

THE PREMISE OF SUCCESSFUL MATHEMATICS STUDIES

Zita Baužienė, Janina Morkūnienė

Kauno kolegija / University of Applied Sciences, Pramonės pr. 20, Kaunas, Lithuania

Abstract

In today's changing world, each person needs to acquire a variety of skills and abilities that will be useful to improve throughout life. It should be noted that in many cases the readiness of a novice student for mathematics studies is insufficient. The article presents results of the qualitative research on first-year students studying mathematics at Kauno kolegija/UAS. The analysed students' reflections are discussed, in which semantic findings are distinguished that allow to detect and determine significant factors of successful mathematics studies. The focus is on the development of students' mathematical literacy, mathematical reasoning, and mathematical competence; the impact of internal and external motivation, the importance of students' mutual cooperation, the importance of the teacher's ability to create an inclusive environment as a prerequisite for successful studies. The harmonious interaction of all these factors, can be considered as the premise of fruitful mathematics studies.

Keywords: mathematical reasoning, mathematical skills, mathematics learning problems

1. INTRODUCTION

In today's world, each individual needs to gain various skills and abilities that will be useful to develop throughout life. It should be noted that, in many cases, the readiness of a novice student for the study of mathematics is insufficient. This is presumed by the results of the TIMMS research (Šileikytė, 2020), which reveal that for Lithuanian students the most difficult areas are that require the application of mathematical thinking and mathematical knowledge. In order to achieve and ensure the quality of studies, it is important to know the level of readiness of novice students and identify superficial and deep obstacles in the study process that may hinder the implementation of the goals set in the mathematics subject study programme and achievement of high results.

Problem of the research – insufficient knowledge and understanding of mathematical concepts hinders the successful study and application of mathematical knowledge and mathematical thinking skills in solving practical tasks.

Object of the research – application of mathematical concepts and knowledge in solving practical tasks.

Aim of the research – to empirically substantiate the importance of knowing and understanding the concepts of mathematics in solving practical tasks.

Hypothesis – insufficient understanding of the concepts of mathematics is one of the most significant factors complicating the learning of mathematics.

Methodology. The research is based on humanistic philosophy, philosophy of pragmatism, and provisions of the constructivism philosophy. A qualitative study was applied to conduct the research: the qualitative content analysis of students' reflections. Qualitative content analysis was employed to analyse reflections provided by students, e.g. findings that describe students' mathematical abilities, e.g. ability to use mathematical concepts and to apply them in solving practical tasks; identifying the obstacles students face in completing certain tasks. The selected findings are grouped into semantic units (subcategories), which are combined according to semantic proximity and general categories are distinguished (Žydzūnaitė, Sabaliauskas, 2017).

2. MATHEMATICAL COMPETENCE

European Reference Framework (2018) outline eight general competences, including mathematical competence, that covers the ability to develop and apply mathematical thinking and insight in solving practical problems. It is noted that mathematical competence to some extent covers the ability and willingness to use methods of mathematical thinking and expression, namely: formulas, models, constructs, diagrams, schemes, etc. As stated in the reference framework, in the field of mathematics, respect for truth and the desire to look for reasons and assess their validity are particularly important.

Three main concepts are distinguished: mathematical reasoning; literacy and competence.

Table 1. Mathematical reasoning, literacy and competence according to R. Norvaiša (2020), E. Jablonka (2003), J. Dudaitė (2007)

Mathematical reasoning	Mathematical literacy	Mathematical competence
• Concepts are defined precisely; • Statements are formulated accurately, there are no ambiguities; • All statements are based on logical accurate reasoning; • Concepts are formed on the basis of available knowledge; • Mathematical knowledge is targeted at solving a specific problem.	Readiness to use mathematical knowledge in practice and beyond mathematics: 1) A sense of numbers; 2) Spatial thinking; 3) Ability to interpret; 4) Critical assessment of everyday situations and problem solving; 5) Dependence of the concept on the provisions of the socio-cultural field.	• Involvement in mathematical research; • Compilation and solution of mathematical problems; • Analysis and construction of non-mathematical context and situation models; • Evaluation and construction of the substantiation of a mathematical statement; • Analysis of different representations of a mathematical object; • Communication within, with and about mathematics; • Use of material tools in mathematical activities.

