Journal of International Scientific Publications:
Materials, Methods & Technologies, Volume 6, Part 1

MODELING PHOTONIC CRYSTAL WAVEGUIDES WITH NONCIRCULAR GEOMETRY USING GREEN FUNCTION METHOD

I. Uvarovaa[1], B. Tsyganok[1], Y. Bashkatov[1], V. Khomenko[2]

[1]National Technical University of Ukraine "Kiev Polytechnic Institute", Peremohy avenue, 37, Kiev, 03056,Ukraine.

[2]Institute of Physics, National Academy of Science of Ukraine, Nauky Avenue, 46, Kyiv, 03028, Ukraine

Abstract

Currently in the field of photonics is an acute problem fast and accurate simulation photonic crystal waveguides with complex geometry. This paper describes an improved method of Green's functions for non-circular geometries. Based on comparison of selected efficient numerical method for finding the eigenvalues for the Green's function method for non-circular holes chosen effective method for our purposes. Simulation is realized in Maple environment. The simulation results confirmed experimentally.

Key words: *photonic crystal, waveguide, modeling, Green function, complex geometry.*

1. INRODUCTION

Photonic-crystal fibers (PCF) - is a relatively new class of optical fibers, using the properties of photonic crystals. In cross-section of the PCF is a quartz or glass microstructure with periodic or aperiodic system or micro-inclusions of micro cylindrical shape oriented along the fiber axis. The defect is the core of the fiber, providing a waveguide mode of propagation (fig. 1). The ability of the PCF to hold and direct the light depends on many physical and geometrical parameters, so their creation have additional abilities for controlling the characteristics of light propagating within the fiber.

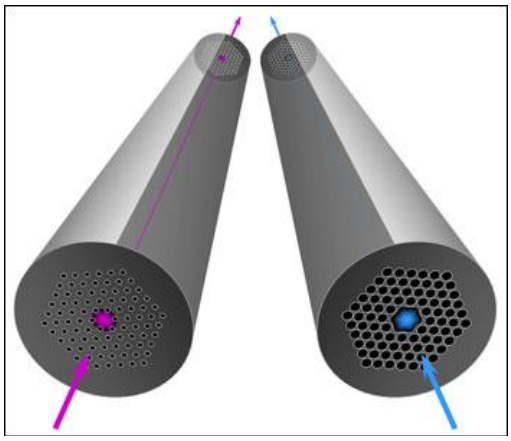


Fig. 1. Photonic crystal fiber

Practical interest to optical waveguides and fibers gave rise to the need to develop methods for their study, allowing numerically analyze the properties of existing models and to predict the prospects of using synthesized by means of computer simulation of waveguide structures.

Historically, the first method used for calculating the modes of the PCF, has become the method of effective index. Although significant progress has been made, the rigorous analysis of light propagation in PCF remains problematic because of the large index contrast, the vectorial nature of the Maxwell equations, and the complicated cross-sections of the hole geometries involved. Today there are several methods to calculate the modes of the PCF. All of them can be divided into three groups: approximate analytical or decomposition methods, integral methods and finite difference methods.

The basic idea exploited in decomposition methods - is the possibility of representing the mode field of the fiber in the form of an expansion in some basis. But it is possible only in simple cases. Integral methods are the grid, that is, in contrast to the methods of the previous group, in this case the solution of the problem of finding the mode field is a grid function, and not given analytically. Finite difference methods, as well as integral methods, give grid solution.

In this work we consider type of integral methods – Green functions method. In the method of Green's function, the problem of finding the propagation constants of modes can also be reduced to a problem on the eigenvalues of the matrix, which is designed to address the special fast algorithm. This method works in the case of complex geometric shapes of PCF, albeit at a slower rate of convergence than in the case of circular holes.

The main goal of this paper is to build effective algorithm for analysis complex geometric shapes of PCF. For this purposes we used method of integral equations for mathematical formulation wave propagation in PCF. For solving this mathematical equation we use Green function. And for finding eigenvalues we used QR-algorithm.

