MAGNETOABSORPTION OF D^- CENTERS IN QUANTUM WELLS AND ELECTRON-PLASMON RESONANCE

Virginia Gurau

State University of Moldova, Department of Physics and Engineering,
60 Alexei Mateevici, 2009 Chisinau, Moldova

Abstract

This paper presents the calculation of far-infrared magneto-absorption spectra of D^- centres, consisting of neutral shallow donors that bind on additional electron in quantum well. The calculation is carried out under the conditions of electron-plasmon resonance when the distance between Landau levels is equal to the energy quantum of magnetic plasma oscillations. Throughout the process, resonant interaction of two-dimensional D^- centre with magnetic plasma is taken into consideration. It is shown that the relative height of the absorption peaks is dependent upon the resonance frequency detuning. It is also found that the theoretical absorption spectra, calculated with account of resonant electron-plasmon coupling, coincide with experimental data obtained in GaAs-$Ga_{0.75}Al_{0.25}As$ multiple quanta well (MQW) structures.

Keywords: *multiple quanta well, two-dimensional D^- centre, magneto-absorption spectra, electron-plasmon resonance, magnetic plasma oscillations*

1. INTRODUCTION

Coulomb many-body interactions of charge carriers play an important role in semiconductor optics. Besides energy spectrum renormalization and band gap shrinkage, they also cause transitions with the participation of several low-frequency optical plasmons [1-5]. The processes of their emission and absorption essentially may modify the dynamics of electrons and holes [1-5]. The collective interactions and multiplasmon optical transitions were investigated theoretically and experimentally [2-5]. Among these papers, the generalized semiconductor Bloch equation for semiconductor polarization is developed with account of memory coherent effects and multiplasmon processes of light absorption and emission [5].

Magnetic plasma differs in many respects from plasma in absence of a magnetic field and the effects of its interaction with an electromagnetic field have been an object of investigation for many years. In this paper, the spectrum of light absorption by impurity centre in a quantum well is calculated in the presence of a constant and homogeneous magnetic field. The interaction of the bound electron with magnetic plasma oscillations is considered.

The quantum state of the electron-plasmon system, in which there are no plasmon and electron, is on the first Landau level and another one in which electron is on the ground Landau level with one plasmon are attributed to just the same energy level of system. The interaction of these two degenerated states leads to the splitting of the first Landau level. Analogous splitting of the second and another excited Landau levels takes place. In turn, the splitting of Landau levels is the origin of the absorption lines splitting and fine-structure manifestation. The fluctuation-dissipation theorem is used to express the correlation function via a structure factor [6] for the evaluation of the correlation function density-density describing a spectrum of interacting elementary excitations.

2. MATERIALS AND METHODS

2.1. Materials

Magneto-absorption spectra are calculated for multiple quanta well structures with two-dimensional D^- centres consisting of neutral shallow donors that bind on additional electron in quantum well. The

absorption spectra of the impurity centre in a quantum well are calculated at the presence of a constant and homogeneous magnetic field. The interaction of the bound electron with magnetic plasma oscillations is considered. The theoretical absorption spectra of D^- centres is compared to experimental data obtained in $GaAs$-$Ga_{0.75}Al_{0.25}As$ multiple quanta well structures. Previously, the magneto-photo-conductivity (MPC) for selectively silicon-doped 10 *nm* well, 10 *nm* barrier $GaAs$-$Ga_{0.75}Al_{0.25}As$ multiple quanta well structures was measured with 150 periods [7]. The samples were doped with *Si* (10^{10} cm^{-2}) in the centre of each quantum well at 1 *nm* from its surface. Therefore, the calculated absorption spectra for multiple quanta well structures with two-dimensional D^- centres truly reflects the basic experimental behaviour of magneto-photo-conductivity [7].

