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NONLINEAR MODEL PREDICTIVE CONTROL OF BIOTECHNOLOGICAL PROCESS FOR ETHANOL PRODUCTION

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Abstract

Nonlinear model predictive control (NMPC) for a biotechnological process has been developed in this paper. The approaches conditions that guarantee stability of the closed-loop has been derived if an NMPC state feedback controller is used together with a full state observer for the recovery of the system state for a biotechnological process. The method is applied for a biotechnological process for Ethanol production. The developed control algorithm ensures maximal Ethanol production at the end of the process and guarantees a feedback on disturbance as well as robustness to process disturbances.

Key words: *biotechnological process, nonlinear model predictive control, output feedback, stability*

1. INTRODUCTION

In many control problems it is desired to design a stabilizing feedback such that a performance criterion is minimized while satisfying constraints on the controls and the states. Ideally one would look for a closed solution for the feedback law satisfying the constraints while optimizing the performance. However, typically the optimal feedback law cannot be found analytically, even in the unconstrained case, since it involves the solution of the corresponding Hamilton-Jacobi-Bellman partial differential equations. One approach to circumvent this problem is the repeated solution of an open-loop optimal control problem for a given state. The first part of the resulting open-loop input signal is implemented and the whole process is repeated. Control approaches using this strategy are referred to as model predictive control (*MPC*), moving horizon control or receding horizon control.

The control of fermentation processes focuses on an open-loop operation owing to their highly nonlinear and inherently difficult dynamic behavior. The optimization is carried out *off-line* and the reactor is fed by the determined optimal feed profile. Once the batch proceeds there is no provision to account for disturbances occurring during the batch

In general one distinguishes between linear and nonlinear model predictive control (*NMPC*). Linear *MPC* refers to a family of *MPC* schemes in which linear models are used to predict the system dynamics and considers linear constraints on the states and inputs and a quadratic cost function. Even if the system is linear, the closed-loop dynamics are in general nonlinear due to the presence of constraints. *NMPC* refers to *MPC* schemes that are based on nonlinear models and/or consider non-quadratic cost-functionals and general nonlinear constraints on the states and inputs.

Since its first invention in the 70s of the last century, linear *MPC* has crystallized as one of the key advanced control strategies. By now linear model predictive control, is widely used in industrial applications especially in the process industry, see for example (Froisy 1994, García 1989, Morari

1999, Qin 2000). The practical success is mainly based on the possibility to take constraints on the states and inputs systematically into account while operating the process optimally.

Many systems are, however, inherently nonlinear. Higher product quality specifications and increasing productivity demands, tighter environmental regulations and demanding economical considerations require to operate systems over a wide range of operating conditions and often near the boundary of the admissible region. Under these conditions linear models are often not sufficient to describe the process dynamics adequately and nonlinear models must be used. This inadequacy of linear models is one of the motivations for the increasing interest in nonlinear model predictive control.

The purpose of this paper is twofold. In the first part a review is provided on the current state of *NMPC*. After a presentation of the basic principle of predictive control some of the key theoretical, computational and implementational aspects of this control strategy is presented. Furthermore, the inherent advantages and disadvantages of *NMPC* are discussed. If it is mainly focus on *NMPC* for continuous time systems using sampled measurement information and do not go into details on discrete time *NMPC* strategies. For more self contained reviews on *NMPC* is referred to (Allgower 1999, Chen 1998, De Nicolao 2000, Mayne 2000).

One important precondition for the application of *NMPC*, is the availability of reliable and efficient numerical dynamic optimization algorithms, since at every sampling time a nonlinear dynamic optimization problem must be solved in real-time. Solving such an optimization problem efficiently and fast is, however, not a trivial task and has attracted strong research interest in recent years).

In this paper we apply the ideas of *NMPC* for a fed-batch fermentation process for an ethanol production from glucose using the bacterium *Saccharomyces cereviziae*.

2. THE PRINCIPLE OF NONLINEAR MODEL PREDICTIVE CONTROL

Model predictive control is formulated as the repeated solution of a (finite) horizon open-loop optimal control problem subject to system dynamics and input and state constraints. Figure 1 depicts the basic idea behind model predictive control. Based on measurements obtained at time t, the controller predicts the dynamic behavior of the system over a prediction horizon T_p in the future and determines (over a control horizon $T_c \leq T_p$) the input such that a predetermined open-loop performance objective is minimized. If there were no disturbances and no model-plant mismatch, and if the optimization problem could be solved over an infinite horizon, then the input signal found at $t=0$ could be open-loop applied to the system for all $t \geq 0$. However, due to disturbances, model-plant mismatch, and the finite prediction horizon the actual system behavior is different from the predicted one. To incorporate feedback, the optimal open-loop input is implemented only until the next sampling instant. In principle the time between each new optimization, the sampling time, can vary. For simplicity of presentation, that it is fixed, i.e the optimal control problem is re-evaluated after the constant sampling time δ. Using the new system state at time $t+\delta$, the whole procedure – prediction and optimization – is repeated, moving the control and prediction horizon forward.