Mathematical reasoning (see Table 1) (Willingham, 2019) is disciplined thinking that encourages cognition and, at the same time, enables understanding. In other words, such reasoning is perceived as critical thinking which is based on original thinking, using the memorised experience of previous situations independently, ignoring instructions, in accordance with recognised agreements, that enable to achieve the desired thinking effect in solving the task.

When discussing the concept of mathematical literacy, there has been a noticeable variation in the interpretation among different countries. A significant point is that the concept of the term is not restricted by the field of mathematics science, but it depends directly on the country's internal culture and period of time. This leads to a change of the concept of mathematical literacy. Mathematics (Dudaitė, 2007) is generalisable, abstract, independent from context and society, apolitical, accurate and predictable, whereas mathematical literacy, on the contrary, is specific, belonging to a particular context, experiencing the influence of public consciousness, politically engaged, approximating and unpredictable.

Mathematical competence (Norvaiša, 2020) is an insightful preparation of an individual to act in a particular situation, which is related to mathematical challenges. It is associated with deep teaching of mathematics, based on mathematical reasoning. To sum up, it should be noted that teaching of

mathematics depends on the goals chosen and the attitudes of the curriculum developers. It can be assumed that if school mathematics programmes were based on the idea of mathematical reasoning, the notion of learning for all would be fulfilled as an objective reality, based on the essence of teaching/learning itself instead of being dependent and constrained by social, political, economic factors that affect members of a particular community in the country on a smaller or a larger scale.

3. OBSTACLES TO STUDY MATHEMATICS

The duality of the subject of mathematics is noticeable: as a science discipline at school, it can be oriented either to academic mathematics or application of mathematics. It is stated that academic mathematics-oriented content is available only to the most gifted students. Such an approach is considered as one of the fundamental problems. As R. Norvaiša (2020) states, teachers of mathematics confuse explanatory descriptions with logical definitions. Explanatory descriptions are not mathematical statements and they are not applicable to evidence; they are not actual logical definitions. It would be important for both pupils and students to know exact logical definitions.

Difficulties in learning mathematics are related to mathematical reasoning. As the concepts of mathematics are abstract, they can be roughly defined using real-world objects (Norvaiša, 2020), in order to understand the content of the meaning of the concept.

According to R. Norvaiša (2020), first-year students encounter challenges in mastering a higher mathematics course. If a secondary school course does not educate an individual to think abstractly, then a course of higher mathematics can be mastered by very gifted students with a lot of effort.

In terms of learning mathematics, N. Munawarah and E. Surya (2017) noticed, that “*Individuals who exhibit learning difficulties may not be intellectually impaired; rather, their learning problems may be the result of an inadequate design of instruction in curricular materials*” (p. 96) and identified the following difficulties:

- Difficult to understand a problem which requires knowledge of mathematics;
- Difficult to ask a question because they do not know what to ask and what actually unclear is;
- Difficult to reason;
- Lack of motivation in learning;
- Fear of involvement, unwillingness to show interest;
- Too complex tasks.

E. P. Kim and M. Olson (2016) note, that the quality of studies depends on the student’s motivation to learn and the methods of motivation encouragement. Possible factors that reinforce external and internal motivation:

- *external motivation* (good grades, praise, a better job in the future),
- *internal motivation* (student’s desire to acquire knowledge and competencies).

When analysing the peculiarities of mathematics teaching/learning, it is important to evaluate the characteristics of a particular generation. At present, the focus is on the representatives of Generation Z (Baužienė, Vosyliūtė, 2021), who are characterised by the following: individualism, activity, use of IT, the need for visual information in learning, and its effective perception in accepting and assimilating knowledge. Therefore, Generation Z is called digital generation. It is very important to consider the characteristics of this generation when organising the study process, in addition to the preparation of tasks, tools and methodology. This can be considered as a significant guideline for the quality of studies, enabling the student to achieve a personal qualitative leap. Taking into account of the aforementioned characteristics of young individual, i.e. student, can be named as the premise of fruitful studies.

