency J_{tk}	Indicator of Changes in Expenses' Structure J_{kk}	
RESOURCES	Indicator of Changes in Relationships Between Results and Resources J_{tr}	Indicator of Changes in Relationships Between Expenses and Resources J_{kr}	Indicator of Changes in Resources' Structure J_{rr}

Fig. 4. The division of the matrix model into six zones, each of which has a specific content

Source: Developed by the authors.

The half-field below the main diagonal of the matrix is divided into six zones, each of which has a specific content (Fig. 3). It is possible to calculate a specific performance indicator for each zone. The following proportions between specific performance indicators should be followed:

$$\begin{aligned}
 J_{tt} &< J_{tk} < J_{tr} \\
 J_{kk} &< J_{kr} < J_{tr} \\
 J_{rr} &< J_{kr}
 \end{aligned}
 \quad (3, 4, 5)$$

Where

J_{tt} - Indicator of Changes in Results' Structure;

J_{tk} - Indicator of Expenses' Efficiency;

J_{tr} - Indicator of Changes in Relationships Between Results and Resources;

J_{kk} - Indicator of Changes in Expenses' Structure;

J_{kr} - Indicator of Changes in Relationships Between Expenses and Resources;

J_{rr} - Indicator of Changes in Resources' Structure.

The linkages between performance indicators of specific zones are demonstrated in Figure 5:

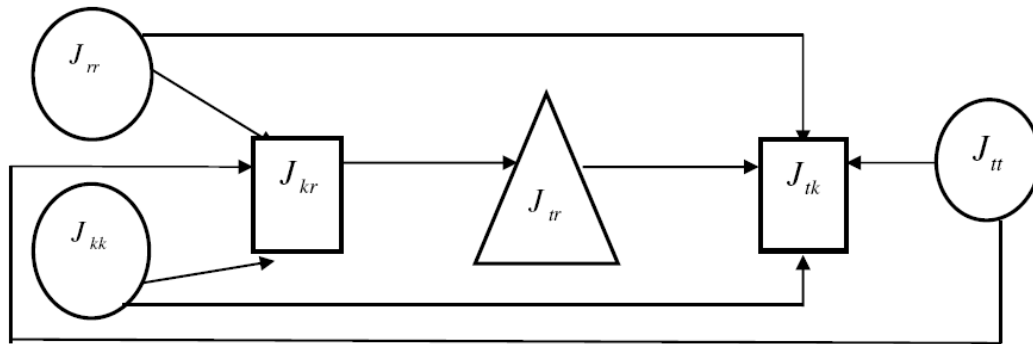


Fig. 5. Relations between indicators of six specific zones

Source: Developed by the authors.

The problem of ordering the initial indicators within each of the three groups (Fig. 2) is much more complex and requires further research.

3.4. The overall performance indicator (OPI)

The matrix approach for measuring economic efficiency / effectiveness stems from the fact that economic efficiency and effectiveness, as multifaceted phenomena, cannot be reflected in a single index number. If the company's economic efficiency /effectiveness has been improved during the review period, then it is reflected in the increase of the average value of these qualitative indicators (ratios), whose value according to general theoretical requirements should increase if efficiency /effectiveness increases. The overall performance indicator (OPI) should synthesize and summarize the changes in one measurement many individual components in the level of efficiency /effectiveness. The good basis for calculating the OPI are the values of the indexes of variable composition.

OPI can be calculated as an arithmetic mean of indexes of variable composition:

$$I_E = \frac{2 \sum I_{ij}}{n^2 - n} \quad (14)$$

where I_{ij} is the index of variable composition and n is the number of initial quantitative indicators.

OPI can be calculated also as a geometric mean of indexes of variable composition:

$$I_E = \sqrt[n^2 - n]{\prod I_{ij}} \quad (15)$$

Theoretically, geometric mean is better to use for calculating of OPI (Alver and Startseva 2013). However, in practice, the arithmetic mean is often used because it is easier to follow.

OPI synthesizes and summarizes the changes in many individual components in the level of efficiency all in one. OPI can be used to compare any entities based on the relative efficiency of their business activities. Alternatively, OPI can be used for ranking any entities based on the relative changes (dynamics) in the effectiveness of their business activities.

4. CONCLUSIONS

Performance measurement systems serve as useful tools that enable managers to improve entity's processes, productivity and performance. Performance measurement is a difficult and complex phenomenon, and today evaluators lack widely recognized performance measurement methods. Index numbers are important tools of performance analysis. They are the indicators of various trends.

