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STACKING SEQUENCE OPTIMIZATION OF COMPOSITE LAMINATED BEAMS FOR THE MINIMIZATION OF MAXIMUM NORMAL AND SHEAR STRESSES

Fatih KARAÇAM, Taner TIMARCI
Trakya University, Department of Mechanical Engineering, 22030, Edirne, TURKEY

Abstract

In this study, the stacking sequences of laminated composite beams under uniform distributed load are optimised for the minimization of maximum normal and shear stresses. Genetic algorithm is used as optimization technique. A unified three-degree-of-freedom shear deformable beam theory is used in analytical solution. Initially, the stacking sequences giving the maximum values of normal and shear stresses are obtained analytically among all possible configurations for various boundary conditions. Then, the minimum stresses and corresponding stacking sequences among the maximum ones are obtained by applying genetic algorithm. In order to improve the efficiency of the algorithm, elitism is included. The processing time taken by both approaches are compared for different boundary conditions.

Key words: *Laminated composite beams, shear deformation beam theory, stacking sequence optimization, genetic algorithm.*

1. INTRODUCTION

Structural elements that are constructed of composite materials such as beams, plates and shells, have wide application fields in engineering. Because of high elasticity modulus and strength, composite materials are popular than conventional materials, besides, by changing the lay up, fiber orientation, matrix and fiber material, the structures can be designed variously depending on desirable mechanical properties. In this case, by choosing the convenient weight, geometry, natural frequency, deflection, strength and buckling load parameter, stacking sequence, fiber orientation and layer thickness can be optimized directly. Because of applicability in various variables and fittness functions, genetic algorithm (GA) is widely used in the optimization of laminated composite structures. Di Scuvia et al, optimized the weight and buckling load of the structure by maximizing the buckling load and minimizing the weight respectively. They searched for the convenient stacking sequences in order to satisfy the results by genetic algorithm and compared the results with the ones obtained by simulated annealing (Di Scuvia et al, 2003). Walker and Smith investigated the bending analysis of eight layered composite plates by a finite element method and by defining a fitness function depending on weight and deflection parameters, they minimized it by use of genetic algorithm (Walker and Smith, 2003).

In this study, considering the shear deformation effects, on the basis of a unified beam theory governing equations of rectangular cross-section composite beam are obtained. The same theory was used in bending analysis of composite beams in previous studies (Karaçam and Tımarcı, 2005). In the study presented, the stacking sequences of laminated composite beams under uniform distributed load are optimised for the minimization of maximum normal and shear stresses. During the optimization process, genetic algorithm is used and elitism is included in order to improve the efficiency of the algorithm. Initially, the stacking sequences giving the maximum values of normal and shear stresses along the beam thickness are obtained analytically among all possible configurations for various boundary conditions. Then; the minimum stresses and corresponding stacking sequences among the

maximum ones are obtained by applying genetic algorithm. Additionally, the processing time depleted in analytical, genetic algorithm solution for various boundary conditions are compared and shown in tables.

2. ANALYTICAL METHOD

In the study, laminated composite beams with a rectangular cross-section, unith width, length of "L" and thickness of "h" are considered (**Figure 1**). The coordinate system is replaced in the mid-plane of the beam as $0 \leq x \leq L$ and $-h/2 \leq z \leq h/2$. The beam is assumed to be constructed of arbitrary number of linearly elastic layers and under of a uniform distributed transverse load (q(x)) on its top plane.