R. Novikienė (2015) identifies the factors that determine the insufficient students' readiness to study mathematics at higher educational institution:

- limited comprehension of the mathematical text, as the final examinations are not focused on it;
- inadequate methodological toolbox of a general education school teacher, enabling them to fully educate pupils of Generation Z;
- pupils of different abilities, belonging to the Generation Z in particular, who exhibit difficulties in summarising, systemising and revealing causal connections.

In order to meet high standards of the study quality, it is necessary to select appropriate teaching methods, as they directly affect and determine the development of personality, manifested in the competencies to be developed (Mačėnaitė, 2018):

- openness and ability to cooperate;
- analysis of one's activity and realisation of one's potential available;
- self-confidence based on self-esteem;
- preparation for professional activity.

To sum up, it can be noted that a negative and intimidating attitude towards mathematics which prevails in the society, that this subject is for talented learners only, forms a preconceived notion which halts and hinders even an attempt to get involved in learning mathematics, and therefore limits the opportunities to reach the heights of academic mathematics studies realisation. In addition, successful studies of mathematics are hindered by the fact that the general education school focuses on mathematical literacy, ignoring mathematical reasoning and competence. Having analysed the scholarly literature, it was found that the following factors have a significant impact on the learning of the subject of mathematics: the level of readiness to study mathematics; learning environment, a secure and collaborative atmosphere; a sense of internal and external motivation; the attitude of the socio-cultural field, the cultural attitude towards the science of mathematics, the meaning and the need of the subject itself for the society, personal life of the individual.

4. RESULTS OF THE RESEARCH

From December 2020 to March 2021, a qualitative research was carried out aiming to analyse students' reflections presented in the course of solving practical tasks, the solution of which requires the application of knowledge and appropriate skills in mathematics. In the reflections, students described personal experiences in solving practical tasks of mathematics. During the research, the findings of the reflections were selected that revealed how students managed to do practical tasks, using data, justifying conclusions obtained, as well as the usefulness and value of the learning experience.

135 first-year students majoring in social sciences at Kauno kolegija participated in the research. The data obtained from the research were depersonalized. The research of students' reflections focused on the findings that justify the students' experience of studying mathematics, that presupposes the level of readiness to study, development of skills and the problems of the study process itself. In addition, the research delves into the findings that allow to detect and identify the socio-cultural field's attitude towards mathematics subject.

Table 2. Significant factors for students' learning of mathematics

Category	Subcategory	Illustration
Level of the student's readiness for the study of the subject of mathematics	Difficulties	<...> <i>the beginning was the most difficult</i> <...>; <...> <i>did not know how to ask</i> <...>; <...> <i>I did not get along with mathematics</i> <...>.
	Preliminary approach	<...> <i>mathematics is not "my friend"</i> <...>; <...> <i>there are no mathematicians in my family</i> <...>; <...> <i>I believe that not everybody can learn mathematics</i> <...>; <...> <i>I don't like mathematics</i> <...>.
Motivation	Internal motivation	<...> <i>"I clenched my teeth", but I put a lot of efforts and I managed to</i> <...>; <...> <i>it's important to overcome fears</i> <...>; <...> <i>strive to get a satisfactory assessment</i> <...>.
	External motivation	<...> <i>not to disappoint parents</i> <...>; <...> <i>I want to get a scholarship</i> <...>; <...> <i>I seek a state-funded place</i> <...>.
Cooperation in the study process	Students' cooperation with each other	<...> <i>we shared the task in parts</i> <...>; <...> <i>having completed certain parts of the independent study project, we joined an online meeting and discussed</i> <...>.
	Student and teacher cooperation	<...> <i>it was awkward with a new teacher in the beginning</i> <...>; <...> <i>I am afraid to ask not to appear stupid</i> <...>; <...> <i>teacher makes jokes</i> <...>; <...> <i>(teacher) tells what's wrong but does not assess immediately</i> <...>.
Learning environment	Organisation of the study process	<...> <i>the studies started not with math, so we managed to get used to new environment</i> <...>.
	Student-friendly atmosphere	<...> <i>friendly atmosphere; teachers encouraged that we would manage to complete the mathematics course</i> <...>; <...> <i>made friends while doing tasks in groups</i> <...>.

