Journal of International Scientific Publications:
Materials, Methods & Technologies, Volume 7, Part 2
ISSN 1313-2539, Published at: http://www.scientific-publications.net

AN EXPERIMENTAL STUDY FOR COMPARISON OF LOAD ESTIMATION METHODS BY USING FORWARD EXTRUSION OF SPUR GEAR FORM

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Abstract

The aim of this research is the comparison of the load estimation methods by means of Upper Bound Analysis and a commercial FEM based programme called DEFORM-3D. For this purpose forward extrusion was chosen as forming method to manufacture a spur gear form. Two different die geometries were chosen for the forward extrusion; including straight tapered and cosine profiled dies. In the experimental study, Al 1070 was used as working material. Experiments using two sets of dies with spur gear form were performed, and the measured forming load results were compared with the predictions of the theoretical analyses by using DEFORM-3D and by using upper bound analysis. Experiments were carried out on the 150 metric ton hydraulic press with constant ram speed of 5 mm/sec. An acceptable agreement was obtained between the theoretical results and the experiments.

Key words: *Gear like elements, extrusion, upper bound, DEFORM-3D*

1. INTRODUCTION

Gears are the most important machine element used in wide range areas such as general gear boxes, automotive industry and so on. A gear is a machine element designed to transmit power and motion from one mechanical unit to another. Spur gears are the simplest type of gears. They are manufactured by using conventional manufacturing methods. Metal forming is a favorable manufacturing method of gears and gear-like components due to their longer lifetime, higher fatigue strength and cost effectiveness when compared to machined gears when it is made of the same material. At last two decades, plastic deformation is getting more famous method because of its advantages. Extrusion is a plastic deformation process in which a billet is placed in a container and pressed by the punch, causing the metal in the container to flow through a die with an opening (Çan et al 2004). Extrusion process can be used in a wide range from light alloys to hard metals. Extrusion process generally eliminates the secondary machining. There are so many parameters which affects process like material properties, extrusion type, frictional conditions and extrusion ratio. Cold extrusion is done at room temperature or slightly elevated temperatures. Cold extrusion parts have good mechanical properties such as fatigue strength because of the severe cold forming and good surface finish with the using proper lubricants (Alves 2001)**.** Design of forward extrusion die profiles are important owing to their effect on metal forming. Researchers commonly try to adopt curved dies for the applications from simple axisymmetric products to complicated sections such as gears and splines (Altınbalık 2008).

Deformation process is complex because of its nature and sometimes analytical methods can't exactly define it. But numerical methods like finite element method and upper bound method give an acceptable solution to the problem. Estimating the deformation load for the extrusion with a different types of die profiles such as curved or conical are the main topics of the number of analytical studies. In these studies researchers used slab method, finite element method and upper bound approach with number of different sections (Chang 1971, Nagpal 1974). The upper bound theorem was firstly

formulated by Prager and Hodge and later modified by Drucker et al to include velocity discontinuities (Bramley 2001). Many attempts have been made to calculate the load using the upper bound method which is a very useful and economical method to obtain knowledge and information on the die designing of metal forming processes. According to Bramley (2001) upper bound analysis can be provide rapid but approximate simulation and preform design when compared with currently available finite element based methods**.** Gunasekera and Hoshino (1985) obtained solutions by using upper-bound methods for the extrusion of polygonal sections from round billet through converging dies. In the mentioned study the surface of dead metal zone has been assumed as a die profile and given by radius function. A kinematically admissible velocity field of a generalized three-dimensional extrusion for an arbitrarily shaped section was derived in which all the velocity components were expressed in general function forms by Han et al (1986). Lee et al (1991) used a general surface model to optimize the die surface by using the upper bound analysis. Gordon et al (2007) presented a series of studies to develop the flexible velocity fields that could be used in an upper bound model for flow of a perfectly plastic material during extrusion through dies of any shape. Upper bound analysis has been used to evaluate the total energy of the process. A theoretical analysis was performed by Maity et al (1996) for the extrusion of square sections from using curved dies by means of the upper bound analysis. Kim et al (1997) proposed a simple kinematically admissible velocity field for the square-die extrusion of non-axisymmetric bars from round billets to determine the final stage extrusion load and average length of the extruded products. Kim also applied the applied velocity field to the square die extrusion of regular polygonal shaped bars, trochoidal gear shaped bars and rectangular shaped bars. Alfozan and Gunasekera (2003) used upper bound element technique and indicated that simulation of forging by using Upper bound method gave favourable results in a much shorter period of time comparing to FEM. Choi et al (1999) analyzed the two types of forgings of helical gears by using the upper-bound method. Choi et al (2000) performed an upper bound solution of spur gear with internal serration and used a combined velocity field. A series of experiment was carried out by Çan and Mısırlı (2008) by choosing a gear-like section for comparison of evaluation of the methods of lateral extrusion and closed-die forging. New kinematically admissible velocity fields for three dimensional extrusion of clover sections were suggested for both to obtain the optimum die length of shaped inlet dies and determination of the extrusion load by Altınbalık and Ayer (2009, 2012). They studied for extrusion of clover sections which are used for trochoidal gears of external gear pumps. Huang et al (2009) investigated cosine, conic, elliptic and hyperbolic profiles both theoretically and experimentally by using upper bound method.

