

STABILITY OF A THIN ELASTIC PLATE AT UNIFORM CORROSION

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Abstract

The study is devoted to investigation of stability loss of a thin-walled rectangular elongated plate, subjected to simultaneous action of permanent longitudinal compressive forces and uniform surface corrosion. The plate is considered as a thin elastic isotropic body with simply supported butt ends on the long side. Subcritical stress state in the plate is described within Kirchhoff-Love approximation and it is membrane-like. The critical time of stability loss of the plate is found. It is shown that the critical time can be established if the solution of the respective problem of the static stability loss for the plate, not subjected to corrosion, and law of corrosion rate are known. Numerical results are received for plates, made from steel and aluminum, with different initial wall thickness.

Key words: corrosion, elastic plate, longitudinal load, stability, critical stress

1. INTRODUCTION

Corrosion is a common phenomenon in engineering leading to deterioration of mechanical properties of materials and even failure, when the load exceed the limiting value specific for the given element at the existing corrosion state (Nicastro (1997), Minoshima et al. (2005)). The phenomenon was widely studied both experimentally and theoretically; the common difficulty in experimental analysis is that it is too time-consuming (Zhou et al. (2007)). Therefore, much thought is being given to modeling of corrosion processes and its effect in different conditions. Typical problems come from civil and mechanical engineering, and among them behavior of corroded reinforced concrete, stress corrosion cracking, corrosion effect on buckling of airframes and ship parts, etc (Vua et al. (2009), Grinfeld (1998), Gutman et al. (2000), Slater et al. (2000)). Increased interest to different aspects of the corrosion impact on the mechanical properties of implants is recognized in modern medicine (Ziębowicz and Marciniak (2006), Maguirea et al. (2005)). Theoretical estimates of mechanical properties of corroded construction elements have great importance for evaluation of their service life and timely repairs. In many cases engineering analyses is based on rather simple but useful models, one of which is an elastic plate subjected to various loads, particularly, longitudinal compressive forces (Donnel (1976)). Usual assumption is that compressive forces are distributed uniformly along the width of the plate, which can be supposed small in comparison with its length. Stability problem for such rectangular plates at uniform compression belongs to classical problems of the theory (Volmir (1963), Ventsel and Krauthammer (2001)). However, the theory doesn't account for the fact that the plate in the process of structure operation can be subjected to the action of corrosion, which changes its thickness and can cause stability loss before the breaking stresses in the plate are reached. To allow for this effect it is necessary to modify the classic criterion of the plate stability loss.

Effect of uniform corrosion on stability loss of a thin-walled circular cylindrical tube was investigated by Bergman et al. (2006). It was shown, in particular, that life-time of the tube in these conditions is completely defined by the classic criterion of stability loss of elastic pipe, modified with the account for corrosion rate law, suggested in (Gutman (1994)). In the present paper the method, developed in (Bergman et al. (2006)), is used for finding of the critical time of stability loss for thin-walled rectangular elongated plate, subjected to simultaneous action of permanent longitudinal compressive forces and uniform corrosion.

2. PROBLEM DESCRIPTION

Corrosion is considered below as a process of uniform dissolution of the plate surface, resulting in a decrease of its thickness h with time t . The rate of this decrease is equal to the rate of corrosion. Experimental data for various metals and steels, received for pure elastic stressed state, have shown that corrosion rate depends essentially from the stressed state of the material, which in our case is characterized by longitudinal compressive stress σ . It is changed with time due to wall thickness decrease. As a result, corrosion rate v follows the relation $v = f[\sigma(t)]$. It was shown in (Gutman (1994)) that this dependency with a good accuracy can be approximated by exponential function, including experimentally established parameters.

The stability problem for a thin-walled rectangular elongated plate subjected to simultaneous action of permanent longitudinal compressive forces and uniform surface corrosion is studied below within Kirchhoff-Love approximation. The formulas for determining of the critical time, corresponding to the moment of stability loss, will be obtained.

3. STABILITY LOSS OF THE PLATE IN THE ABSENCE OF CORROSION

Let consider in linear approximation the problem of stability loss for rectangular plate of the length a , width b and wall thickness $h(t)$, elongated in x -direction ($a \gg b$), subjected to uniformly distributed compressive longitudinal forces with intensity N (Fig. 1).