The analysis of the findings revealed (see Table2), that the difficulties experienced by students in studying mathematics at higher school occur on the very first days of studies. This is evidenced by the following statements of students: *"I don't know what will be next, whether I will overcome"*, *"the most difficult is the beginning"*. The fragments of the findings reveal the fear caused by the uncertainty and /or by distrust of one's knowledge of mathematics, the ability to apply them and even to fully participate in the study process in general: to communicate and cooperate with fellow students and teachers. Furthermore, it is possible to notice the effect of the sociocultural field on the students' preconceived negative and intimidating attitude towards mathematics and their personal opportunities to study successfully.

It was difficult for some students to learn mathematics at secondary school, so the negative experience raised preliminary internal doubts about their ability to study mathematics at higher education institution and achieve intended learning outcomes as provided in the study programmes. This causes personal tension. This is evidenced by the following findings: *"I am not a gifted student in mathematics"*; *"not everybody can be good at mathematics"*, *"no one in our family succeeded in math"*.

It should be noted that learning of mathematics is encouraged by the students' desire to get a satisfactory grade, as it has a direct impact on the continuation of studies, as well as the opportunity to maintain or obtain a state-funded place at higher school. Students are also obliged to learn by the desire to get a scholarship, to justify expectations and internal ambitions, etc. One of the factors that determines success is mutual cooperation between students and the teacher, who not only performs his/her direct

pedagogical work, but also can build interpersonal relationships, enhance students' self-confidence, motivate, and engage learners, creating a positive and supportive learning atmosphere, in which all the participants of the study process are equally important. The findings of students' reflections reveal that the importance of learning environment is unquestionable. It must be based not on the fear of asking or making a mistake, but, on the contrary, on an interpersonal relationship that creates a sense of trust in each other, a positive attitude and engagement of all the study process participants, as it fosters each individual's self-esteem. Students' reflections are justified as follows: *"teacher makes jokes and encourages, and I am braver to ask"*, *"After the first conversation with the teacher, I realised that I could ask and he would help me find mistakes and would not rush to assess"*. In one of the excerpts, it becomes clear that the feeling of fear and tension arises when the student asks and risks being assessed, thus it is important to ensure the opportunity of interpretation without assessment, as well as the opportunity to correct mistakes. This creates a safe learning environment that encourages which, in turn, is very important and achievable trait for a developed personality.

Summarising the discussion on significant factors that determine students' learning experience of mathematics, it should be noted that a successful study process is based on internal and external motivation, positive and fruitful communication between students, inclusive environment created by the teacher. All this changes the student's negative preconceptions about the subject of mathematics, denies the cautious and intimidating attitude prevailing in the socio-cultural field that mathematics is for exceptionally gifted students only, and enables to achieve personal results significant for the student's growth, development and preparation for the future profession.

Table 3. Experiences of learning mathematics

Category	Subcategory	Illustration
Experience in preparing self-study assignments	Positive experiences	<i><...>I learned to work in group <...>;<...> good achievements gave me confidence that I could also learn math <...>; <...> while preparing a self-study project, once faced with ambiguity, it was necessary to consult with the teacher, I had to learn and dare to ask<...>; <...> it's hard to understand what you don't understand and formulate a question to be understood <...> but I managed to <...>;</i>
	Negative experiences	<i><...> the group mates were indifferent, so I had to take some actions <...>; <...> I was scared that I did not understand anything and I was afraid to ask both group mates and teachers <...>.</i>
Value of the experience acquired	Positive experiences	<i><...> I found out with my group-mates, what (...) is <...>; <...> I asked the teacher and it seemed so simple <...>; <...> I failed at school (...) I missed a lot, but now I read on the internet and I understand, because I need <...>;</i>
	Negative experiences	<i><...> search for particular mathematics (...) takes a lot of time, as there is much information online <...>; <...> it is time-consuming and requires a lot of efforts in order to get a good result <...>.</i>