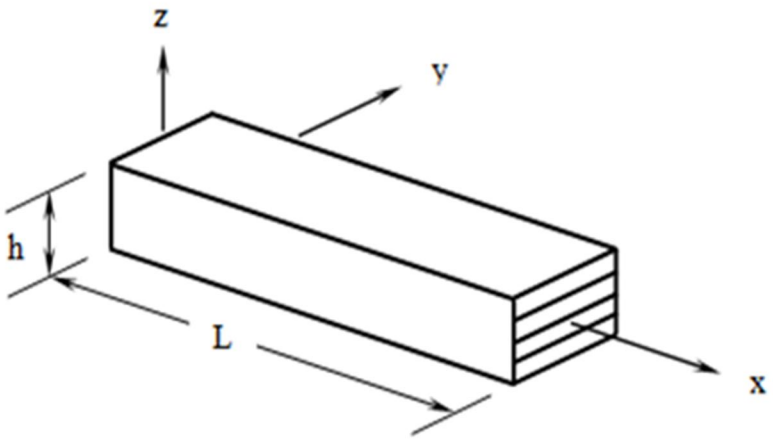


Figure 1. Beam geometry and coordinate system

A unified beam theory at which classical and various shear deformation beam theories can be obtained as a special case by a shape function which is depending on the thickness coordinate is used in the study (Karaçam, 2005). The displacement fields used for the beam are in parallel with general shear deformation shell theory (Soldatos and Tımarcı, 1993) and given as follows;

$$U(x,z) = u(x) - zw_{,x} + \emptyset(z)u_1(x)$$
$$W(x,z) = w(x) \qquad (1)$$

The terms of U and W mentioned above, represent the displacement fields along x and z axis where u, w and u_1 represent the unknown displacement functions of the middle surface of the beam. ",x" denotes the differentiation with respect to x.

The shape function (ϕ (z)) is chosen as a cubic function of layer thickness that correponds to parabolic shear deformation beam theory which satisfies the stress-free conditions at the top and bottom surface of the beam.

$$\phi\,(z) = z(1 - \frac{4z^2}{3h^2}) \tag{2}$$

The displacement fields yield to the following kinematic relations:

$$\varepsilon_x = u_{,x} - zw_{,xx} + z(1 - \frac{4z^2}{3h^2})\,u_{1,x}$$
$$\gamma_{xz} = (1 - \frac{4z^2}{h^2})\,u_1 \tag{3}$$

According to Hooke's Law, the state of stresses for each k^{th} layer is given as below. The terms of $Q_{ij}^{(k)}$ are the reduced stiffnesses which include the material properties such as elasticity modulus (E_{ij}), shear modulus (G_{ij}) and Poisson ratio (υ_{ij}) (Jones, 1975):

$$\begin{bmatrix} \sigma_x^{(k)} \\ \tau_{xz}^{(k)} \end{bmatrix} = \begin{bmatrix} Q_{11}^{(k)} & 0 \\ 0 & Q_{55}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \gamma_{xz} \end{bmatrix} \tag{4}$$

By use of these relations into force and moment definitions (Tımarcı and Soldatos, 1995), the constitutive equations can be obtained as below:

$$\begin{bmatrix} N_x \\ M_x \\ M_x^a \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & B_{111} \\ B_{11} & D_{11} & D_{111} \\ B_{111} & D_{111} & D_{1111} \end{bmatrix} \begin{bmatrix} u_{,x} \\ -w_{,xx} \\ u_{1,x} \end{bmatrix}$$
$$Q_x^a = A_{55}\,u_1 \tag{5}$$

The superscript "a" stands for the additional shear effect. A_{ij}, B_{ij} and D_{ij} are the material rigidities which correspond to extensional, coupling and bending rigidities respectively. The terms more than two indices occur because of shear deformation (Soldatos and Tımarcı, 1993). "Q_x^a" is the resultant of the transverse shear force. The governing equations for laminated composite beams under uniform distributed load are given as follows:

$$N_{x,x} = 0$$
$$M_{x,xx} = q(x) \tag{6}$$
$$M_{x,x}^a - Q_x^a = 0$$

By the use of constitutive equations into the governing equations, a set of differential equations and their derivatives depending on material rigidities and displacement functions are obtained as below:

$$\begin{aligned} A_{11}u_{,xx} - B_{11}w_{,xxx} + B_{111}u_{1,xx} &= 0 \\ B_{11}u_{,xxx} - D_{11}w_{,xxxx} + D_{111}u_{1,xxx} &= q(x) \\ B_{111}u_{,xx} - D_{111}w_{,xxx} + D_{1111}u_{1,xx} - A_{55}u_1 &= 0 \end{aligned} \tag{7}$$