2.2. Methods

According to the Kubo formula [8], the light absorption coefficient can be defined by the following expression:

$$\alpha(\omega)=\frac{4\pi e^2 N_c}{\omega\hbar cn(\omega)}(1-e^{-\frac{\hbar\omega}{k_0T}})\sum_n \rho_i \left|(\hat{p}_y)_{fi}\right|^2 \mathrm{Re}\int_0^\infty e^{i(\omega-\omega_{fi})t} I_{fi}(t)dt \quad (1)$$

Here e and m_0 are the electron charge and mass, k_0 is the Boltzmann constant, T is the lattice temperature, $\hbar\omega_g = E_g$ is the band gap energy, N is the centre's concentration, c is the light speed, and $n(\omega)$ is the refractive index. The light polarization vector is directed along the y-axis laying in the quantum well plane. The case of a constant and homogeneous magnetic field with the intensity H oriented along the z-axis of the size quantization that is normal to the interface is considered. Indexes i and f symbolize the quantum numbers of the optical electron. The wave functions of the system are presented in multiplicative form. The transition frequency ω_{if} and the initial state population probability ρ_i are found in the one-particle approach without considering the bound on a centre electron interaction with the lattice and magnetic plasma oscillations, which has the expression:

$$V(r)=\sum_q V_q \rho_q P_{-q}, \quad \rho_q = \exp(iqr), \quad (2)$$

where V_q is the Fourier transform of electron Coulomb interaction with a charge density P_{-q}, caused by lattice and magnetic plasma oscillations. The function I_{fi} describes the effects of Coulomb many-body interactions on the absorption spectrum; for the coupled electron-plasmon system, it is determined by the equation:

$$I_{fi}=\langle i(r_s)|\langle i(r_\sigma)|\langle f(r'_s)| < \exp\{-\frac{i}{\hbar}\int_0^t [V(r_s)-V(r_s')]ds - \int_0^\lambda V(r_\sigma)d\sigma\} > |i(r_s)\rangle|i(r_\sigma)\rangle|f(r'_s)\rangle \quad (3)$$

Here $\lambda = 1/k_0T$, $V(r_s)$ is the interaction (2) in Heisenberg representation. If $V(r)=0$ is obtained $I_{fi}=1$. Averaging <….> in the formula (3) is performed with account of quantum states of system without a considered bound electron and the Feynman ordering operators' rule is used [9]. The interaction V_σ (3) has been neglected in many semiconductor kinetic phenomena studies. One should also note that it is necessary to account the correlator $<V_\sigma V_s>$, because it significantly influences the time dependence of the generating function I_{fi} (3) calculated using the cumulant expansion method [10] and $\langle \exp A\rangle \approx \exp[\langle A\rangle + 0.5(\langle A^2\rangle - \langle A\rangle^2)]$. This approximation gives precise results in the linear harmonic approach. It is then necessary to obtain the density – density correlator to calculate function (3). The density – density correlator can be expressed in terms of the structural factor [6]; within a quasi-static approach, it is determined by the equation as follows:

$$\left\langle \mathrm{P}_q(s)\mathrm{P}_{-q}(s_1)\right\rangle = \frac{\hbar}{\pi v_q}\int_{-\infty}^{\infty}\exp\{-i\Omega(s-s_1)\}F_q(\Omega)d\Omega, F_q(\Omega) = (n(\Omega)+1)\,\mathrm{Im}\{\frac{\varepsilon_\infty q^2}{q_i\varepsilon_{ij}^*(q,\Omega)q_j}\} \quad (4)$$

Here $n(\Omega) = \{\exp(\lambda\hbar\omega)-1\}^{-1}$, ε_∞ is the high-frequency permittivity and ε_{ij} is the permittivity tensor. The quantum averaging (3) is calculated using the first cumulant approach. As a result, for the generating function, the following expression is obtained:

$$I_{fi}(t) = \exp\{-\frac{1}{\pi\hbar}\sum_q v_q\int_{-\infty}^{\infty}F_q(\Omega)[\sum_n|\rho_{fn}(q)|^2\Phi_t(\Omega-\tilde{\omega}_{fn})+\sum_n|\rho_{in}(q)|^2\Phi^*{}_t(\Omega-\tilde{\omega}_{in})+ \quad (5)$$

$$+\sum_n|\rho_{in}(q)|^2\Phi_{i\hbar\lambda}(\Omega-\tilde{\omega}_{in})-2\rho_{ii}(q)\rho^*{}_{ff}(q)\,\mathrm{Re}\,\Phi_t(\Omega)+2i(\ \rho_{ii}(q)-\rho_{ff}(q)\ )\rho^*{}_{ii}(q)\frac{\sin\Omega t}{\Omega^2}]d\Omega\}.$$