2. MODELING

2.1. Formulation of equations for modeling

For the modal analysis of PCF, we assume the longitudinal axis is the *z*-axis, and that the structure of the PCF is defined by its cross-section in the *xy*-plane. Section of PCF is shown in Fig. 1. More precisely, the fiber consists of a glass matrix denoted by V_0 with refractive index *n* which is perforated by a finite number of air holes with non-circular geometry denoted by V_1-V_6.

Propagation mode can be obtained as vectors electric and magnetic field in *z* direction:

$$E(x,y,z,t) = E^{i(\beta z-\omega t)}\left(\frac{i\omega\mu}{k^2-\beta^2}\partial_y H_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_x E_3^\beta, \frac{-i\omega\mu}{k^2-\beta^2}\partial_x H_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_y E_3^\beta, E_3^\beta\right),$$

$$H(x,y,z,t) = H^{i(\beta z-\omega t)}\left(\frac{-i\omega n^2\varepsilon}{k^2-\beta^2}\partial_y E_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_x H_3^\beta, \frac{i\omega n^2\varepsilon}{k^2-\beta^2}\partial_x E_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_y H_3^\beta, H_3^\beta\right),$$

where ω is a frequency, β – propagation constant, ε,μ - permittivity and permeability of free-space.

Assuming above mentioned and continuity of tangential components of the **E** and **H** fields we can summarize the eigenvalue problem which determines a mode with propagation constant b as follows:

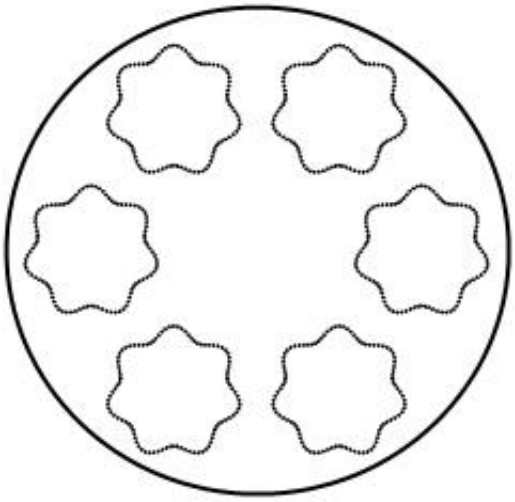


Fig. 2. Cross-section of PCF

Propagation mode can be obtained as vectors electric and magnetic field in *z* direction:

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$$H(x,y,z,t) = H^{i(\beta z-\omega t)}(\frac{-i\omega n^2\varepsilon}{k^2-\beta^2}\partial_y E_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_x H_3^\beta, \frac{i\omega n^2\varepsilon}{k^2-\beta^2}\partial_x E_3^\beta + \frac{i\beta}{k^2-\beta^2}\partial_y H_3^\beta, H_3^\beta),$$

where ω is a frequency, β – propagation constant, ε,μ - permittivity and permeability of free-space.

Assuming above mentioned and continuity of tangential components of the **E** and **H** fields we can summarize the eigenvalue problem which determines a mode with propagation constant b as follows:

$$\nabla^2 E + (k^2-\beta^2)E = 0$$

$$\nabla^2 E + (k^2-\beta^2)E = 0$$

$$[E] = 0$$

$$[H] = 0$$

$$\left[\frac{\beta}{k^2-\beta^2}\frac{\partial H}{\partial t}\right] = -\left[\frac{k^2/k_v}{k^2-\beta^2}\frac{\partial E}{\partial n}\right]$$

All equations can be found in standard papers that deal with modeling PCF using integral method.

