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AN ITERATIVE METHOD FOR CALCULATING THE THERMAL CONDUCTIVITY OF SOIL FREEZING

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Abstract

This paper proposes an approximate method for calculating the nonlinear thermal conductivity of soil freezing. We consider the coexistence of three zones: the melt, the phase and the frozen zone. On the basis of a priori estimates of the direct and the dual problem of limited proven an approximate value of the thermal conductivity.

1. INTRODUCTION.

Nowadays the climate of the earth is changing, the globe temperature is rising, and resulting in the continuous thawing of permafrost areas' frozen soil and many countries have realized the seriousness of the problem. To ease the impact of melting of frozen soil on people's daily lives and economic construction, many countries are making great effort on the study. The cornerstone of the frozen soil affects the exchange of heat between nature and thermodynamics of frozen soil. During the passage of high heat through the frozen ground occurs temperature exchange between the environment and the soil directly through the contact surface, in this case, the earth's surface. To study this process it is necessary to know the thermodynamic parameters of frozen soil, only then we can see the heat process of frozen soil transparently. Coefficient of thermal conductivity - a rather important factor in the basic parameters of the thermodynamics of frozen soil, and this is characteristic of the soil, showing the rate of change of temperature of frozen soil. In this paper is described a method for calculating the thermal conductivity of frozen soil under the influence of ambient temperature.

2. OVERVIEW

There are many methods to determine the thermal conductivity of nonmetallic materials, which caused by huge variety of properties (structure, particle size, the structure of the skeleton), states (tight, loose, wet, dry).

The most common are methods based on creating the image of a stationary thermal conditions, because of of their relative simplicity: to determine the thermal conductivity and a sufficiently accurate measurement of heat flow through the sample and record the temperature difference between two isothermal surfaces in it [1]. In some cases determined the ratio of the thermal conductivity standard and test samples. В этом случае метод становится относительным [2]. Regime between the stationary and non-stationary is a regular thermal regime, the theory and practical implementation of which have been proposed in the works of G. Kondratiev and his students [3]. The temperature field inside the system under regular condition is found by solving the heat equation at appropriate initial

boundary conditions. Further extension of the regular regime method followed by the line of renunciation of need of constancy of heat transfer coefficient, this line of development led to the creation of devices, called biocalorimeters[4]. Transition between the initial, purely non-stationary regime and the subsequent - often stationary regime is not only regular but also quasi-stationary thermal conditions. In this regard, the regular mode is very similar to the quasi-stationary regime, and only thanks to a number of features peculiar to it, can be separated from the quasi-stationary regime.If all the methods of the regular regime assume heat exchange of the body with the environment maintained at a constant temperature, the quasi-stationary methods mean the heat exchange with the environment of the body whose temperature varies with time linearly.

Regular mode occurs after a certain period of time after the start of the heat transfer process in the system material-environment where the solution of the heat equation, obtained in a series, can be limited by one member. Quasi-stationary mode also occurs after a certain period of time after the start of the heat transfer process in the same system material-environment, when the temperature of any point of the material begins to change at a constant rate. The initiative to develop methods of quasi-stationary regime belongs to A.V. Lykov. A.V. Lykov suggested the use of the principle of quasi-stationary regime in order to find the thermal diffusivity of damp materials [5].

A method described here for determining the thermophysical characteristics of the material is based on the registration process of attenuation of thermal waves in the material on the surface which is created by the experimental temperature wave. This wave is transmitted through the process heat into the body's depth with delay in phase and with reduced amplitude. In this case is suggested that the process of moving waves continues for so long that the initial temperature distribution ceases to affect its course. This affects the nature of quasi-stationary thermal regime described by the solution. Such a task is characterized by the absence of initial conditions.

Method for determination of thermal diffusivity of a long and narrow rod, at the ends of which temperature was changing periodically, proposed by Angstromom[6]. Heating (heating with steam) and cooling (water flow) was carried out at regular intervals. After some time, at any point of rod was set a periodic temperature distribution, independent from the initial distribution. Registration of the temperature with time at two neighboring points x1 and x2 allows to find the thermal diffusivity.