Here,

$$\Phi_t(\omega) = \{1-i\omega t-\exp(-i\omega t)\}/\omega^2. \quad (6)$$

Formulas (1-5) determine the absorption coefficient of light by the impurities with account of multi-quantum transitions. Elementary excitations frequencies with which the bound electrons interact are determined by zeros of permittivity tensor of considered system in magnetic field. These general results are used for the analysis of the frequency dependence of two-dimensional D- centres absorption coefficient.

The linear part of function Φ regarding parameters t and λ determine contributions to the energy of optical electron bound on D- centre caused by its interaction with lattice vibrations (polaron effect) and plasma. Interaction with magnetic plasma and lattice vibrations lowers the energy levels of the bound electrons. According to experimental data [7], the binding energy of the electron with D- centre increases with the increase of magnetic field approximately under the square root dependence. The theoretical results obtained in papers [11-12] for binding energy $E_b = \hbar\omega_b$ without taking into account interaction (2), agree qualitatively with the experimental dependence of E_b on H, though obtained quantities E_b are a little lower than experimental values. The quantitative agreement of the theory with the experiment can be achieved not only by improving variation functions [11], but also by considering the contribution in E_b from the interaction with lattice vibrations and plasma.

The analysis of experimental data [7] carried out in theoretical work [11] has shown that lines shape of absorption spectra has Lorentz form, as follows:

$$L(\omega-\tilde{\omega}_{if}) = \frac{\Delta_{if}/2\hbar}{(\omega-\tilde{\omega}_{if})^2+(\Delta_{if}/2\hbar)^2}. \quad (7)$$

Here $\hbar\tilde{\omega}_{if} = \tilde{E}_i - \tilde{E}_f$ is the transition energy found with account of centre interaction with lattice vibrations and plasma. The absorption line half-width depends on imaginary part of permittivity tensor and has a formula:

$$\Delta_{if} = \sum_q v_q\left\{\sum_n|\rho_{fn}(q)|^2F_q(\omega_{fn})+\sum_n|\rho_{in}(q)|^2F_q(\omega_{in})-2\rho_{ff}(q)\rho_{ii}(-q)F_q(0)\right\}. \quad (8)$$

The absorption coefficient $\alpha(\omega)$ frequency dependence is determined first by energy spectrum of D^- centre. However, the spectrum of the magnetic plasma and crystal elementary excitations is also important. In case of a quantum well, when the magnetic field is directed perpendicularly at its interface, according to known results [6] for two-dimensional plasma the following expression is obtained:

$$\frac{\varepsilon_0 q^2}{q_i \varepsilon_{ij}^{*}(q,\Omega) q_j} = \frac{\Omega^2 - \omega_c^2}{\Omega^2 - \omega_c^2 - \omega_p^2}, \omega_c = \frac{eH}{mc}, \omega_p^2 = \frac{4\pi n e^2}{\varepsilon_0 m} . \quad (9)$$

This expression has a pole in point $\Omega = \tilde{\omega}_p = (\omega_c^2 + \omega_p^2)^{1/2}$, which determines the frequency of a so-called upper hybrid mode $\tilde{\omega}_p$. At low temperatures, only the spontaneous emission of magnetic plasma quanta is essential.

3. RESULTS

Absorption spectra generated by formulas (1-9), as was previously shown [13], truly reflects the basic experimental behaviour of magneto-photo-conductivity (MPC) [7], represented in figure 1. Authors of paper [7] have measured MPC for selectively silicon-doped 10 nm well, 10 nm barrier *GaAs-Ga* $_{0.75}$ *Al* $_{0.25}$ *As* MQW structures with 150 periods. A series of peaks quasi-periodic with the magnetic field was observed for D^- centres and neutral donors (peak *A*) at the laser energy $E = 17.58$ *meV*. Peak *B* is due to the photo-ionization transition from D^- state to the N=1 Landau level. The harmonic structure at a lower magnetic field is due to the transitions to N=2, 3, 4 levels. Peak *B* dominates in the spectrum. These experimental data agree with the theory (1-9).

