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APPLICABILITY OF LINEAR PROGRAMMING IN MANAGING SUPPLIES FOR LARGE CONSTRUCTION PROJECTS

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Abstract

Construction projects are highly varied. Next to some small buildings of general use, there are large construction projects in civil engineering or industrial construction, road construction projects underway including all accompanying infrastructure. In order to complete construction projects, it is necessary to supply required materials from which the constructions are raised. Proper planning of the supplies enables contractors to carry out work according to previously approved deadlines. When roads are built, the distance between suppliers and a site where given materials will be used change as the work progresses. It is not an uncommon development that the supplier used at the beginning turns to be located too far away as the work continues. In order to plan the supplies well, it is advisable to use mathematical methods which support the decision-making process. This article suggests using linear programming, especially the transport problem, which enable one to find a solution easily.

Key words: *management in the construction industry, managing a construction project, supporting decision-making, supplies of construction materials*

1. INTRODUCTION

Being a component of the market economy, the construction industry is governed by certain general rules. However, it has many specific characteristics, for instance it has a highly varied structure. Next to some small buildings of general use, there are also large building projects in civil engineering or industrial construction. Whole housing estates are built alongside typical facilities. Large industrial compounds are raised as well. Recently, in Poland, there have been many road building projects including all necessary infrastructure. Building projects require supplies of adequate amounts of building materials, which will be used for raising planned constructions. Good planning of the supplies of materials enables contractors to complete work according to previously established deadlines. When buildings are raised on a specific site, finding suppliers is relatively simple and depends on prices as well as a distance to the site. However, when construction work stretches over many kilometers, distances between suppliers and a site where their materials will be used change as the work progresses. It is not an uncommon development that the supplier whose services we use at the beginning of the project proves to be located too far in respect of the subsequent site of the construction work (fig. 1). Once we add to this situation the fact that, as the work progresses, time passes and prices of building materials change, we may discover that finding good suppliers could prove to be quite a difficult task.

2. REVIEW OF REFERENCES

In a situation so complicated from the point of view of managers, the relevant literature suggests several methods supporting the decision-making process, methods which aid both construction

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companies and contractors. Despite its specific character, a construction company is governed by the same rules as companies which belong to other branches of economy. As in other firms, building companies need early warning systems (Kapliński 2007) and methods for evaluation of a financial situation and risk. There are many different methods suggested, for example: a discriminant analysis (Kapliński 2008), neural networks (Baetge, Krause. 1993); and network methods (Szafranko 2008), multifactor modeling (Zavadskas et al. 2008). It is also important to make proper decisions regarding the execution of the construction process itself. Some of the aforementioned methods are applicable here as well, for example when choosing an alternative design (Nowak 2005) selecting contractors and analyzing tender bid documentation (Plebankiewicz 2009), (Abidali, Haris 1995), and cost estimates of designs (Zavadskas et al. 2004). Optimalisation of decision-making processes is often attained with an aid of simple mathematical methods (Sakalauskas, Zavadskas 2009), e.g. linear programming (Koźniewski, Orłowski 2005).

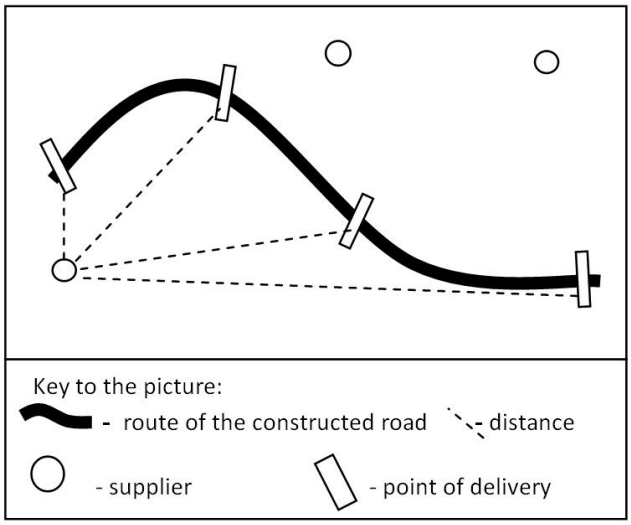


fig. 1. Diagram of the set of supplies changing as the work progresses (the author's own elaboration)