2.2 Representation of Helmholtz equation using Green function

Using Green function for Helmholtz equation ($G_j(P,Q) = \frac{i}{4}H_0^1(\sqrt{k_j^2-\beta^2}\,\|P-Q\|)$) we can consider electric and magnetic field inside

$$E_j(P) = \int_{\Gamma_i} (s^E(2,j) G_j(P,Q) \sigma_j^E(Q) + d^E(2,j) \frac{\partial G_j(P,Q)}{2n_Q} \mu_j^E(Q)) ds_Q$$

$$H_j(P) = \int_{\Gamma_i} (s^E(2,j) G_j(P,Q) \sigma_j^H(Q) + d^H(2,j) \frac{\partial G_j(P,Q)}{2n_Q} \mu_j^H(Q)) ds_Q$$

and outside holes

$$E_0(P) = \sum_{j=1}^{M} \int_{\Gamma_i} (s^E(2,j) G_j(P,Q) \sigma_j^E(Q) + d^E(2,j) \frac{\partial G_j(P,Q)}{2n_Q} \mu_j^E(Q)) ds_Q$$

$$H_0(P) = \sum_{j=1}^{M} \int_{\Gamma_i} (s^E(2,j) G_j(P,Q) \sigma_j^H(Q) + d^H(2,j) \frac{\partial G_j(P,Q)}{2n_Q} \mu_j^H(Q)) ds_Q$$

Here, M denotes the number of holes.

After this equation we can write integral equations for modeling. They are bulky and need merely to write a computer algorithm in the program, which implement a simulation. We are not going to direct them because of their uselessness for the presentation of material.

2.3. Finding modes and eigenvalues

Next task is to find eigenvalues of β. For noncircular geometries this task can be difficult and solves only using some specific methods to resolve this problem.

In many papers using the method of Muller, which significantly increased the computer time and have a negative impact on the convergence of the method.

We use to find the QR-method to find both the real and complex eigenvalues. In the QR-algorithm based on the idea of the matrix in the form A=QR, where Q- an orthogonal matrix, and - R an upper triangular.

3. NUMERICAL RESULTS

3.1. Dispersion curves

To ensure the most accurate and actual results the parameters have been chosen one of the most well-known fiber Thorlabs HC-800B:

- Center Wavelength: 820 nm
- Mode Field Diameter: 5.5 µm
- Numerical Aperture: ~0.20
- Effective Mode Index: ~0.99
- Attenuation: <0.3 dB/m
- Bandwidth: 770 - 870 nm
- Core Diameter: 7.5 ± 1 µm
- Air Fill (in Holey Region): >90%

- Diameter of Holey Region: 45 µm
- Silica Cladding Diameter: 130 ± 5 µm
- Coating Diameter (Fiber O.D.): 220 ± 50 µm.

Resulting dispersion curves are shown on Fig.3 and Fig.4.

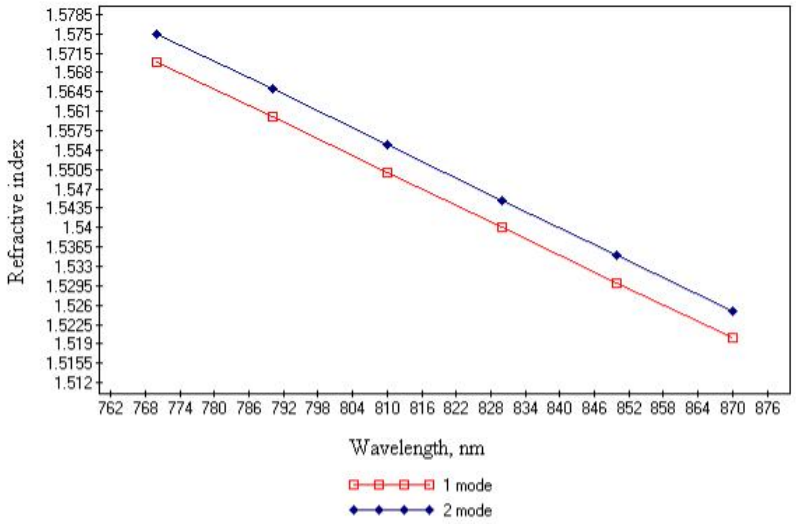


Figure 3. Dependence real part of refractive index on wavelength obtained by effective Green method.