On the other hand strains and stresses in complex bodies such as gears can also be obtained by FEM. Nowadays many studies is being chosen to determine the deformation load for the extrusion of metal through a variety of die shapes by using FEM. Among the many traditional FEM based programmes DEFORM-3D is a powerful process simulation system that enables designers to analyze metal forming, heat treatment, machining and mechanical joining processes on the computer. Fang et al (2009) used FEM for simulating the extrusion process to manufacture profiles of industrial significance through multi-hole pocket dies by DEFORM-3D. In order to examine the influences of the various parameters on the extrusion behaviour of an aluminum billet during its axisymmetric extrusion through a conical die Chen et al (2007) carried out a study by using DEFORM-3D. FE simulation of porthole die extrusion process has been investigated by Jo et al (2006) by using DEFORM-3D in the non-steady extrusion process. Fang et al (2009) performed to combine the considerations on die design and process optimization for the alloy to manufacture a complex solid profile with large differences in wall thickness, with DEFORM-3D. He et al (2010) carried out a theoretical investigation to understand the non-steady state extrusion process by means of the behaviour of metal flow, the extrusion load and the velocity field based on the FEM simulation called DEFORM-3D.

In the presented study, a series of experiments for gear-like extrusion were carried out to obtain the optimum die surface design and compared the measured experimental loads values with those calculated from UB and a commercial FEM programme called DEFORM-3D. The die geometries were chosen in the present investigation as cosine curved and straight tapered.

2. THEORETICAL ANALYSIS

2.1. Mathematical Modeling of Profiled Dies

Especially in the last two decades, researchers have performed many studies on curved dies such as linearly converging die profile and cosine die profile. Curved dies cover an area of application from simple axisymmetric products to complicated sections such as gears and splines (Song and Im 2004).

In this study gear form was selected for forward extrusion and the die inlet geometry has been design by two different types as straight tapered transition and cosine curved transition. Gear form has been defined mathematically as shown in Fig.1 and the inlet and exit geometry of the dies were obtained as;

$$R_{outlet}(\theta, z) = A + Bcos(6\theta) \quad (1)$$

$$R_{inlet}(\theta, z) = C + Dcos(6\theta) \quad (2)$$

where R_{outlet} is the outlet profile function and R_{inlet} is inlet profile function.

Parameters A, B, C and D in the inlet and outlet sections and are varied the shape of the die. Different billets and product sections can be multiplied related with these parameters. In the presented study these parameters have been chosen as;

C=14, D=0, A=8.7, B=3.5.

Transition profile between inlet and outlet profile of the die is expressed by Eq. (3) and Eq.(4).

$$R_{cos}(\theta, z) = \frac{(R_{inlet}+R_{outlet})}{2} + \frac{(R_{inlet}-R_{outlet})}{2} cos\left(\frac{\theta z}{L}\right) \quad (3)$$

and straight tapered transition profile is;

$$R_{tap}(\theta, z) = R_{outlet} + (R_{inlet} - R_{outlet})\frac{(L-z)}{L} \quad (4)$$

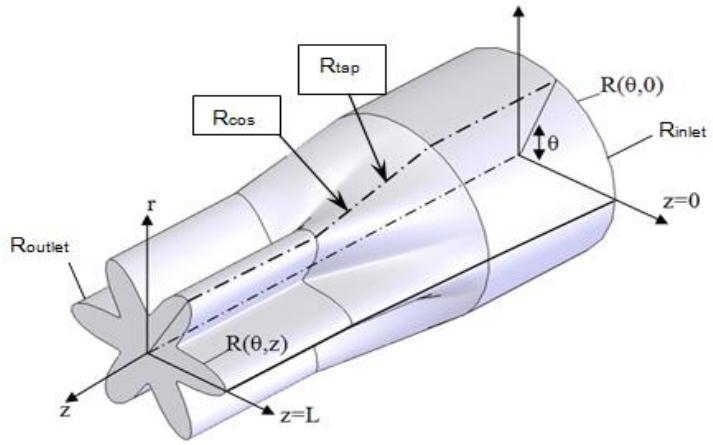

Figure 1. Schematic representation of the curved extrusion dies with the axes of reference and profile functions