Application of mathematical methods in management and decision-making processes is often named operational research. Operational research is defined as scientific methods for solving problems in executive decision-making (Figueira et al. 2005; Lin et al. 2008; Ustinovichius et al. 2007). The range of applicability of operational research encompasses:

- making mathematical, economical and statistical descriptions or models of decisions in situations characterised by a large complex uncertainty,
- analysis of dependences which determine likely consequences of the choice of decisions and formulation of appropriate effectiveness measures in order to assess the results.

The course of actions in operational research (Wagner 1976; Cheng, Lin 2006;) can be divided into stages:

1. formulating the problem and the aim of the activity,
2. building and testing a model,
3. determining values of the model's parameters,

4. solving the pre-defined model,
5. interpreting the results,
6. implementing the results.

The first steps are very important, i.e. formulating the problem and determining numerical values of the analyzed phenomenon. Formulating the problem and the aim of the activity relies on becoming aware that there is a certain problem, recognising its constituents and interrelations between them as well as conditions and available means for an action to be undertaken. Another significant element is the choice of a criterion for optimalisation. A good choice of this criterion determines the value and usability of the subsequent results. In order to solve the model, it is necessary to know specific numerical values of the parameters. Although it is sometimes very difficult to determine the values of the parameters, it has to be done for the sake of solving the problem. Likewise, it is often necessary to correct the assumptions as the process of solving the problem progresses (fig. 2), so as to achieve a logical reply.

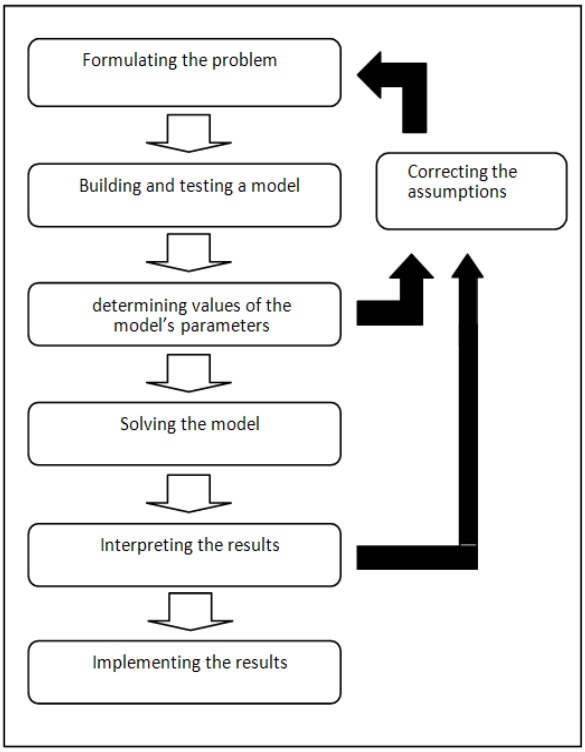


Fig. 2. Diagram of the flow of operational actions (the author's own elaboration)

3. MATHEMATICAL MODEL OF THE PROBLEM

Among the linear programming methods, there is transport problem. It is most often applied to derive the optimum planning of transport of uniform material between a certain number of suppliers D_i (who have available supplies in the amount of a_i) and customers O_j (who order supplies in the amount of b_j). Optimalisation can be applied to the total cost of transport (the most frequent case), time or distances covered while completing the orders.

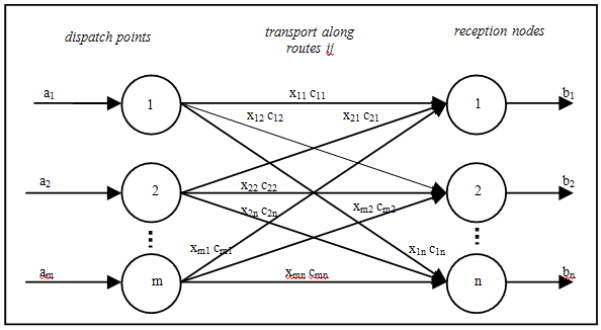


Fig. 3. The network diagram of the problem (the author's own elaboration)

The transport problem also has a simple network interpretation (fig. 3). The network is two-part and complete, i.e. all its nodes can be divided into two groups: supplies nodes – dispatch points (numbered *1,2,3, ... m*) and reception nodes (numbered *1,2,3, ... n*). Each supplies node had *n* arches running from this node to each reception point (transport along routes *ij*). The arch must be described by the parameters which determine such items as unit costs of transport, time of transport or distances. Most often, it is a unit cost.