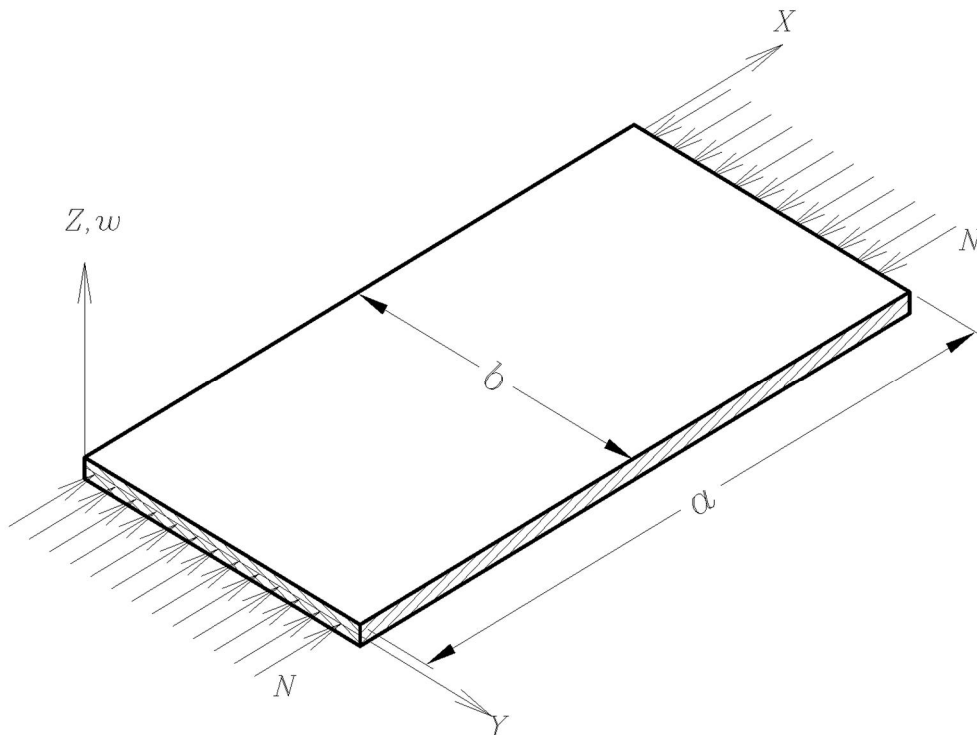


Fig. 1. The loading of the plate.

Assuming that changes of the wall thickness $h(t)$ are quasi-static, the equations of static stability loss can be applied. It is assumed also that the plate is simply supported along the long butt ends and it is possible to neglect by the boundary conditions on the short sides (Volmir (1963)). The initial stress state of the shell is supposed to be a membrane-like. Then the governing equation for stability loss of the plate within Kirchhoff - Love approximation can be written in the form (Goldenveizer (1961)):

$$\frac{E h^2}{12(1-\nu^2)} \nabla^4 w + \sigma_x \left(\frac{\partial^2 w}{\partial x^2} \right) = 0 \quad (1)$$

where $\sigma_x = \frac{N}{h}$ is the longitudinal compressive stress. In Eq. (1) E and ν are the Young modulus and Poisson coefficient, respectively; x and y - rectangular coordinates of the plate middle surface and w - its normal deflection. Differential operator ∇^4 is defined by the expression:

$$\nabla^4 f = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 f \quad (2)$$

where $f = f(x, y)$ is a function having partial derivatives of the corresponding order. Boundary conditions for simply supported butt ends on the long sides of the plate have the form:

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \quad (y = 0; y = b) \quad (3)$$

The boundary conditions (3) are satisfied automatically, if the solution of Eq. (1) is searched in the form:

$$w = c \cdot \sin \frac{\pi x}{l} \cdot \sin \frac{n y}{b} \quad (4)$$

where n is the number of half-waves along the short side of the plate. It is assumed that the wave formation in elongated plate along x -direction is free, and the half-wave length is denoted by l . Substitution of Eq. (4) into Eq. (1) yields:

$$N = \frac{E h^3 \pi^2}{12(1-\nu^2)} \cdot \left(\frac{b}{l} + \frac{n^2 l}{b} \right)^2 \quad (5)$$

To obtain the critical value of compressive stress $\sigma_x = \frac{N}{h}$ it is necessary to find the minimum of parameter N from Eq. (5), which leads to the relations (Volmir (1963)):

$$n = 1, \quad \frac{dN}{d(l/b)} = 0 \quad (6)$$

Solution of (6) has the form:

$$l = b, \quad \sigma^* = \frac{N_c}{h} = \frac{\pi^2 E}{3(1-\nu^2)} \cdot \left(\frac{h}{b} \right)^2 \quad (7)$$

Here σ^* and N_c are critical values of the compressive stress σ and parameter N , respectively.

It follows from Eq. (7) that the bent surface of the elongated plate after stability loss consists from square parts of the size $b \times b$, and the deflection within each of these parts is described by multiplication of two sinusoidal half-waves in x and y directions, respectively.