Eq.(3) gives cosine profile between inlet and outlet sections where Eq.(4) defines the tapered profile. If the cross section shape is not given in an analytical form shown as Eq.(3) and Eq.(4), the die surface function cannot be determined. Every parameter defined for the profile is unique. These equations are so important to predict the material flow of the extrusion process. Transition profile influences both material flow and extrusion load. The lengths of the dies have been determined to be L=25 mm. Schematic illustrations of the die assembly and profiles of dies for forward extrusion are shown in Fig.2.a. and Fig.2.b.

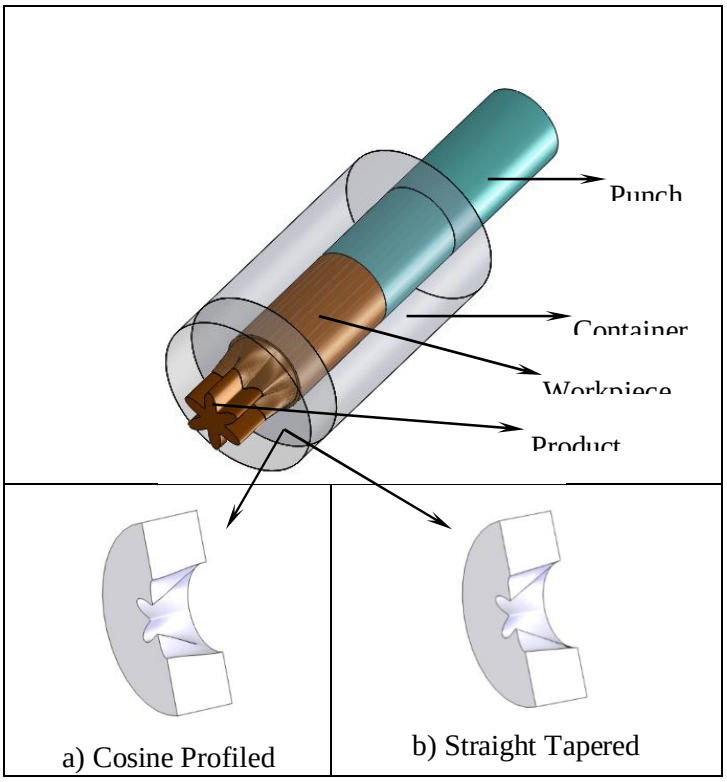

Figure 2. Schematic illustration of the die assembly for extrusion process

2.2. Generalized Velocity Fields

Velocity components are related to inlet, outlet sections and die transition profile. Following equations are available for cosine die profile.

V_z is expressed as Eq. (5) in general forms.

$$V_z = \frac{R_0^{\,2} V_0}{R_{\cos}^{\,2}} \tag{5}$$

Eq. (5) can be written as Eq. (6) with parameters;

$$V_z = \frac{CV_0}{2\left[C + A + B\cos(4\theta) + (C - A - B\cos(4\theta))\cos\left(\frac{\pi z}{L}\right)\right]} \quad (6)$$

Radial velocity component V_r is;

$$V_r = \frac{V_z\left(R_{\cos}(\theta,z)\frac{\partial R_{\cos}(\theta,z)}{\partial z}\right)r}{(R_{\cos}(\theta,z))^2} \quad (7)$$

$$V_r = \frac{-CV_0(C - A - B\cos(4\theta))\sin\left(\frac{\pi z}{L}\right)\pi}{\frac{L}{2}\left[C + A + B\cos(4\theta) + (C - A - B\cos(4\theta))\cos\left(\frac{\pi z}{L}\right)\right]^2}$$

(8)

Angular velocity is;

$$V_\theta = r\Omega(\theta,z)$$

(9)

$$V_\theta = \left[\frac{\left(-39.47r^2CV_0\sin\left(\frac{\pi z}{L}\right)\right)\left(\left[\left(-C-A-B+(A-B-C)\cos\left(\frac{\pi z}{L}\right)\right)\left(-C-A-B+(A+B-C)\cos\left(\frac{\pi z}{L}\right)\right)\left(B-A-C+(A-B-C)\cos\left(\frac{\pi z}{L}\right)\right)\right]^{\frac{1}{2}}\right)}{\left(L\left((B-A-C)+(A-B-C)\cos\left(\frac{\pi z}{L}\right)\right)\left(-(A+B+C)+(A+B-C)\cos\left(\frac{\pi z}{L}\right)\right)\left[(A^2-B^2+C^2+2AC)+2(B^2+C^2-A^2)\cos\left(\frac{\pi z}{L}\right)+(A^2-B^2+C^2-2AC)\cos\left(\frac{\pi z}{L}\right)^2\right]^{\frac{1}{2}}\right)}\right] *$$