The problem involves determination of such values of transport x_{ij} so as to minimize the total cost of transport

$$F(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \longrightarrow \min \qquad (1)$$

where x_{ij} – searched volume of transport,

c_{ij} – unit cost of transport

between supplier *i* and buyer *j*.

Under the conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2,\ldots,m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2,\ldots,n \qquad (3)$$

$$x_{ij} \geq 0 \qquad i = 1,2,\ldots,m \qquad j = 1,2,\ldots,n$$

Both the assumptions for the previously described situation and the solution can be written in the form of a matrix.

The matrix presentation of the input data:

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$$\overline{C}_{ij} = \begin{vmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & c_{m3} & \dots & c_{mn} \end{vmatrix} \qquad \overline{a}_i = \begin{vmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{vmatrix} \tag{4}$$

$$\overline{b}_j = \begin{vmatrix} b_1 & b_2 & \dots & b_m \end{vmatrix} \tag{5}$$

The matrix presentation of the solution:

$$\overline{x}_{ij} = \begin{vmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{vmatrix} \tag{6}$$

In order to check whether the obtained solution is an optimal one, optimality indices Δij. for the set transport programme should be established.

The matrix of optimality:

$$\overline{\Delta}_{ij} = \begin{vmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \dots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \dots & \Delta_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \Delta_{m1} & \Delta_{m2} & \Delta_{m3} & \dots & \Delta_{mn} \end{vmatrix} \tag{7}$$

It can be demonstrated that optimality index Δij for variable x_{ij} can be presented as

$$\Delta ij \; \square\square\square c_{ij} - (\square u_i \; \square\square v_j) \qquad i=1,2,...,m \; ; \; j=1,2,...,n \tag{8}$$

We now establish the values of all $m + n$ of dual variables (i.e. potentials) u_i and v_j. For this purpose, we take advantage of the property stating that for base variables (routes with transports) $\Delta ij = \square 0$. There are $m + n - 1$ such variables. It is therefore possible to construct a $m + n - 1$ set of equations with $m + n$ unknown u_j and v_j and unit costs of transport c_{ij} as free terms. Using dual variables calculated in the above way, it is possible to compute the remaining values Δij.

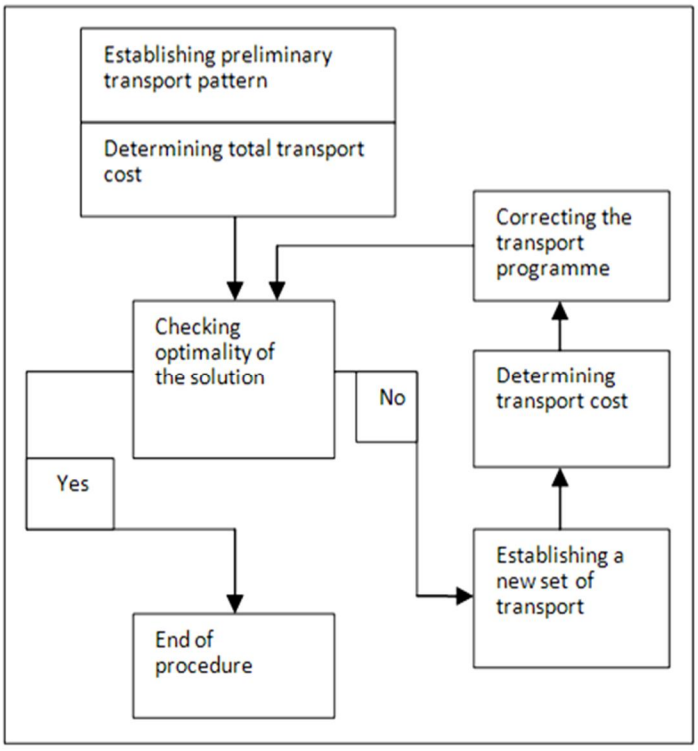


Fig. 4. Diagram for calculations in a transport problem (the author's own elaboration)