The Eq. (7) was obtained with the assumption that the plate wall thickness is specified, whereas the critical value of the intensity of the distributed longitudinal forces N is the unknown quantity. It follows from the Eq. (7) that if we assume, on the contrary, that the value of the intensity of the longitudinal forces N is specified, then the shell wall thickness h^* , corresponding to stability loss, will be equal to:

$$h_* = \sqrt[3]{\frac{3 N (1 - \nu^2) b^2}{\pi^2 E}} \quad (8)$$

4. EFFECT OF THE UNIFORM CORROSION

It is assumed now that the plate is subjected to simultaneous action of uniform corrosion and permanent longitudinal distributed forces with intensity $N = h_0 \sigma_0$, where index 0 corresponds to $t = 0$. The rate of the shell thickness decrease at each moment of time t is equal to general corrosion rate:

$$\frac{dh}{dt} = f(\sigma) \quad (9)$$

Here $f(\sigma)$ is a sufficiently smooth function of longitudinal compressive stress:

$$\sigma \equiv \sigma_x = \frac{N}{h} \quad (10)$$

which can be described by the following relation, grounded both theoretically and experimentally in (Gutman (1994)):

$$f(\sigma) = -v_0 \cdot \exp\left(\frac{V \cdot \sigma}{R_0 \cdot T}\right) \quad (11)$$

Here v_0 is the corrosion rate for a non-stressed plate, V is the molar volume of the material, R_0 is the universal gas constant and T is the absolute temperature. Then the critical time t^* , corresponding to the moment of the plate stability loss, follows from the Eqs. (9)-(11) and is defined by the formula:

$$t^* = (h_0 \sigma_0 / v_0) \int_{\sigma_0}^{\sigma^*} \sigma^{-2} \exp[(-V\sigma)/(R_0 T)] d\sigma \quad (12)$$

In Eq. (12) σ^* is the critical stress defined by the relation (7) with h_0 used instead of h . Note that in the Eq. (12) it is assumed that the intensity of longitudinal forces N for a given initial wall thickness h_0 and specific plate material satisfies the inequality:

$$\sigma_0 \leq \sigma^* \quad (13)$$

If we introduce the dimensionless parameters:

$$T^* = (t^* \nu_0) / h_0; \quad \tau = \sigma / \sigma^*; \quad \tau_0 = \sigma_0 / \sigma^* \quad (14)$$

then Eq. (12) takes the form:

$$T^* = \int_{\tau_0}^1 \tau_0 \cdot \exp[-(V\tau\sigma^*)/(R_0T)] \cdot \frac{d\tau}{\tau^2} \quad (15)$$

Here the value of T^* can be treated as a relative durability of the shell and the value of τ_0 as a safety coefficient for stability.

5. NUMERICAL RESULTS

Numerical simulations were performed for thin-walled plates on the basis of the formula (15) in order to study the dependency of the relative durability T^* from the safety coefficient for stability τ_0 . The results are shown in Figs. 2 and 3. The plots in the Fig. 2 correspond to steel and aluminum plates with the following parameter values: $E = 2 \cdot 10^{11} \text{ N/m}^2$, $\nu = 0.3$, $V = 7 \cdot 10^{-6} \text{ m}^3/\text{mol}$ (steel) and $E = 7 \cdot 10^{10} \text{ N/m}^2$, $\nu = 0.34$, $V = 3 \cdot 10^{-5} \text{ m}^3/\text{mol}$ (aluminum). It was adopted for all curves on the Fig. 2 $\frac{h_0}{b} = 0.02$, $T = 27^\circ \text{C}$, $R_0 = 8.317 \text{ N} \cdot \text{m}/(\text{mol} \cdot \text{grad})$.