The boundary conditions at the edges of the beam when x=0 and x=L, are given as follows (Soldatos and Tımarcı, 1993):

Simply Supported (S)	$N_x = w = M_x = M_x^a = 0$
Cantilever (A)	$u = w = w_{,x} = u_1 = 0$
Free (F)	$N_x = M_x = M_{x,x} = M_x^a = 0$

(8)

By integrating and solving the equations (7) simultaneously and using the boundary conditions at both edges, three unknown displacement functions with eight integration constants (C_k) are obtained as below:

$$\begin{aligned} u_1(x) &= C_1 e^{-px} + C_2 e^{px} - (qx + C_3)\frac{D_{111}}{A_{55}D_{11}} = 0 \\ u(x) &= -\frac{B_{111}}{A_{11}}u_1(x) + C_7 x + C_8 = 0 \\ w(x) &= \frac{D_{111}}{pD_{11}}\left[-C_1 e^{-px} + C_2 e^{px} - \frac{p}{A_{55}}\left(\frac{qx^2}{2} + C_3 x\right)\right] + \frac{1}{D_{11}}\left(\frac{qx^4}{24} + C_3\frac{x^3}{6}\right) + C_4\frac{x^2}{2} \\ &\quad + C_5 x + C_6 = 0 \end{aligned} \tag{9}$$

$$p = \sqrt{\frac{-A_{55}A_{11}D_{11}}{D_{111}^2 A_{11} - D_{1111}A_{11}D_{11}}} \tag{10}$$

3. GENETIC ALGORITHM

A genetic algorithm is an evolutionary optimization technique using Darwin's principal of "survival of the fittest" to improve a population of solutions. The principals of GA are firstly proposed by John Holland from Michigan University. For optimization problems, he used evolutionary laws in genetic algorithm. Instead of generating a single solution, GA generates a various set of solution which are totally independent from each other. If the population size is suitably large, GA is not at the risk of being stuck in a local optimum (Ghiasi et al, 2009). They are also derivative-free optimization methods, as they do not need functional derivative information to search for a set of parameters that minimizes or maximizes a given objective function. By using such algorithms in optimization problems, robustness and flexibility can easily be obtained (Goldberg, 1989).

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An important limitation of traditional GA's, especially in larger populations, is the time waste to distinguish the convenient fitness function. In order to avoid this and increase the convergency rate, additional methods such as artificial neural networks (ANN) are used (Walker and Smith, 2003, Apalak et al, 2008). Besides, in addition to the classical genetic operators, elitist approaches can be used to create new generations in genetic algorithm (Almedia and Awruch, 2009). In elitism, the better individuals of a percentage of each generation are used in next generations. Thus; regardless of the problem type (minimization or maximization problem), better populations will be obtained in each generations with an increasing convergency rate. However, a larger population which also means a high processing time or mutation genetic operator must be chosen in elitism in order to provide the diversity among generations.

There are many uses of genetic algorithm in engineering designs. The algorithm is generally based on natural selection, crossover and mutation which are also called as the genetic operators. It depends on a random population which represents the potential solution. In each generation, the fitness values of each individuals are calculated repeatedly. The process continues until the stopping criteria is obtained. If not, the genetic operators mentioned above, are applied to improve the efficiency and the convergency rate. In genetic algorithm, reproduction is usually the first operator applied to the population. The individuals chosen are selected to be parents to crossover and produce offspring. Crossover is the genetic operator at which the chromosomes are combined to each other in order to produce new chromosomes (offsprings). Mutation is another genetic operator used in genetic algorithm. It is generally used to maintain the genetic diversity from one generation to another. It basically depends on altering one or more genes in a choromosome. A basic genetic algorithm flowchart is given in **Figure 2**.