The condition for achieving optimality is that for all the set values the non-negativity condition $\Delta ij \geq 0$ be fullfilled. The procedure is illustrated by the diagram in fig. 4.

4. A CASE STUDY

The problem will be analysed based on an example of supplies of crushed stone aggregate needed to build 20-cm thick road foundation under a 35-km long section of regional road. It is assumed that the materials will be delivered at points located along the route at 5-km intervals. Three suppliers located at different distances along the road under construction have declared to be able to supply the materials. The prices of the materials are likewise varied. Another point for consideration is the cost of transport, which depends on the distance to cover in order to bring the materials (Tab. 1).

Because aggregate is transported using heavy-duty vehicles and the material itself is delivered in large quantities, this part of work has a significant effect on the wear-out of the road infrastructure and burdens the traffic with heavy lorries. Therefore, the analysis should also include an estimated duration of the total transport distance.

In order to plan the deliveries, the demand for the material has been estimated for all the stages in the construction project. The total amount of the material needed to complete the work is 36 000 m^3 (72 000 tons). The calculation is based on the work timetable and cost analysis, from which the demand for aggregate during each stage of construction work was derived. (Tab. 2).

Tab. 1. Set of deliveries including prices and cost of transport (the author's own elaboration)

No	Supplier	Price PLN/t	Distance km	Price for Transport PLN/t	Custo-mer
1	Nr 1	65	5	22,5	1
		68	7	31,5	2
		70	13	58,5	3
		72	17	76,5	4
		75	20	80,0	5
		79	25	100	6
		82	30	120,0	7
2	Nr 2	56	17	76,5	1
		57	12	54,0	2
		60	10	45,0	3
		60	25	67,5	4
		64	20	90,0	5
		69	22	99,0	6
		70	27	121,5	7
3	Nr 3	55	32	128,0	1
		57	29	116,0	2
		59	22	88,0	3
		62	19	76,0	4
		67	15	60,0	5
		68	11	44,0	6
		68	6	24,0	7

Table 2 Demand for material at particular work stages (the author's own elaboration)

No	Point of delivery	Demand (t)
1	1	10 000
2	2	10 000
3	3	12 000
4	4	12 000
5	5	10 000
6	6	10 000
7	7	6 000

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In order to adjust the example to the model's conditions, a ficticious buyer O_8 was added and the suppliers' resources were established on a slightly higher level. This way we ensure more flexibility in planning the transport. The problem is presented in Tab. 5 and the preliminary solution obtained with the minimal index method can be found in Tab. 6.

The value of the objective function is 7939 thousand PLN.

$$F_1(\mathbf{x}) = 10 \times 87,5 + 10 \times 99,5 + 12 \times 105 + 12 \times 127,5 + 2 \times 155 + 1 \times 154 + 9 \times 127 + 10 \times 112 + 6 \times 92 + 3 \times 0 = 7939 \ tys.zł$$

The value of the objective function (Tab. 6) suggests some modifications in the pattern of supplies. In the preliminary solution, buyer 5, located in the middle section of the constructed road, should be supplied from 3 sources, which may lead to some chaos in the organization of work. It is therefore suggested to slightly shift the fronts of work, which would lead to some correction in the supplies. (Tab. 3)

Tab. 3 Corrected demand for material (the author's own elaboration)

No	Point of delivery	Demand (t)
1	1	10 000
2	2	12 000
3	3	12 000
4	4	13 000
5	5	19 000
6	6	10 000
7	7	6 000

The solution which includes the suggested correction can be seen in Table 7, and the value of the objective function is calculated below:

$$F_2(\mathbf{x}) = 10 \times 87,5 + 12 \times 99,5 + 12 \times 105 + 13 \times 127,5 + 0 \times 155 + 0 \times 154 + 9 \times 127 + 10 \times 112 + 6 \times 92 + 3 \times 0 = 7801,5 tys.zł$$

The value of the objective function after correction is 7801.5 thousand PLN and is lower than the one obtained before, which proves that the correction was advisable.