$$\left[\begin{array}{l} A^2\cos\left(\frac{\pi z}{L}-1\right)-2AC\cos\left(\frac{\pi z}{L}\right)+ \\ B^2\left(1-\cos\left(\frac{\pi z}{L}\right)\right)+C^2\left(1+\cos\left(\frac{\pi z}{L}\right)\right) \end{array}\right]$$

(10)

For the straight tapered profile equations can be written as;

$$V_z = \frac{R_{inlet}V_0}{R_{tap}(\theta,z)}$$

(11)

$$V_z = \frac{CV_0}{(A + B\cos(6\theta) + (C - A - B\cos(6\theta)))\left(\frac{L-z}{L}\right)}$$

(12)

$$V_r = \frac{V_z\left(R_{tap}(\theta,z)\dfrac{\partial R_{tap}(\theta,z)}{\partial z}\right)r}{\left(R_{tap}(\theta,z)\right)^2}$$

(13)

$$V_r = -\frac{CV_0(C-A-B\cos(6\theta))r}{L\left[A+B\cos(6\theta)+(C-A-B\cos(6\theta))\left(\dfrac{L-z}{L}\right)\right]}$$

(14)

$$V_\theta = r\Omega(\theta,z)$$

(15)

$$V_\theta = \left[\frac{\left(-2\pi V_0 r^2 L(CL+(A-B-C)z)\left[(CL+(A+B-C)z)(CL+(A-B-C)z)\right]^{\frac{1}{2}}\right)\left(\left[CL(A-C)+z\left(A^2-B^2-C^2-2CA\right)\right]\right)}{\left((CL+(A-B-C)z)(CL+(A+B-C)z)\left[C^2L^2+2CLz(A-C)+z^2\left(A^2-B^2+C^2-2AC\right)\right]^{\frac{1}{2}}\right)}\right]$$

(16)

2.3. Upper Bound Solution

In this study, the upper bound method is used to predict the extrusion load. At the beginning, velocity components are input for the numerical calculation and they are used for the strain rate components and then the upper bound solution on power is determined by the numerical integration.

$$\varepsilon_{rr} = \frac{\partial V_r}{\partial r}, \qquad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{V_r}{r}, \qquad \varepsilon_{zz} = \frac{\partial V_z}{\partial z} \qquad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right)$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r}\right) \qquad \varepsilon_{\theta z} = \frac{1}{2}\left(\frac{\partial V_\theta}{\partial z} + \frac{1}{r}\frac{\partial V_z}{\partial \theta}\right)$$

$$\varepsilon_{eff} = \frac{\sqrt{2}}{3}\sqrt{\frac{1}{2}\left(\varepsilon_{rr}^{\ 2} + \varepsilon_{\theta\theta}^{\ 2} + \varepsilon_{zz}^{\ 2}\right) + \varepsilon_{r\theta}^{\ 2} + \varepsilon_{rz}^{\ 2} + \varepsilon_{\theta z}^{\ 2}}$$

(17)

Internal deformation energy, shear and friction energy can be calculated numerically from equations (18), (19) and (20). after the strain-rate components are obtained both cosine and tapered profiles.

$$W_d = \int_v \bar{\sigma}\dot{\bar{\varepsilon}}\,dv$$

(18)

σ is the flow stress of the material.

$$W_s = \int_{\Gamma_s} \frac{\bar{\sigma}}{\sqrt{3}}|\Delta v|\,ds$$

(19)

The friction, which exists between the deformation zone in the workpiece and the die, is characterized by a constant friction factor approach. The frictional shear stress is given by $\tau = m\frac{\sigma_0}{\sqrt{3}}$ where the constant friction factor m, can have a value between 0 and 1.

$$W_f = \int_{\Gamma_f} m\frac{\overline{\sigma}}{\sqrt{3}}|\Delta v|\,ds$$

(20)

Total energy component for the extrusion process is obtained by summing the internal deformation energy and the power dissipated on all frictional and velocity discontinuity surfaces in the Eq.21

$$W_{total} = W_d + W_s + W_f$$

(21)

Extrusion load can be calculated using the Eq.22

$$F_{ext} = \frac{W_{total}}{V_0}$$

(22)

V_0 is the velocity of the ram.