Non-stationary methods for determining the thermo-physical characteristics of materials based on the theory of solving the heat equation with non-stationary heat flux. Problems for solving inverse problems of unsteady heat conduction equation studied by many scientists, in particular Alifanov OM, Artyukhin EA, Rumyantsev SV [7] J. Beck, B. Blackwell, St. Clair Charles [8], Kabanikhin SI, Bektemesov MA, AT Nurseitova [9].

Coefficient of thermal conductivity is determined by the composition of the soil structure and texture of the rock. There are numerous experimental data, which determined the thermal conductivity of different materials in the laboratory, and compiled tables of thermal conductivity of the basic materials and rocks, often occurring in practice and in industry. But still have not been developed sufficiently reliable method for determining the characteristics of the soil in the field.

In the study of heat transfer in complex systems such as soils, generally, you can go two ways, "microscopic," consisted in analyzing the mechanism of the phenomenon, and "macroscopic" based on the phenomenological description of the phenomenon, which distracted from the discrete structure of the soil and treat it as a solid body, characterized by the average "macroscopic" parameters.

"Macroscopic" approach allows to describe the thermal regime of freezing and thawing of soil the most simply, i.e., to answer the question, the most interesting for Geocryology. Therefore, this work is devoted to the processes of heat conduction, namely from those positions.

3. FORMULATION OF THE PROBLEM

In order to solve the problem, namely, to determine the thermal conductivity of thawed, frozen product and to determine the change λ in the zone of phase transformation of the soil, it is necessary to solve the following problem /10,11/:

$$\gamma_0 c(z)\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_n(\theta)\frac{\partial\theta}{\partial z}\right),\ 0<z<H, \tag{1}$$

$$\left.\frac{\partial\theta}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial\theta}{\partial z}\right|_{z=H}=0, \tag{2}$$

$$\theta|_{t=0}=\theta_0(z),\ 0\le z\le H, \tag{3}$$

$$\left[\lambda_n\frac{\partial\theta}{\partial z}\right]_{z=h(t)}=p\frac{dh}{dt},\ [\theta]_{z=h(t)}=0,\ \ p=q_0\gamma_0\,\Delta W_i\text{, при } z=h(t), \tag{4}$$

$$\left[\lambda_n\frac{\partial\theta}{\partial z}\right]_{z=h_1(t)}=0,\ [\theta]_{z=h_1(t)}=0\ . \tag{5}$$

Also use additional information on the part of the boundary, namely, the measured temperature of soil at the surface $T_g(t)$. Here $\theta(z,t)$ -the temperature of the soil, $T_b(t)$ -air temperature. The parameters in (1)-(5): $\gamma_0, c(z), \lambda(\theta)$ - respectively specific gravity, heat coefficient and thermal conductivity of the soil.

Problem (1)-(5) is solved in the region $Q=(0,H)\times(0,T)$. The temperature of the soil θ depends on two variables, *z* and *t*. Where $z\in(0,H),\quad t\in(0,T)$. Axis Oz is directed upwards, the origin of coordinates is at the low-lying layer of soil. The boundaries of the transition from one zone to another are determined by the temperature basis, i.e. $\theta(h(t),t)=\theta_t$ boundary of melt and phase zones; $\theta(h_1(t),t)=\theta_f$ boundary of phase and frozen zones. Parameters present in (1) are functions of z and θ, that is

$$c=c(z),\gamma_0=\gamma_0(z),\ \lambda=\lambda(\theta).$$

We will consider the frequent case in practice /12/

$$\lambda(\theta) = A(z) + B(z)\theta .$$

Required to determine the functions $A(z)$ и $B(z)$.

3.1 Differential scheme

Interval (0,H) is divided into N equal parts with a step $\Delta z = H / N;$ and interval (0,T) is divided into m equal parts with a step Δt=T/m. Then the area $Q = (0,H)\times(0,T)$ goes into the grid

$$Q_N^m = \left\{ z_i = i \cdot \Delta z;\ i = 0,1,2,...,N;\ \ t_j = j \cdot \Delta t,\ \ \mathrm{j} = 0,1,2,...,m \right\}.$$

Grid analogue of $\theta(z_i t_j)$ is denoted by Y_i^j, and grid analogue of $\theta(z_i, t_j + \Delta t)$ is denoted by Y_i^{J+1}.