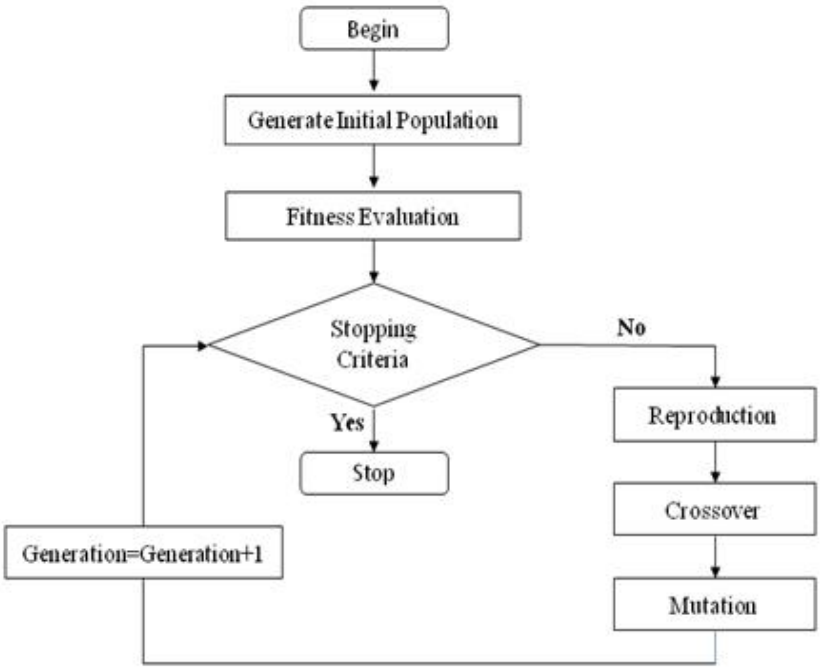


Figure 2. Genetic algorithm flowchart

4. NUMERICAL RESULTS

In the study presented, the composite beam is assumed to be constructed of graphite/epoxy and under a uniform distributed load and various boundary conditions. Mechanical properties of graphite/epoxy are given as follows; E_{11}=241,5 GPa, $E_{22}=E_{33}$=18,89 GPa, G_{23}=3,45 GPa, $G_{12}=G_{13}$=5,18 GPa, υ_{23}=0,25, $\upsilon_{12}=\upsilon_{13}$=0,24.

The maximum normal and shear stresses are calculated at certain sections of the beam where the maximum moment and shear forces may occur. The normal stresses are calculated at the edges of the beam where x=0 or L and z=-h/2 in "Clamped-Clamped" (CC) and "Clamped-Free" boundary conditions and in the middle of the beam where x=L/2 and z=-h/2 in "Simply Supported (at both edges)" (SS) boundary conditions. The shear stresses are calculated in the middle of the thickness of the beam where x=L/4 and z=0 for CC and CF boundary conditions and at both edges where x=0 and z=0 for SS boundary conditions.

In **Table 1** and **Table 2**, the comparison of normal and shear stresses for different cases are presented respectively. Initially, the maximum normal and shear stresses and corresponding stacking sequences are calculated among 1000 possible configurations analytically. Then, by use of genetic algorithm, the minimization of maximum normal and shear stresses are carried out for a population of 100 individuals and 20 generations. In order to develop the algorithm and increase the convergency rate, elitism is included into the algorithm. By the minimization of maximum normal and shear stresses for various boundary conditions and loading type, the laminated composite beams can be designed more safely in order to prevent the failure.