2.4.DEFORM-3D Finite Element Method (FEM) Simulation Procedure

The study performed rigid-plastic finite element simulations using a commercial FEM code DEFORM-3D v10.2 was used to simulate the extrusion process. DEFORM-3D developed by Scientific Forming Technology Corporation (SFTC)® in the US. In the present simulations, the complete models of each extrusion experiment were modeled to obtain more intensive simulation results. The dimensions of aluminium billets used in the calculations are a diameter 28 mm. and a length of 50 mm. The die set is modeled as rigid body while the extrusion billet which is aluminium is a plastic material and represented by a solid mesh approximately 6000 nodes and 26000 tetrahedral elements. The friction factors (m) between the extrusion billet and the contact surfaces are constant and assumed as 0.4. The velocity of ram is 5 mm/s as the same as press and the stroke is 30 mm as in the simulations and in the experiments also. As a result of the large and localized deformations, frequent remeshing is done during the course of the simulations. Load-stroke curve is taken from the simulations and compared with experimental results. Deform 3D model is shown in Fig.3 with meshed workpiece. This illustrates the end of the extrusion. As seen in Fig.3, workpiece is extruded and exited from the die as a six toothed gear product from a circular billet.

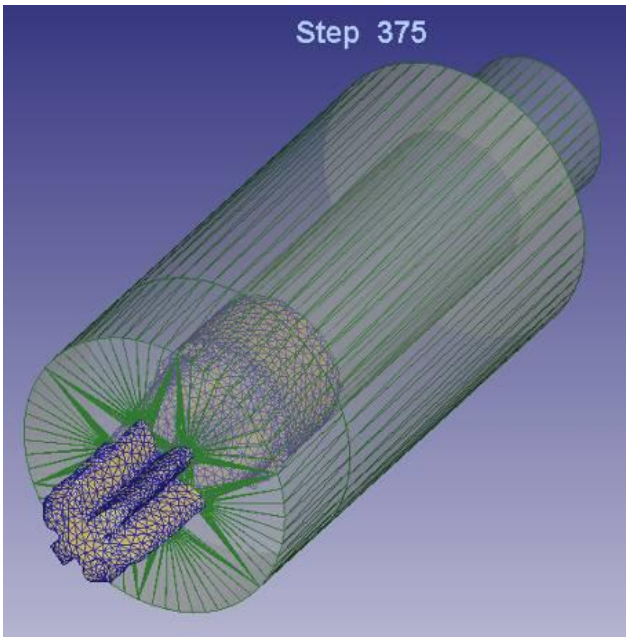

Figure 3. FEM model of the process

3. EXPERIMENTAL STUDY

3.1. Sample Preparation

Al1070 used as experimental material for investigation. The stress-strain relationship of the material determined from compression test is;

$$\sigma = 144\varepsilon^{0.162} \text{ MPa} \quad (23)$$

Billets have been cut from the bar and machined to 28 mm diameter and 45 mm in length. An extrusion container with internal diameter of 28.2 mm having 60 mm outer diameter and a punch having 28.2 mm diameter were machined. The die transition geometries chosen in the present investigation were cosine curved and straight tapered as mentioned before. The dies were made by W-EDM because of their geometrical complexity and the other die parts were machined at CNC. Dies, containers and the punches were made from 1.2344 DIN hot worked tool steel and hardened to 54 HR_c. Photographical view of the dies with each part is shown in Fig.4.a. and with the experimental set-up as Fig.4.b . Extrusion dies are also shown in Fig.4.c. Experiments were done on the 150 metric ton hydraulic press with constant ram speed of 5 mm/sec. Billets were cleaned with acetone before experiments in order to obtain the similar friction conditions. Photographical view of the extruded part is given in Fig.4.d. The hydraulic press was equipped with a pressure-current transducer in order to measure and record the experimental load. Required stroke of the press was adjusted by a vernier ruler which has an inductive switch at the end. Thus, the upper plate of the press reached the adjusted position of 30 mm punch stroke, the experiment was stopped by means of the software. Graphics of the load-stoke were drawn by software.