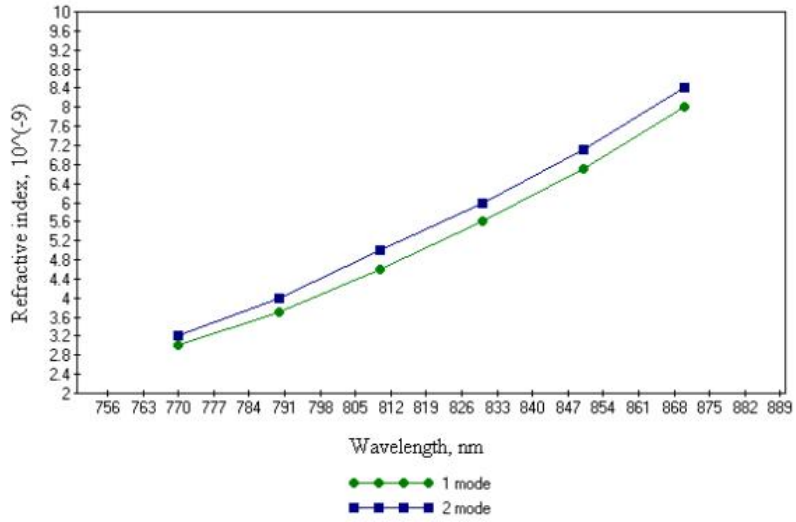


Figure 4. Dependence imaginary part of refractive index on wavelength obtained by effective Green method.

According to the charts, you can easily determine the dispersion relations and the fiber length for the zero-dispersion wavelength. The dependence of the real part of refractive index on the wavelength has

the form obtained by the formula Sellmeyers formula. It is easy to see that zero-dispersion wavelength is equal 816 nm that is near to the value given in datasheet.

3.2. Spectrum

Obtained spectrum is shown on Fig. 5.

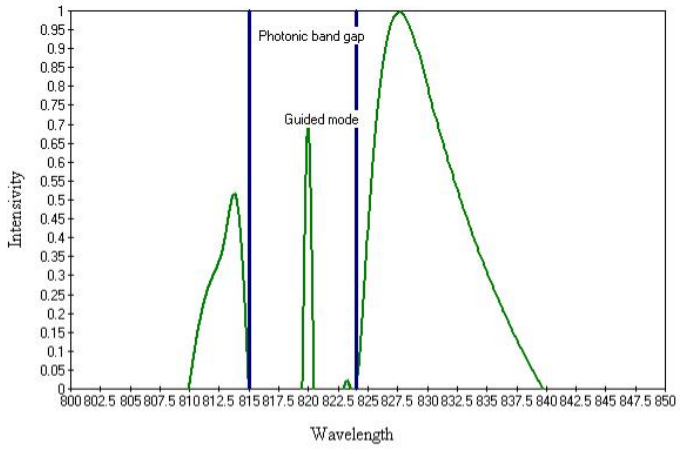


Figure. 5. Spectra in the output of PCF.

From this spectrum we can easily determine the width of the PBG - 9 nm. It is a characteristic value for the PCF. It is also easy to see the defect mode, which is spread signal.

3.3.Convergence

To obtain accuracy in modeling optical objects we need to investigate convergence. Figure 6 represented the dependence of errors in refractive index calculating on a number point per hole.