Table 1

Comparison of Normal Stresses and Stacking Sequences (Pa) (Increment 10°, L/h=50, Generation=20, Population=100)

Boundary Condition	Analytical Result & Stacking Sequence	GA Result & Stacking Sequence	GA Result (with Elitism) & Stacking Sequence
CC	$3{,}8508.10^{8}$	$5{,}8068.10^{6}$	$6{,}2632.10^{5}$
	10°/40°/60°	80°/40°/60°	60°/60°/60°
CF	$4{,}0197.10^{9}$	$4{,}8398.10^{8}$	$1{,}8475.10^{6}$
	20°/60°/60°	20°/0°/70°	20°/80°/60°
SS	$1{,}9115.10^{7}$	$3{,}6808.10^{6}$	$3{,}6808.10^{6}$
	10°/80°/80°	0°/80°/80°	0°/80°/80°

The minimization of normal and shear stress values are carried out in four ways. As it can easily be seen from Table 1 and Table 2, the maximum values of normal and shear stresses and corresponding stacking sequences are obtained from analytical solution for all boundary conditions. In order to obtain the minimum value of the maximum stresses at certain section, genetic algorithm is applied. In each generation, the maximum normal and shear stresses and their corresponding stacking sequences are kept in memory and after the 20^{th} generation, the stacking sequence giving the minimum among the maximum stresses is selected. It is obvious from the tables that the results of genetic algorithm with elitism included converges to minimum better than the other solutions.

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Table 2

Comparison of Shear Stresses and Stacking Sequences (Pa) (Increment 10°, L/h=50, Generation=20, Population=100)

Boundary Condition	Analytical Result & Stacking Sequence	GA Result & Stacking Sequence	GA Result (with Elitism) & Stacking Sequence
CC	$3,6561.10^6$ 0°/60°/80°	$1,2318.10^5$ 40°/0°/90°	$4,4131.10^4$ 0°/10°/50°
CF	$4,0197.10^9$ 20°/60°/60°	$2,6892.10^8$ 40°/70°/90°	$5,3396.10^6$ 80°/0°/70°
SS	$6,9898.10^5$ 10°/80°/80°	$4,9528.10^4$ 80°/10°/30°	$1,5659.10^4$ 0°/90°/90°

During the optimization of stacking sequences, the processing time is also calculated. In **Table 3** and **Table 4**, the comparison of processing time for normal and shear stresses are given respectively. The results are obtained for all three combinations of boundary conditions. Initially, the processing time is obtained analytically for all possible configurations. Then, for a population of 100 individuals and 20 generations, the processing time is obtained by genetic algorithm. It can be observed from **Table 3** and **Table 4** that the time values obtained by GA solution are higher than the analytical solution. Individuals with low fitness values in each generation may cause to this. In genetic algorithm solution, the minimum of normal and shear stress values are generally obtained at 7^{th} and 9^{th} generations. When elitism is included into the algorithm, the processing time decreases because of better populations with high fitness values. In elitist solution, the minimum values are generally obtained in 2^{nd} and 3^{rd} generations.

Table 3

Comparison of Processing Time (s) for Normal Stresses

Boundary Condition	Analytical Solution	GA Solution	GA (with Elitism) Solution
CC	338	652	92
CF	366	637	224
SS	367	408	87

Table 4

Comparison of Processing Time (s) for Shear Stresses

Boundary Condition	Analytical Solution	GA Solution	GA (with Elitism) Solution
CC	330	579	85
CF	344	398	76
SS	318	994	74

5. CONCLUSIONS

In the study; by use of a unified shear deformation beam theory, the stacking sequences of composite laminated beams under a uniform distributed load are optimized in order to minimize the maximum normal and shear stresses for different boundary conditions. For each layer, the fiber angles are assumed to be changing between 0° and 90° with an increment of 10°. Since the composite beams considered are quite thin, the differences in numerical results have been observed between analytical, and genetic algorithm solutions. The minimum values of the stresses are obtained with a developed genetic algorithm in which elitism is included. During the optimization process, in order to observe the efficiency of the algorithm, the processing time for all solutions are also calculated.

The application of evolutionary methods for engineering designs has demonstrated to be powerful and sufficient enough for leading the search process to an optimal solution. The study presented can be extended as an optimization problem by taking failure into consideration for different layer thickness, number of layers, stacking sequences and material properties. Besides; by the minimization or maximization of parameters such as deflection, frequency, buckling load, normal and shear stresses and using together, the study can be investigated as a multi-objective optimization problem.

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