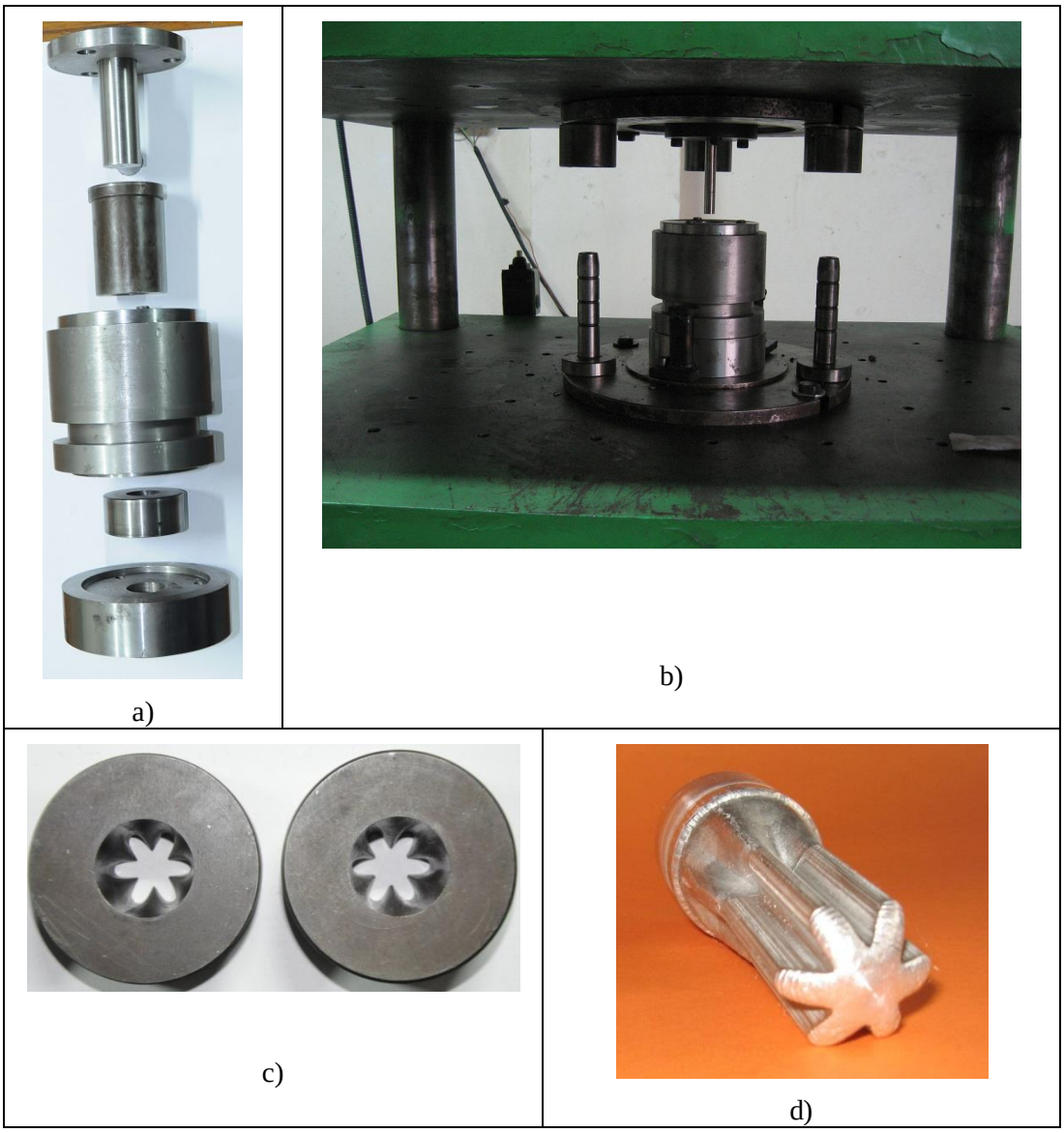

Figure 4. Photographic view of the dies with each part

a) forward extrusion die set b) experimental set-up c) dies d) extruded part

4. RESULTS AND DISCUSSION

Comparison of forming loads versus the punch stroke between experimentally measured and theoretically calculated load values which obtained from DEFORM-3D and UB solution for straight tapered die are shown in Fig. 5. As seen from the diagram the forming load increases with increasing the punch stroke due to the increase in frictional surfaces, as expected. On the other hand, the theoretical predictions are always higher for UB solution and mostly lower for DEFORM-3D than experimentally measured load values. As it is known that the prediction of the maximum extrusion load is important for determining the press capacity and tool material selection. Because of the assumed kinematically admissible velocity fields don't validate the last stage of the process, UB

solution is found to be lower at the end of the process. The value of the maximum load calculated by the UB deviates from the value obtained experimentally only with 10% and this discrepancy is acceptable one. As it can be seen from the diagram calculated FEM results is about 18% deviate than the experimental results for the maximum extrusion load. What leads to this difference is the assumptions made in the program don't entirely match the actual material properties.