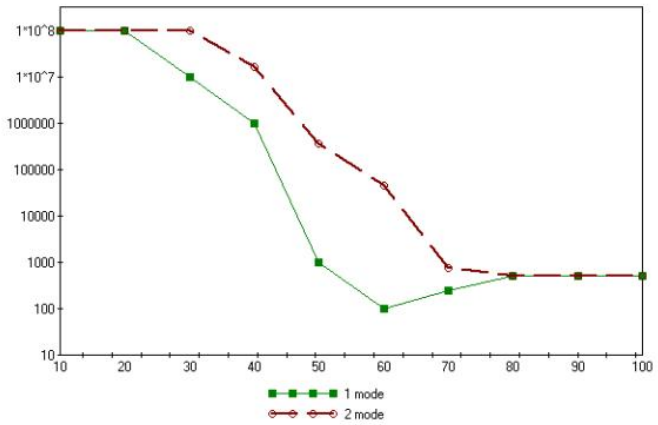


Figure 6. Convergence study of our method: dependence of refractive index errors (10^{-12}) on number of points.

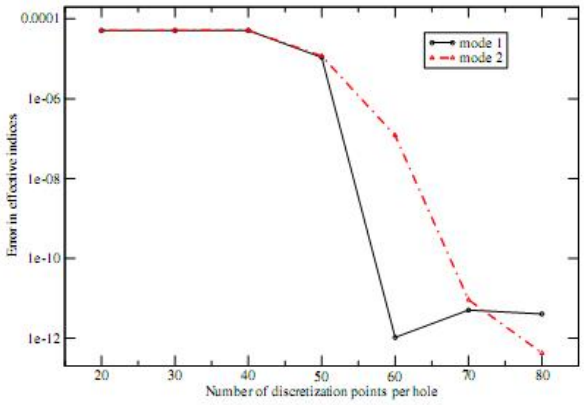


Figure 7. Convergence curves obtained by scientist of Madmax Optics Inc:H. Cheng, W.Y. Crutchfield, M. Doery and L. Greengard

It is shown that choose 70 and higher number of points per hole as modeling discretization parameter can obtain reliable results of calculating refractive index and simulation. That curves has visible lower values that the same curves shown in familiar works (Fig. 7).

4. EXPERIMENTAL RESULTS AND COMPARISON

4.1. Experimental setup and spectrum

For experimental comparison we used experimental set up at Institute of Physics NASU, which is used for generation supercontinuum (SC) radiation in PCF with non circular geometry.

The laser radiation was supplied by a femtosecond laser complex (Coherent Mira Optima900-F) at the Institute of Physics of the NASU.A typical scheme of the experimental setup is shown in Fig.8. Experimental results of generation are shown on fig. 9.

4.2. Comparison

For obtain reliability of modeling results we have to compare modeled SC and experimental SC. Curves are shown on fig. 10.

It is shown that nonlinearity coefficient and pulse power all produce broader supercontinuum. Pumping the laser at a wavelength around the zero dispersion velocity (ZDV) of the fiber means that dispersive effects are small compared to nonlinear ones. At a wavelength lower than the ZDV of the fibred, the pulse does not experience much broadening. So it would be better to pump the laser into the fiber at a wavelength just above the ZDV of the fibred.

It can be seen from Fig. 10 that higher nonlinearity coefficients cause greater break-up of the pulse. Higher pulse powers mean that the nonlinear effects are more emphasized.