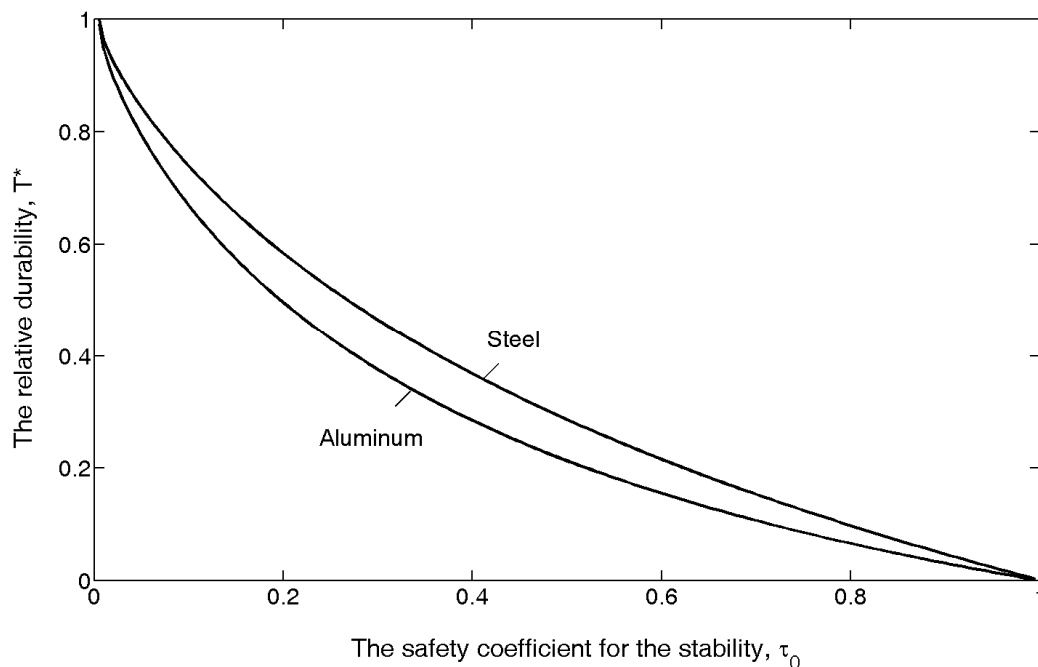


Fig. 2. Dependency of the relative durability T^* on the safety coefficient for stability τ_0 for steel and aluminum plates.

In Fig. 3 the curves are presented for the steel made plates with different values of the relative wall thickness $\frac{h_0}{b}$ (the values of T , R_0 are the same as for the data on the Fig. 2). The graphs show that the relative durability decreases with an increase of the safety coefficient for stability; the value of T^* for steel for all values of the parameter τ_0 is higher than for aluminum; and the rate of decrease of the relative durability with the safety coefficient for stability is larger for thicker plates. For very thin plates the dependency $T^* = T^*(\tau_0)$ is almost linear.

6. CONCLUSION

The critical time of stability loss of thin-walled rectangular elongated plate, subjected to simultaneous action of permanent longitudinal compressive forces and uniform corrosion, can be predicted from solution of the corresponding static stability problem for the same plate without corrosion, by accounting for the plate thickness changes due to corrosion and its rate dependency on mechanical stresses in the material. The analytic dependency of the relative durability of the plate from the safety coefficient was found and studied numerically for different initial wall thickness and different elastic materials. It was shown, that the structure life time for the studied system is completely governed by the criterion of stability loss for elastic plate and the law of corrosion rate.

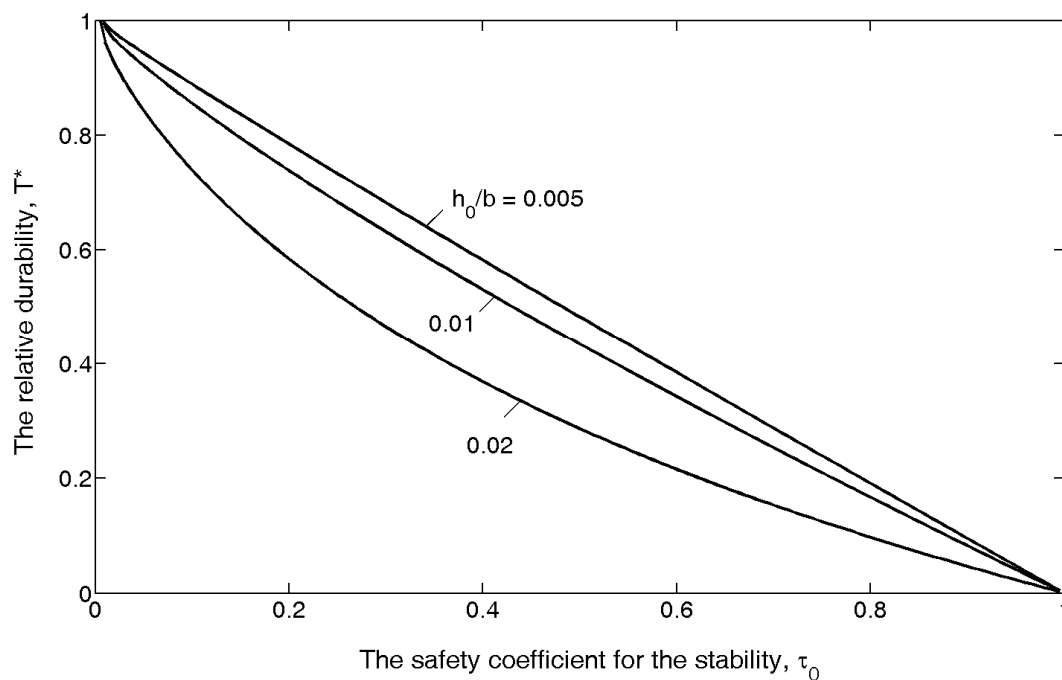


Fig. 3. Dependency of the relative durability T^* on the safety coefficient for stability τ_0 for steel plates with different initial thickness.

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