The Estonian academician Uno Mereste developed an alternative branch of application of index number theory in business analysis, which allows studying the interaction and mutual influence of any number of interrelated economic phenomena using analytical factor indexes. His methodology of performance analysis called *Integrated System of Performance Analysis (ISoPA)* allows an analyst to evaluate the business results of an individual entity or corporate group from many different perspectives at the same time and compare them with the results of either earlier periods or other entities or groups.

The assessment of the value of the company's overall performance (i.e. the overall performance indicator or *OPI*) and the findings of the analysis depend on the selection and sequence of the initial indicators in the matrix model. Therefore, the paper pays special attention on ordering the initial indicators, what is significant for constructing the efficiency and effectiveness matrices.

The cognitive opportunities offered by index numbers are insufficiently used in practice to solve business problems. The authors believe that the use of *ISoPA* methodology in exploring the interrelations of the key performance indicators of business entities' is one of the most promising ways to improve the level of analysis.

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Appendix

Description of Matrices M-1 to M-13 as used in the Standard Methodology of *ISoPA*

M-1 Average values of the qualitative indicators in the basis period '0'

M-2 Average values of the qualitative indicators in the current period '1'

M-3 Composite indexes

Comment: As this matrix includes only one row, it is essentially a vector. The value of M-3 lies in the fact that it forms the basis for the construction of analytical component indexes in the following matrices. The composite indexes express the change in both qualitative and quantitative factors. There is a way to arithmetically verify the composite indexes through the relation $M-5_{ij} \times M-4_{ij} = M-3_j$. M-3 has a corresponding matrix, namely M-8 that expresses the same information in terms of absolute change.

M-4 Indexes of the quantitative factors

Comment: Each element in the matrix shows the relative influence of the quantitative factor on the respective resultant phenomenon. It shows how much the value of resultant indicator γ_j has changed compared to the basis period because of the change in the value of quantitative indicator α_i . Its corresponding matrix is M-9.

M-5 Indexes of the qualitative factors (constant composition)

Comment: Each element in the matrix shows the relative influence of the qualitative factor on the respective resultant phenomenon. It shows how much the value of the resultant indicator γ_j has changed compared to the basis period because of the change in the value of the qualitative factor β_j . Its corresponding matrix is M-10.

M-6 Indexes of variable composition (structure)

Comment: The matrix shows the relative change in average values of qualitative indicators. It shows how much a qualitative indicator β_{ij} has changed on average compared to the basis period. In the *ISoPA*, the M-6 contains indicators that are associated with the concept of economic efficiency and used to calculate the overall performance indicator (OPI). Its corresponding matrix of absolute change is M-11.

M-6a Indexes of the constant composition

Comment: The values of elements in M-6a are equal to the values of corresponding elements of M-5, although with different formula. Therefore, usually they are not calculated separately. These indexes show how much of the overall change in the values of respective qualitative indicators were caused by changes in individual values of the same components. Its corresponding matrix is M-12.

M-7 Indexes of the structural changes

Comment: This matrix shows how much of the overall change in the qualitative component β_j was caused by changes in the structure of the quantitative component α_i . Its corresponding matrix is M-13.

M-8 Absolute increases/decreases of the initial indicators

Comment: As this matrix includes only one row, it is essentially a vector. It shows the absolute changes in each of the resultant indicators compared to the basis period and is connected with the matrix M-3.

M-9 Influence of the changes in quantitative factors on the absolute changes in the value of the resultant quantitative indicators

Comment: This matrix shows how big was the absolute change in the value of the resultant quantitative indicator γ_j due to the change in the value of the quantitative indicator α_i . There is a linkage between matrices M-9 and M-4.

M-10 Influence of the change of qualitative factors on the absolute change in the value of the resultant quantitative indicators

Comment: This matrix shows how big was the absolute change in the value of the resultant quantitative indicator γ_j due to the change in the value of the qualitative indicator β_{ij} . There is a linkage between matrices M-10 and M-5.

M-11 Overall absolute increase/decrease of the average value of the qualitative indicators

Comment: There is a linkage between matrices M-11 and M-5.

M-12 Influence of individual absolute changes in the component parts of qualitative indicators on their average value

Comment: There is a linkage between matrices M-12 and M-5.

M-13 Influence of structural changes in the quantitative factors on the average values of qualitative indicators

Comment: There is a linkage between matrices M-13 and M-7.