In the area Q_N^m we consider the problem

$$\gamma_0 C(z_i)\frac{Y_i^{J+1} - Y_i^J}{\Delta t} = \frac{1}{\Delta z}\left(\lambda_{i+1}^j \frac{Y_{i+1}^{J+1} - Y_i^{J+1}}{\Delta z} - \lambda_i^j \frac{Y_i^{J+1} - Y_i^{J+1}}{\Delta z} \right), \qquad (6)$$

$$Y_0^{j+1} = 0,\ \ \lambda_N^j \frac{Y_N^{j+1} - Y_{N-1}^{j+1}}{\Delta z} = -\alpha(Y_N^{j+1} - T_b^{j+1}) , \qquad (7)$$

$$Y_i^0 = \theta_0(z_i),\ \ i = 0,1,2,..., N . \qquad (8)$$

The coefficient of thermal conductivity $\lambda(\theta)$ is determined iteratively. Number of iterations is denoted by n Given the initial approximations $A_n(z_i)$ and $B_n(z_i)$. The corresponding solution of the system (6)-(8) is denoted by $Y_{ni}^{j+1} = Y^n$. We introduce the notation $\lambda_n = A_n(z_i) + B_n(z_i)Y_i^{j+1}$. For the function $Y^{n+1} - Y^n = \Delta Y_i^{j+1}$ we obtain the difference problem

$$\gamma_0 c \Delta Y_{i\bar{t}}^{j+1} = \left(\Delta\lambda Y_{n+1,i,z}^{j+1} + \lambda_{i+1}\Delta Y_{iz}^{j+1} \right)_{\bar{z}}, \qquad (12)$$

$$\Delta Y_0 = 0 , \quad \Delta\lambda_N Y_{n+1,N,\bar{z}}^{j+1} + \lambda_N \Delta Y_{N\bar{z}}^{j+1} + \alpha \Delta Y_N^{j+1} = 0 , \ \ \Delta Y_i^0 = 0 , \qquad (13)$$

where $\Delta\lambda = \lambda_{n+1} - \lambda_n$. We multiply (12) to an arbitrary grid function $U_i^j \Delta t \Delta z$ and sum by internal points of the grid Q_N^m /11/. Then, after a single application of the formula for summation by parts in variable i and j taken the relation:

$$\sum_{i=1}^{N-1} \gamma_0 C \Delta Y_i^m U_i^m \Delta z - \sum_{i=1}^{N-1} \gamma_0 C \Delta Y_i^0 U_i^0 \Delta z - \sum_{i=1}^{N-1} \sum_{j=0}^{m-1} \Delta Y_i^{j+1} \gamma_0 C U_{i\bar{t}}^{j+1} \Delta z \Delta t =$$

$$= \sum_{j=0}^{m-1} \left(\Delta\lambda_N Y_{n+1,N,\bar{z}}^{j+1} + \lambda_N \Delta Y_{N\bar{z}}^{j+1} \right) U_N^j \Delta t - \sum_{j=0}^{m-1} \left(\Delta\lambda_1 Y_{n+1,1,\bar{z}}^{j+1} + \lambda_1 \Delta Y_{1\bar{z}}^{j+1} \right) U_0^j \Delta t -$$

$$\sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta\lambda Y_{n+1,i\bar{z}}^{j+1} U_{i\bar{z}}^j \Delta t \Delta z - \sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta Y_{i\bar{z}}^{j+1} \lambda_i^j U_{i\bar{z}}^j \Delta t \Delta z .$$