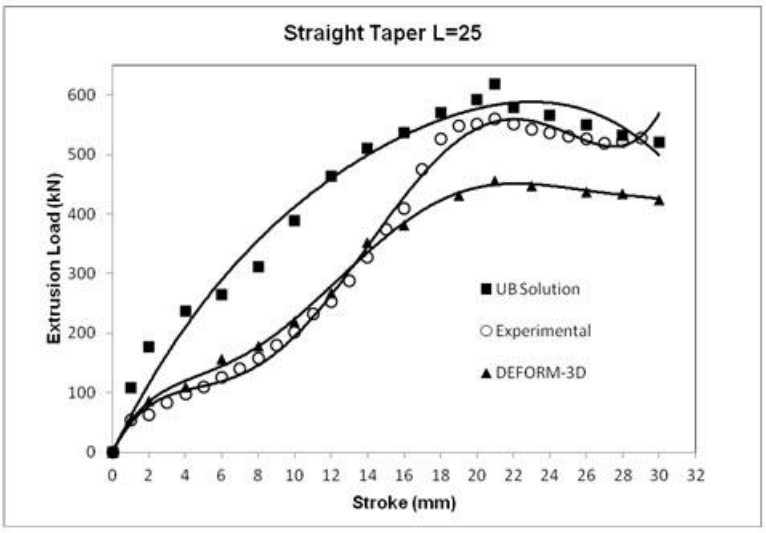

Figure 5. Comparison of theoretical and experimental extrusion load against punch stroke for straight taper die

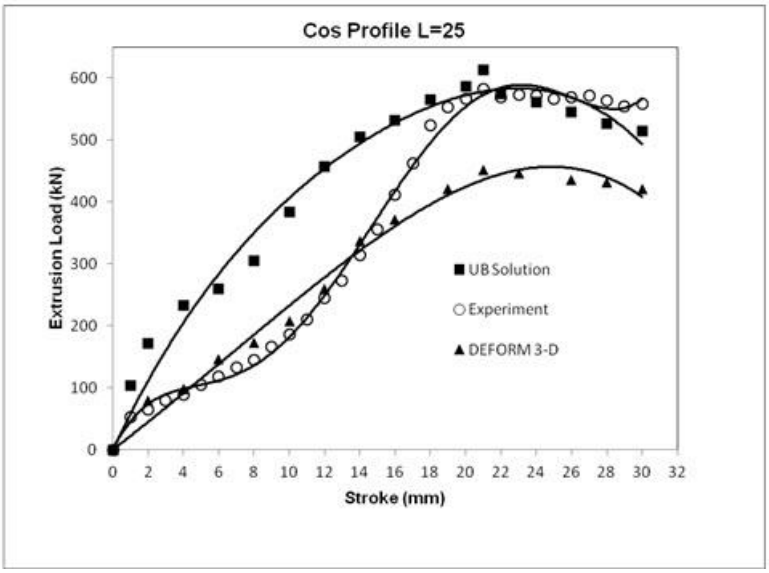

Figure 6. Comparison of theoretical and experimental extrusion load against punch stroke for cosine curved die

In order to investigate to die transition profile on the deformation characteristics of the gear-like form experiments were repeated for the cosine transition profiled die. Similar diagram was obtained and given in Fig.6. As seen in Fig.6 UB solution gives about 5.1% higher and DEFORM-3D solution gives 22% lower extrusion load than the experimental results of cosine transition profiled die. As the diagrams are examined together it can be said that the particularly UB solutions are quite compatible with experimental results for two the different die transition profile. But, DEFORM-3D solutions gives higher values compared to measured values only during the first half of the process. DEFORM-3D is a well equipped simulation software for metal forming and to obtain the results with simulation is quietly short compared to other commercial FEM based metal forming softwares and it is useful for industrial applications. Conversely UB method is mathematical based method and can define the process mathematically and it needs special mathematical definitions.

In order to detail investigation, each component of upper bound analysis for the straight tapered die is given in Fig.7. In Fig.7 it can be seen that 78% of total extrusion force is spent for friction, only 15% of it is spent for deformation and 7% of it is spent for shear at the beginning of the process. For the maximum extrusion load 66% of total load is spent for deformation, 21% of it is spent for shear and only 13% of it is spent for friction. Similar diagram is shown in Fig.8 for the cosine profiled die. In the diagram, it is seen that 82% of total extrusion force is spent for friction, only 13% of it is spent for deformation and 5% of it is spent for shear at the beginning of the process. For the maximum extrusion load, same as the straight tapered die, 66% of total load is spent for deformation, 21% of it is spent for shear and only 13% of it is spent for friction.