The analysis of students' reflections revealed (see Table 3) that students lack certain abilities that are important in performing practical tasks of the independent study project. The findings show that a frequent student is characterised by mathematical literacy: the ability to find necessary data and select appropriate formulas, but problems occur while analysing, interpreting and drawing conclusions. *"It is hard not to count, but to compare <... >to write, <...> it was the most difficult"*; *"how to describe, how*

and how much is it needed”, “I need an example how to describe and compare data”, – students’ statements presuppose that the complexity of analysis, interpretation, generalisation of available data and preparation of conclusions is determined by failure to understand the meaning of concepts and absence of knowledge in the exact meaning of their content. This suggests that at general education school, concepts are taught by explanations or by memorising the wording of the meaning of the content, not by analysing the exact meanings. Therefore, pupils, prospective students, who fail to understand the mathematics concepts at secondary school, later, already studying at higher education institution, exhibit learning difficulties that are determined by previous poor learning experiences and knowledge gaps. As the result, in many cases, the teacher of higher school has ‘to go back’ and explain what students should already know when starting their studies. While studying mathematics, students, by their own efforts, make a certain qualitative leap and manage to master the subject. They manage to understand what seemed to be completely unclear at the beginning of their studies. This is evidenced by the following findings of students’ reflections: “I worked hard and I figured out my gaps in knowledge....., it turns out that I was able to overcome the mathematics tasks given to me”. It should be noted, that the research revealed the most difficult topics for students are differential and integral calculus, which require not only performing calculations using specific formulas, but also understanding of essential concepts used in mathematics subject, knowing of their meanings and ability to apply them in practice. These insights can be based on the following reflections: “while applying formulas I find a derivative or an integral, but I cannot explain what is the meaning “; “where are derivatives applied in practice”.

Students that exhibit mathematical reasoning skills in doing practical tasks of the independent study project, use the methods of mathematical and statistical analysis and interpret the obtained data “Once I read the task, I understood what formulas I will apply and what obtained calculations will reveal, then I will compare the results... and this will be the answer to the question of the task”, “I explained to my friends that it is not possible to calculate the average of relative sizes”, “I said my group-mates what formulas we would apply and explained why”. The fragments of students’ reflections show that those students who understand the concepts and are able to apply them in practice, not only succeed in completing the tasks, but also share their knowledge with other group-mates. In this way, a good learning practice of mathematics is created and developed. It is important to emphasise that these experiences are shared by students of Generation Z, which is characterised by aspiration to achieve results as fast as possible. Performance of practical content tasks of mathematics requires not only knowledge and skills, but also attentive, meticulous and time-consuming work. This suggests that available competence of mathematical thinking enables different individuals to achieve intended learning outcomes.

CONCLUSIONS

The scientific literature proves the importance and necessity of knowledge and skills in mathematics, and, at the same time, the complexity of mastering experienced by the young Generation Z. Mathematical literacy, mathematical reasoning and mathematical competence are three significant concepts that allow to describe and define an individual’s mathematical competence.

The research conducted at Kauno kolegija, in which first-year students of social sciences participated, empirically substantiates that the representatives of Generation Z, in the beginning of their studies, have a preconceived intimidating attitude towards mathematics. The analysed reflections of students revealed that such an approach is often formed by the general attitude of the socio-cultural field that mathematics is an area of theoretical and practical work for exceptionally gifted people. Therefore, students feel they are unable to achieve intended learning outcomes. However, the reflections, prepared while discussing the experience of performing self-study assignments, demonstrate that a proper organisation of the study process and an inclusive environment based on a sense of security enables the students to achieve personal and general goals, to implement requirements of the study programme; also develops a personal self-esteem, significant both in the further process of studies and in the future professional career. As mentioned in the scientific works used in this article, in many cases, students are characterised only by mathematical literacy, as the competence of mathematical reasoning is not acquired or sufficiently developed. It opens up the opportunity for developers of the mathematics curricula in general education schools to focus on the ability to know concepts in the abstract way. On the contrary, the teaching of

mathematics should be focused on the development of mathematical reasoning and mathematical literacy.

The research confirms the hypothesis, that insufficient understanding of the concepts of mathematics is one of the most significant factors complicating the learning of mathematics. The most important concern both of secondary school and higher school teacher community should be a proper organisation of the teaching process, preparation of curriculum and teaching tools, methodologies as a guideline to achieve successful and fruitful teaching of mathematics. Taking into consideration students' abilities, characteristics, learning modes and needs is considered as a premise of successful studies.