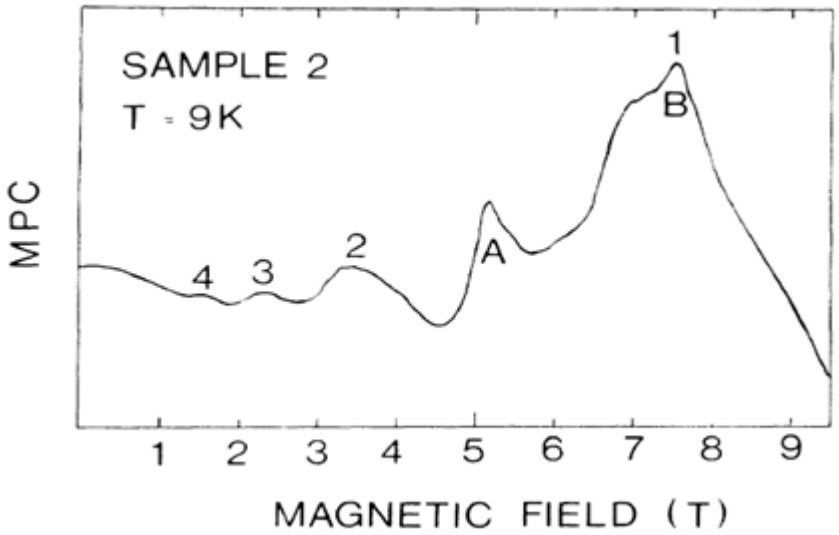


Fig. 1. Magneto-photo-conductivity (MPC) of centres D^- at the transition on 1, 2, 3, 4 Landau levels for sample 2 (arbitrary units), laser energy $\hbar\omega = 17{,}58$ *meV* [7].

However, some experimental features are not reflected by the theory given in [13]. There is a low field shoulder on the experimental peak B, and a high field one on the peak D- $\rightarrow$ N = 2. In paper [7] the first shoulder is assigned to the contribution of near interface donors. However, peaks showing the ionization transitions D- $\rightarrow$ N = 0 and D- $\rightarrow$ N = 2 also should have low field shoulders in that case. According to experimental data [7], peaks MPC D- $\rightarrow$ N = 0 at various values of laser energy $\hbar\omega$ have the symmetric shape without a hint on any structure, whereas there is a high field shoulder on the peak D- $\rightarrow$ N = 2.

I argue that shoulders on peaks are caused by resonant interaction of D- centre with magnetic plasma oscillations. The analogous phenomena (Larsen-Johnson effect) also occur in the resonance interaction of electron and phonon subsystems [14-18]. If cyclotron frequency is near to the frequency of longitudinal optical phonons, a split of Landau levels occurs. Polaron pinnings were observed in

measurements of an interband magneto-absorption [14] and cyclotron - phonon and cyclotron resonance [15-17].

Multi-plasmon transitions are considered in theory (1-9), but it is impossible to explain shoulders in MPC spectra [7] by emission of real quanta $\hbar\widetilde{\omega}_p$. To explain experimental results, I propose considering electron-plasmon system quantum states $|n,N\rangle$, where *n* is the number of plasmons, and *N* is the quantum number of Landau level. At low concentrations of plasma $(\widetilde{\omega}_p \cong \omega_c)$ the state $|0,1\rangle$, in which there are no plasmon, an electron is on the first Landau level and a state $|1,0\rangle$, in which an electron is on zero Landau level plus quantum $\hbar\widetilde{\omega}_p$ have the close values of energy $E_1 \cong E_0 + \hbar\widetilde{\omega}_p$. Under this resonant condition the usual perturbation theory [19] is not applied. Using the superposition of these two degenerated states [19] one can easily include the interaction (2) into the consideration. As a result, the energy spectrum of electron-plasmon system in the region of first Landau level consists of two levels, whose values are determined by the following formulas:

$$\begin{aligned} &\widetilde{E}_1 \cong E_1 + \frac{1}{2}\hbar(\delta \pm \widetilde{\Omega}),\ E_1 = E_0 + \hbar\omega_c, \\ &\delta = \widetilde{\omega}_p - \omega_c,\ \widetilde{\Omega} = \left\{\delta^2 + G_{10}^2\right\}^{1/2},\ G_{jn}^2 = \frac{2\omega_p^2}{\hbar\widetilde{\omega}_p}\sum_q v_q \left|\rho_{jn}(q)\right|^2 \end{aligned} \tag{10}$$

Magneto-plasma analogue of Rabi frequency $\widetilde{\Omega}$ is dependent on the value of resonance detuning $\delta = \widetilde{\omega}_p - \omega_c$ and resonant interaction of zero and first Landau levels (constant *G*). With account of multi-quantum transitions, the expression for absorption coefficient of D^- centre at low temperatures is found as follows:

$$\begin{aligned} &\alpha(\omega,H) = \alpha_0 \frac{\omega_p}{\omega}\left(1 - e^{-\frac{\hbar\omega}{k_0 T}}\right)\frac{\gamma^{5/2}}{(1+\gamma)(1+\gamma+\gamma^2)} \\ &\{L(\omega-\omega_b) + (I_1 b + N_0 d)L(\omega-\omega_b-\omega_c+0.5(\widetilde{\Omega}-\delta)) + (I_1 d + N_0 b)L(\omega-\omega_b-\omega_c-0.5(\widetilde{\Omega}+\delta)) + \ldots\} \end{aligned} \tag{11}$$

The values of peaks height are determined by the following coefficients:

$$\begin{aligned} &I_1 = \frac{4}{\gamma} + \frac{3}{2}\left(\frac{1-\gamma+\gamma^2}{1+\gamma+\gamma^2} - \frac{1-\gamma}{1+\gamma}\right), \gamma = \frac{c\hbar}{eHa^2}, \\ &N_0 = \sqrt{\frac{\pi}{8}}\frac{e^2}{\varepsilon_0 a\hbar\widetilde{\omega}_p}\left(\frac{\omega_p}{\widetilde{\omega}_p}\right)^2, b = \frac{\widetilde{\Omega}+\delta}{2\Omega}, d = \frac{\widetilde{\Omega}-\delta}{2\Omega} \end{aligned} \tag{12}$$

Simple Gaussian variational wave function for bound electron was used to derive formulas (12). At performance of inequalities $N_0 < 1$, $I_1 > 1$ the relative intensity of two absorption peaks is determined by the ratio *b/d*. It is always more than unity ($b/d > 1$) since $\delta > 0$. Therefore, the high magnetic field peak dominates according to experimental spectrum MPC, represented on figure 1. In the region of the second Landau level, there is an interaction of four levels.

Figure 2 shows the result of the numerical calculation of the absorption spectrum dependence on the magnetic field with account of first and second Landau levels splitting. The interaction of second Landau level, *N*=2, with energy level $\widetilde{E}_1 \cong E_1 + \frac{1}{2}\hbar(\delta - \widetilde{\Omega})$ was considered. For these levels, the value of resonance detuning $\delta_1 = 1.5\delta - 0.5\widetilde{\Omega}$ is less than the value of δ.