Consider that $U_i^m = 0$ and $U_0^j = 0$. Taking into account initial-boundary conditions (13) we obtain the equality:

$$-\sum_{i=1}^{N-1} \sum_{j=0}^{m-1} \Delta Y_i^{j+1} \gamma_0 C U_{i\bar{t}}^{j+1} \Delta z \Delta t = -\sum_{j=0}^{m-1} \alpha \Delta Y_N^{j+1} U_N^j \Delta t -$$

$$\sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta\lambda Y_{n+1,i\bar{z}}^{j+1} U_{i\bar{z}}^j \Delta t \Delta z - \sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta Y_{i\bar{z}}^{j+1} \lambda_i^j U_{i\bar{z}}^j \Delta t \Delta z .$$

Again applying formulas for summation by parts to the last sum, we conclude that

$$\sum_{i=1}^{N-1} \sum_{j=0}^{m-1} \Delta Y_i^{j+1} \left(\gamma_0 C U_{i\bar{t}}^{j+1} + \left(\lambda_i^j U_{i\bar{z}}^j \right)_z \right) \Delta z \Delta t = -\sum_{j=0}^{m-1} \alpha \Delta Y_N^{j+1} U_N^j \Delta t - \sum_{j=0}^{m-1} \sum_{i=1}^{N} \Delta\lambda Y_{n+1,i\bar{z}}^{j+1} U_{i\bar{z}}^j \Delta t \Delta z -$$

$$-\sum_{j=0}^{m-1} \Delta Y_N^{j+1} \lambda_N^j U_N^j \Delta t .$$

Grid function U_i^j is selected the way that next equality will be possible

$$\gamma_0 C U_{i\bar{t}}^{j+1} + \left(\lambda_i^j U_{i\bar{z}}^j \right)_z = 0 .$$

Then

$$\sum_{j=0}^{m-1}\left(\lambda_N^j U_{N\bar{z}}^j + \alpha U_N^j\right)\Delta Y_N^{j+1}\Delta t = -\sum_{j=0}^{m-1}\sum_{i=1}^{N}\Delta\lambda Y_{n+1,i\bar{z}}^{j+1}U_{i\bar{z}}^j\Delta t\Delta z\,.$$

Assuming that

$$\lambda U_{N,\bar{z}}^j + \alpha U_N^j = -2\left(Y_N^{j+1} - T_g^{j+1}\right)$$

we obtain an important relation necessary for further study. That is,

$$2\sum_j \Delta Y_N\left(Y_N^{j+1} - T_g^{j+1}\right)\Delta t = -\sum_{i,j}\left(\Delta A_i + \Delta\left(Y_i^j * B_i\right)\right)Y_{n+1,i\bar{z}}^{j+1}U_{\bar{z}}^j\Delta z\Delta t \qquad (14)$$

and the conjugate problem:

$$\gamma_0 c U_{i\bar{t}}^{j+1} + \left(\lambda U_{iz}^j\right)_z = 0,\ i = 1,2,...,N-1;\ j = m-1,m-2,...,1; \qquad (15)$$

$$U_i^m = 0,\ i = 1,2,...,N; \qquad (16)$$

$$U_{1\bar{z}}^j = 0,\ \lambda U_{N,\bar{z}}^j + \alpha U_N^j = -2\left(Y_N^{j+1} - T_g^{j+1}\right),\ j = m-1, \text{m - } 2,...,0\,. \qquad (17)$$

Coefficient of thermal conductivity of multilayer soil is determined from the minimum of the functional:

$$J(\lambda) = \sum_{j=1}^{m}\left|T_g^{j+1} - Y_N^{j+1}\right|^2\Delta t,$$

where T_g^{j+1} – the measured soil temperature at the earth's surface, and Y_N^{j+1} – the estimated value of soil temperature on the surface of the earth. Taking into account (14) for the difference of functional formed the formula:

$$J(\lambda_{n+1}) - J(\lambda_n) = -\sum_j\left(\Delta Y_N\right)^2\Delta t + \sum_{i,j}\Delta A_i Y_{\bar{z}}^{j+1}U_{\bar{z}}^j\Delta z\Delta t + \sum_{i,j}\Delta\left(Y_i^j * B_i\right)Y_{\bar{z}}^{j+1}U_{\bar{z}}^j\Delta z\Delta t\,.$$