REFERENCES

1. Baužienė Z., Vosyliūtė A. (2021). Students' Abilities in Search, Systematization and Application of new Information for Independent Study Assignments. // *Proceedings of IAC 2021 in Budapest*. 68-76. Access online: <https://www.conferences-scientific.cz/file/9788088203216> (retrieved 21-03-2021).
2. Dudaitė J. (2007). Matematinio raštingumo samprata. // *Acta Paedagogica Vilnensia*, 18 t., 170–187. Access online: <https://www.zurnalai.vu.lt/acta-paedagogica-vilnensia/article/view/9667/7578> (retrieved 04-03-2021).
3. Europos Sąjungos Taryba, (2018). Tarybos rekomendacija 2018 m. gegužės 22 d. dėl bendrųjų mokymosi visą gyvenimą gebėjimų. Access online [https://eur-lex.europa.eu/legal-content/LT/TXT/HTML/?uri=CELEX:32018H0604\(01\)&from=SK](https://eur-lex.europa.eu/legal-content/LT/TXT/HTML/?uri=CELEX:32018H0604(01)&from=SK) (retrieved 10-12-2020).
4. Kim, E. P., Olson, M., (2016). Exemplary Chinese University Professors: Qualities and Impact on Students. Access online: <https://doi.org/10.22492/ije.4.1.07> (retrieved 20-05-2020).
5. Jablonka E. (2003). Mathematical literacy. In A. J. Bishop et al.(Ed.), *Second International Handbook of Mathematics Education*, 75–102. Dordrecht: Kluwer Academic Publishers.
6. Mačėnaitė L. (2018). Atvejo analizės metodo taikymas matematikos dėstyme. // *Lietuvos matematikos rinkinys. Lietuvos matematikų darbai, ser. B* 59 t., 40-45. Access online: <https://www.zurnalai.vu.lt/LMR/article/view/14984/14003> (retrieved 20-01-2021).
7. Munawarah N., Suryab E. (2017). An Analysis of the Difficulties in Learning Mathematics by Using Scientific Approach at SMA Negeri 3 Manyak Payed. // *International Journal of Sciences: Basic and Applied Research (IJSBAR)* Volume 33, No 3, 94-104. Access online: <https://core.ac.uk/download/pdf/249335831.pdf> (retrieved: 21-03-2021).
8. Norvaiša R. (2020). Matematikos mokymosi tikslai: matematinis kompetentingumas ar matematinis raštingumas? // *Lietuvos matematikos rinkinys. Lietuvos matematikų darbai, ser. B* 61 t., 8-14. Access online: <https://www.journals.vu.lt/LMR/article/download/22472/22180/> (retrieved 10-02-2021).
9. Novikienė R. (2015). Ž kartos vaikų matematikos mokymo(si) organizavimo problemos. // *Lietuvos matematikos rinkinys. Lietuvos matematikų darbai, ser. B* 56 t., 85-89. Access online: <https://www.zurnalai.vu.lt/LMR/article/view/17717/16884> (retrieved 20-01-2021).
10. Sičiūnienė V., Grakauskaitė-Karkocienė D., Strolienė E. (2016). Divergentinio mąstymo ugdymas per matematikos pamokas. // *Mokytojų ugdymas. Nr. 26 (I)*, 77-93. Access online: <https://etalpykla.lituanistikadb.lt/object/LT-LDB-0001:J.04~2016~1507101433090/J.04~2016~1507101433090.pdf>, (retrieved: 21-02-2021).
11. Willingham D. T. (2019). How to Teach Critical Thinking. NSW Department of Education, Australia. Access online: http://www.danielwillingham.com/uploads/5/0/0/7/5007325/willingham_2019_nsw_critical_thinking2.pdf (retrieved: 12-02-2021).

12. Šileikytė D. (2020). Kas paaikškėjo atlikus TIMSS tyrimą? // Švietimo naujienos. Prieiga internetu: <https://www.svietimonaujienos.lt/kas-paaiskejo-atlikus-timss-tyrima/> (retrieved 13-02-2021).
13. Žydžiūnaitė V., Sabaliauskas S., (2017). Kokybiniai tyrimai. Principai ir metodai. Vilnius: Vaga.