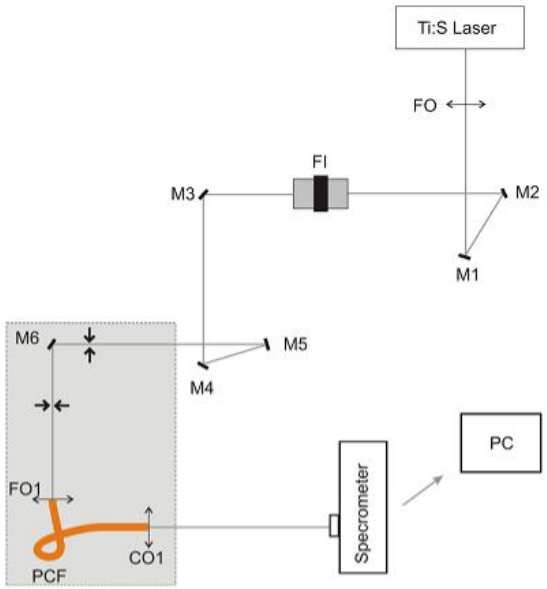


Fig. 8.SC generation scheme: a femtosecond Ti:S laser900Mira Optima-F, FI–Faraday optical isolator, M – mirrors, PCF – photonic-crystal fiber, FO–focusing lenses, CO–collimating lens, PC–personal

computer.

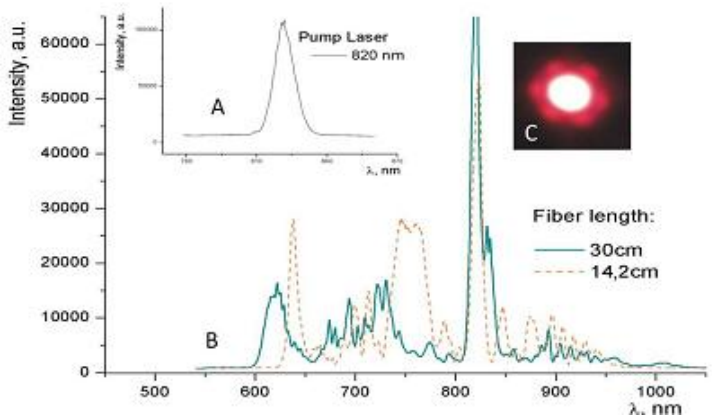


Fig. 9. Emission spectra of a supercontinuum generated by femtosecond pulses of a Ti: Sa laser at a wavelength of 820 nm, a power of 500 mW (A) in microstructured photonic-crystal fibers: Thorlabs NL–2.4–800 with lengths of 14.2 cm and 30cm (B). Insertion(C) experimentally recorded intensity distribution of radiation leaving a fiber.

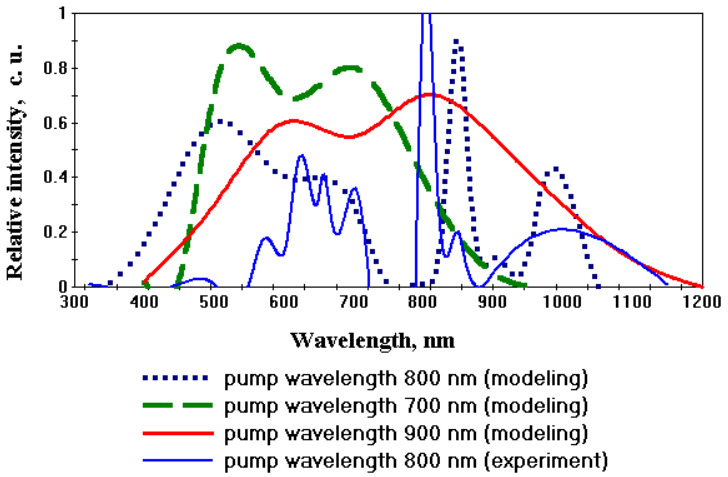


Figure. 10. Results of modeling output spectra of PCF fiber and comparison with experimental curve.

5. CONCLUSIONS

Rigorous analysis of light propagation in PCFs remains problematic. For fast and precise analysis of light propagation in PCF we built numerical method based on Green functions. This method has convergence problem when modeling complex noncircular geometries of holes. But we applied some numerical and mathematical approaches solve this problem.

Using this method it can be easily obtained dispersion curves and spectra.

ACKNOWEDGHEMENT

The authors acknowledge help reserchers of Institution of Physics NASU Department Quantum and Coherent Optics and Department of Radiation Processes(especially Natalia Kachalova, Valerii Voitsekhovich and Boris Danilchenko).

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