The matrix of the solution obtained:

$$\overline{X} = \begin{vmatrix} 10 & 12 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 12 & 13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 10 & 6 & 0 \end{vmatrix}$$

5. ANALYSIS OF THE SOLUTION

In the above solution, supplies will be carried out by 3 subsequent suppliers.

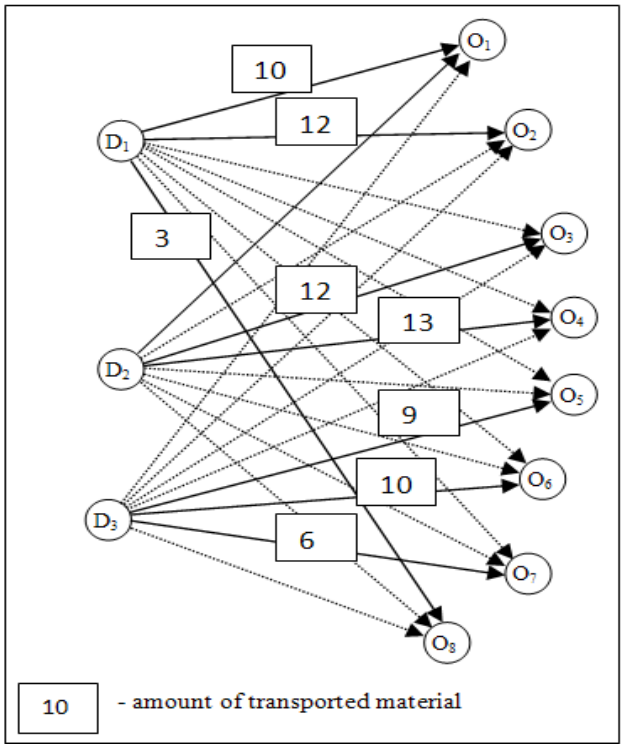


Fig.5 The network diagram of the solution(the author's own elaboration)

Because supplier 1 was the most expensive, it is suggested that he supplies only points 1 and 2. Points 3 and 4 will be supplied by the second supplier. The third supplier, the least expensive one, appears at stage 5 and carries out the supplies to points 6 and 7. Because the first supplier is the most expensive one, he is left with surplus material, which in this solution will be purchased by the ficticious customer. In the transport pattern thus obtained, distances and related costs of transport are also important. The solution confirms that as the work progresses, suppliers should be changed and even if the material transported over a shorter distance is slightly more expensive, it will eventually prove to be a better choice in terms of the total cost of supplies. The solution can also be illustrated as a network model. The solid line shows these connections for which the supplies are actually transported.

The diagram on fig.5 clearly illustrates the example of a road construction project used as an example in this paper.

6. TESTING THE OPTIMALITY OF THE SOLUTION

Determination of the potentials and indices of optimality Δij has been performed in the form of tables which correspond to the matrix presentation of the problem. Because all the optimality indices fullfil the non-negativity condition Δ ij ≥ 0, the solution obtained is optimal. For the analysis to be complete, a comparison of the total length of transport for both solutions was performed.

The comparison of the distances covered by transported supplies shows that in solution 2, with the corrected preliminary assumptions, the total sum of distances covered by transport units carrying the material was lower than in the first solution. This is a very important observation because transport of

aggregate involves heavy lorries and the material itself is delivered in huge quantities. Transporting aggregate for road building can have serious negative influence on the state of repair of existing roads and brings heavy burden to the traffic system.