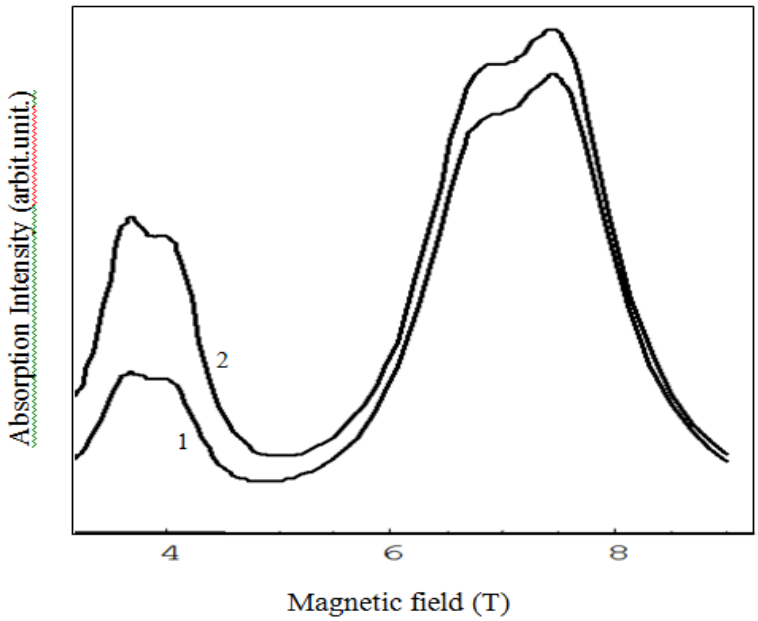


Fig. 2. The dependence of absorption coefficient of D^- centre on the magnetic field, taking into account the interaction of the first and the second Landau levels with the upper hybrid mode of the magnetic plasma oscillations, calculated at the photon energy $\hbar\omega$=17.58 *meV*, $\hbar\omega_p$=2 *meV*, a = 10 *nm*, $\hbar G_{10}$= $\hbar G_{21}$ = 1.64 *meV* and using the experimental data of the binding energy [7]. Line 1 corresponds to N_0 = 0.01, line 2 corresponds to N_0= 0.5.

Hence the value of δ_1 can be negative. It results that the shoulder on peak $D^- \rightarrow N = 2$ appears located on the region of the strong fields according to previous experimental data [7].

4. DISCUSSION

Theoretical magneto-absorption spectra of D^- centres calculated with account of resonant electron-plasmon coupling and multi-quantum transitions truly reflects the basic experimental behaviour of magneto-photo-conductivity obtained previously [7]. Authors observed for D^- centres a series of peaks quasi-periodic with the magnetic field, a dominant peak due to the photo-ionization transition from a D^- state to the N=1 Landau level and a harmonic structure at a lower magnetic field due to the transitions to the N=2, 3, 4 Landau levels. These experimental data agree with the developed theory of resonant electron-plasmon coupling and multi-quantum transitions given by formulas (1-9). However, some experimental features obtained previously [7], are not reflected in the theory. There is a low field shoulder on the experimental dominant peak due to the photo-ionization transition from a D^- state to the N=1 Landau level and a high field one on the peak $D^- \rightarrow N = 2$. In paper [7] the first shoulder is assigned to the contribution of near interface donors. However, peaks showing the ionization transitions $D^- \rightarrow N = 0$ and $D^- \rightarrow N = 2$ also should have low field shoulders in that case. According to experimental data [7], peaks MPC $D^- \rightarrow N = 0$ at various values of laser energy have the symmetric shape without a hint on any structure, whereas there is a high field shoulder on the peak $D^- \rightarrow N = 2$. I argue that shoulders on peaks are caused by the resonant interaction of D^- centre with magnetic plasma oscillations. It is impossible to explain shoulders in MPC spectra [7] by emission of real plasma quanta. To explain experimental results, I found the energy spectrum of electron-plasmon system in the region of the first Landau level in the conditions of electron-plasmon resonance when the distance between Landau levels is equal to the energy quantum of magnetic plasma oscillations and expression for absorption coefficient of D^- centre at low temperatures with account of multi-quantum transitions. The numerical calculation of dependence of an absorption spectrum on a magnetic field with account of splitting of first and second

Landau levels showed that the shoulder on peak $D^- \rightarrow N = 2$ appears located on the region of the strong fields according to experimental data [7].

5. CONCLUSIONS

In this paper, it was found that the theory of resonant electron-plasmon coupling and multi-quantum transitions in the case of resonant interaction of D^- centre in quantum well with magnetic plasma oscillations is in accordance with experimental data for behaviour of magneto-photo-conductivity obtained in multiple quantum well structures.

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