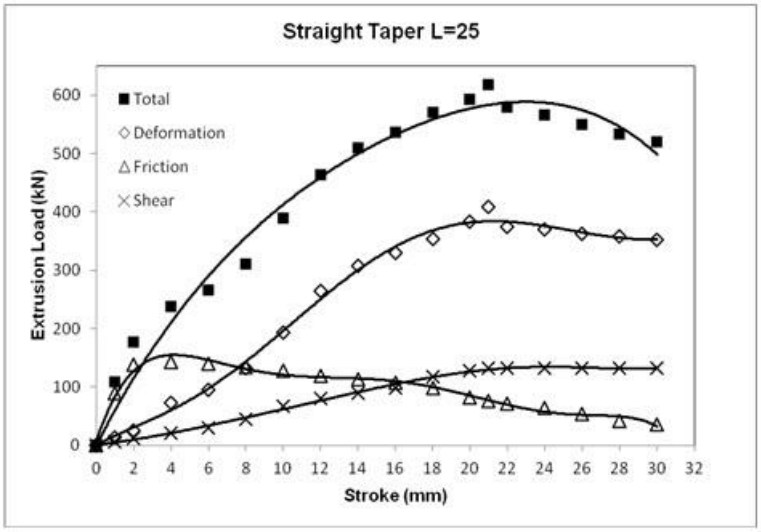

Figure 7. Relationship between stroke and components of extrusion load obtained from UB for straight taper die

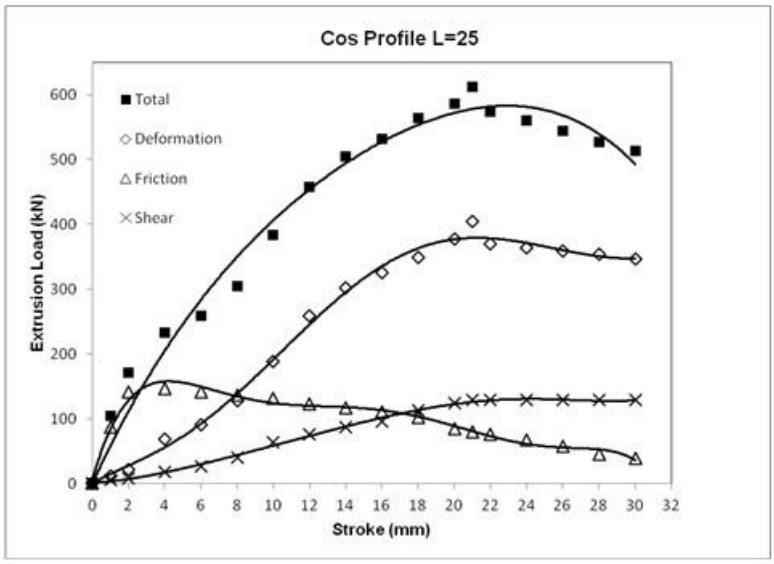

Figure 8. Relationship between stroke and components of extrusion load obtained from UB for cosine curved die

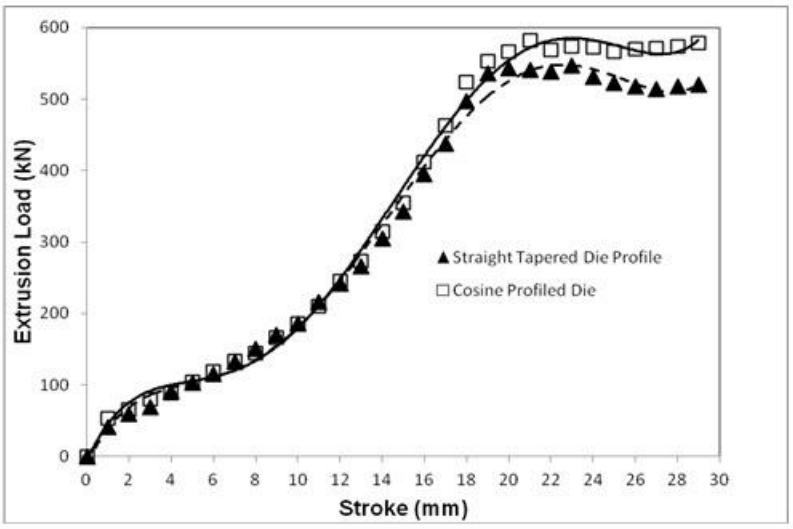

Figure 9. Comparison of experimental extrusion load against punch stroke for two different profiled dies

In order to compare the effect of die transition profile in terms of deformation load experimentally obtained results versus punch stroke is given in Fig.9. According to Fig.9 straight tapered die needs about 6.5% less deformation load at maximum load point. Because mentioned die has less interface area as compared with the cosine profiled die. Besides, higher load values for cosine profile are for only during the second half of the process.

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