Tab. 3 Determination of the values of potentials u_i i v_j (the author's own elaboration)

Position in the matrix	Cij	ui	vj	Δij
13	128,5	0	-106	22,5
14	148,5	0	-128,5	20
16	179	0	-140	39
17	202	0	-120	82
21	132,5	1	-87,5	46
22	111	1	-99,5	12,5
26	168	1	-140	29
27	191,5	1	-120	72,5
28	0	1	0	1
31	183	28	-87,5	123,5
32	173	28	-99,5	101,5
33	147	28	-106	69
34	138	28	-128,5	37,5
38	0	28	0	28

Tab. 4. Determination of optimality indices Δij (the author's own elaboration)

Position in the matrix		Cij	ui	vj
11	1	87,5	0	-87,5
12	2	99,5	0	-99,5
15	3	155	0	-155
18	4	0	0	0
23	5	105	1	-106
24	6	127,5	1	-128,5
25	7	154	1	-155
35	8	127	28	-155
36	9	112	28	-140
37	10	92	28	-120

Tab. 5. Presentation of the problem in the form of a transport table (the author's own elaboration)

	O1		O2		O3		O4		O5		O6		O7		O8		ai
D1	87,5		99,5		128,5		148,5		155		179		202		0		25
D2	132,5		111		105		127,5		154		168		191,5		0		25
D3	183		173		147		138		127		112		92		0		25
bj	10		10		12		12		12		10		6		3		

Tab. 6. Initial solution obtained with the minimal coefficient method(the author's own elaboration)

	O1		O2		O3		O4		O5		O6		O7		O8		ai
D1	87,5	**10**	99,5	**10**	128,5		148,5		155		179		202		0	**3**	25 (22) (12)(2)
D2	132,5		111		105	**12**	127,5	**12**	154	**1**	168		191,5		0		25 (13)(1)
D3	183		173		147		138		127	**9**	112	**10**	92	**6**	0		25 (19)(9)
bj	10		10		12		12		12		10		6		3		

Table 7 Initial solution for corrected assumptions (the author's own elaboration)

	O1		O2		O3		O4		O5		O6		O7		O8		ai
D1	87,5	**10**	99,5	**12**	128,5		148,5		155	**0**	179		202		0	**3**	25 (22) (12)(0)
D2	132,5		111		105	**12**	127,5	**13**	154	**0**	168		191,5		0		25 (13)(1)
D3	183		173		147		138		127	**9**	112	**10**	92	**6**	0		25 (19)(9)
bj	10		10		12		12		12		10		6		3		

Tab. 8 Calculation of the length of transport in km. (the author's own elaboration)

Connec-tions	km	xij	Variant 1-deliveries	Total km	xij	Variant 2-deliveries	Total km
11	5	10	400	2000	10	400	2000
12	7	10	400	2800	12	480	3360
15	20	2	80	1600	0	0	0
18	0	0	0	0	0	0	0
23	10	12	480	4800	12	480	4800
24	15	12	480	7200	13	520	7800

25	20	1	40	800	0	0	0
35	15	9	360	5400	9	360	5400
36	11	10	400	4400	10	400	4400
37	6	6	240	1440	6	240	1440
			sum	30440			29200

7. CONCLUSIONS

The method discussed in this article can easily aid optimalisation of supplies to construction projects which have specific characteristics. When building roads, for example, the work is conducted over a length of tens of kilometers. It is then extremely important that suppliers are adequately chosen so that their transport vehicles would not travel unnecessarily long distances. Linear programming is a very convenient and simple method which enables the user to do calculations on a computer with very popular software.

However, when using mathematical methods to support decision making, one should remember to check the calculations constantly. Constant analysis of the work done, starting with the stage of analysing the problem, determining the parameters and selecting an algorhythm of the procedure, and going through such stages as the interpretation of the results and implementation of the solution obtained, is recommendable.

As a result of the calculations performed, we generate a series of figures and it is very important to read them well. At this point, it is recommendable to return to the assumptions and established data regarding the decision variables and to the values of the parameters determined at the beginning

Implementation of the results of the calculations is possible having carefully analysed the solution. It is equally important that the persons making decisions on the basis of the results of operational research participate in the work of the team involved in the analysis of the whole problem. Another significant aspect is not to treat the result of a mathematical procedure literally. Mathematical methods are just a supporting tool in a decision-making process and it is for us, people, to make the final decision.

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