If

$$\Delta A_i = -\beta_1\sum_{j=0}^{m-1}Y_{\bar{z}}^{n+1}U_{\bar{z}}^{j,n+1}\Delta t,\quad \Delta\left(Y_i^j * B_i\right) = -\beta_2 Y_{\bar{z}}^{n+1}U_{\bar{z}}^{j,n+1}$$

then

$$J(\lambda_{n+1}) - J(\lambda_n) = -\beta_1 \sum_i \left(\sum_J Y_{\bar{z}}^{n+1} \overline{U}_{\bar{z}}^{n+1} \Delta t \right)^2 \Delta z - \beta_2 \sum_{i,j} \left(Y_{\bar{z}}^{n+1} \overline{U}_{\bar{z}}^{n+1} \right)^2 \Delta z \Delta t +$$

$$+ \sum_j (\Delta Y_N)^2 \Delta t + \sum_{i,j} \Delta\lambda \Delta_i Y_{i\bar{z}}^{j+1} U_{\bar{z}}^{j} \Delta z \Delta t \,. \qquad (18)$$

Where β_1, β_2 - sufficiently small values gleaned from the condition of convergence of the iterative process. Then, the approximate value of the thermal conductivity of soil is defined as:

$$A_{n+1}(h_k) - A_n(h_k) = -\frac{\beta_1}{h_{k+1} - h_k} \sum_{i,j} Y_{i,\bar{z}}^{n+1} \overline{U}_{i,\bar{z}}^{n+1} \Delta t \Delta z$$

$$B_{n+1}(h_k) = B_0(h_k) \frac{\sum_i Y_i^0 \Delta z}{\sum_i Y_i^j \Delta z} - \frac{\beta_1}{\sum_i Y_i^j \Delta z} \sum_n \sum_i Y_{i,\bar{z}}^{n+1} \overline{U}_{i,\bar{z}}^{n+1} \Delta z, \quad k = 1,2,3. \qquad (19)$$

Here $d_1 = h_2 - h_1$ - thickness of the melt zone; $d_2 = h_3 - h_2$ - thickness of the phase zone; $d_3 = h_4 - h_3$ - thickness of the frozen zone. And $h_1 = 0$, $h_4 = H$ and $d_k > 0$, k = 1,2,3. In (19) summation is carried out with respect to each zone separately.

In this paper have been proven:

Theorem 1. If $\theta_0(z) \in W_2^1(0,H), T_g(t), T_b(t) \in W_2^2(0,T)$, then for the solution of problems (6)-(11) next estimation is appropriate

$$\max_j \left(\|Y_t\|^2 + \|Y\|^2 \right) + \sum_j \left(\left\| \sqrt{\lambda} Y_{\bar{x}t} \right\|^2 + \left\| \sqrt{\lambda} Y_{\bar{z}} \right\|^2 \right) \Delta t \le \left\| Y_t^0 \right\|^2 + \|\theta_0\|^2, \quad \max_i \|Y_t\| \le \max_i \left| Y_t^0 \right|.$$

Theorem 2. If $\theta_0(x) \in W_2^1(0,H)$, $T_g(t), T_b(t) \in W_2^2(0,T)$, then for the solutions of problems (15)–(17) we have estimation

$$\max_j \left(\| U \|^2 + \|U_t\|^2 \right) + \sum_j \left(\left\| \sqrt{\lambda} U_{\bar{z}} \right\|^2 + \left\| \sqrt{\lambda} U_{\bar{z}t} \right\|^2 \right) \Delta t \le C_1 \left(1 + \sum_i \frac{\Delta z}{\lambda(z_i)} \right).$$

Theorem 3. If $\theta_0(x) \in W_2^1(0,H), T_g(t), T_b(t) \in W_2^2(0,T)$, then iteration process(19) converges and we have the estimates

$$0 < C_2 \le A_{n+1}(z_k) \le C_3 < \infty, \; 0 \le C_4 \le B_{n+1}(z_k) \le C_5 < \infty, \quad k = 1,2,3.$$

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