In the following, $\|\cdot\|$ denotes the Euclidean vector norm in $\mathbf{R}^n$ (where the dimension n follows from context) or the associated induced matrix norm. Vectors are denoted by boldface symbols. Whenever a semicolon ";" occurs in a function argument, the following symbols should be viewed as additional parameters, i.e. $f(x;\gamma)$ means the value of the function f at x with the parameter γ.

Models in *MPC* are used for predicting the process output over a prediction horizon. Control actions are calculated over a control horizon in such a way that the predicted process output is as close as possible to the desired reference signal and the first control action in sequence is applied in each step

(**Fig. 1**) (Morari 1999, Chen 1998).

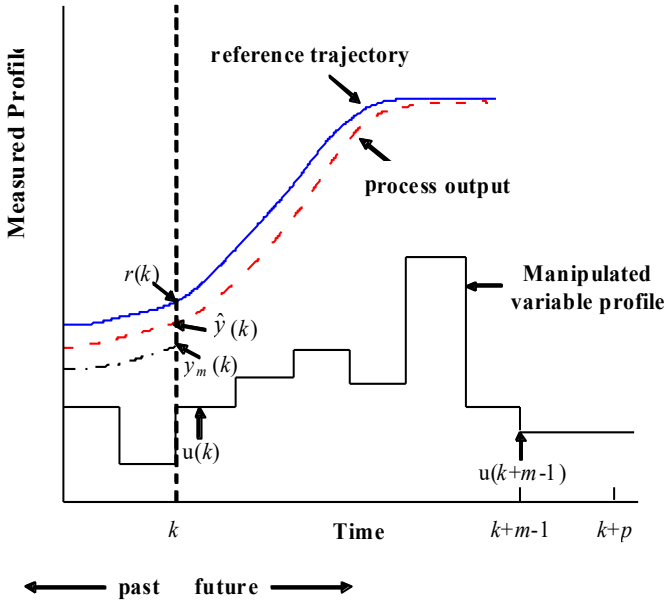


Fig. 1: Principle of model predictive control

In **Fig. 1** the open-loop optimal input is depicted as a continuous function of time. To allow a numerical solution of the open-loop optimal control problem the input is often parametrized by a finite number of "basis" functions, leading to a finite dimensional optimization problem. In practice often a piecewise constant input is used, leading to Tc/δ decisions for the input over the control horizon.

Summarizing, a standard *NMPC* scheme works as follows:

1. Obtain estimates of the states of the system;
2. Calculate a constraint-conforming optimal input minimizing the desired cost function over the prediction horizon using the system model and the current state estimate for prediction;
3. Implement the first part of the optimal input until the next sampling instant;
4. Continue with 1.

While it can be desirable for computational and performance reasons to choose unequal lengths of the prediction and control horizon, assuming that in the following that $T_c=T_p$ for our presentation.

2.1. Mathematical formulation of state feedback NMPC

Consider the stabilization of time-invariant nonlinear systems of the form:

$$\dot{x}(t) = f(x(t),u(t)), x(0)=x_0 \quad (1)$$

subject to the input and state constraints:

$$u(t)\leq \mathbf{U}, \forall t \geq 0 \quad (2)$$

$x(t) \leq \mathbf{X}, \forall t \geq 0$ (3)

With respect to the vector field $f: \mathbf{R}^n \times \mathbf{R}^m \rightarrow \mathbf{R}^n$, assuming that it is locally Lipschitz continuous in the region of interest (typically the region of attraction) and satisfies $f(0, 0)=0$. Furthermore, the set $\mathbf{U} \subseteq \mathbf{R}^m$ is compact, $\mathbf{X} \rightarrow \mathbf{R}^n$ is connected, and $(0, 0) \leq \mathbf{X} \times \mathbf{U}$. Typically $\mathbf{U}$ and $\mathbf{X}$ are (convex) box constraints of the form:

$$\mathbf{U} = \left\{ u \in \mathbf{R}^m \mid u_{\min} \leq u \leq u_{\max} \right\} \quad (4)$$

$$\mathbf{X} = \left\{ x \in \mathbf{R}^n \mid x_{\min} \leq x \leq x_{\max} \right\} \quad (5)$$

with the constant vectors u_{min}, u_{max} and x_{min}, x_{max}.

In sampled-data *NMPC* an open-loop optimal control problem is solved at discrete sampling instants t_i based on the current state information $x(t_i)$. Since considering that a constant sampling time δ, the sampling instants t_i are given by $t_i=I, \delta$, $i=0, 1, 2, \ldots$ When the time t and t_i occurs in the same setting, t_i should be taken as the closest previous sampling instant $t_i < t$.

The open-loop input signal applied in between the sampling instants is given by the solution of the following optimal control problem:

Problem 1: $\min_{\bar{u}(\cdot)} J\left(x(t_i), \bar{u}(\cdot)\right)$, subject to: $\dot{\bar{x}}(\tau) = f\left(\bar{x}(\tau), \bar{u}(\tau)\right), \quad \bar{x}_i = x(t_i)$ (6a)

$$\bar{u}(\tau) \in \mathbf{U}, \quad \bar{x}(\tau) \in \mathbf{X}, \quad \tau \in \left[t_i + T_p\right] \quad (6b)$$

$$\bar{x}(t_i + T_p) \in \Im \quad (6c)$$

The bar denotes predicted variables (internal to the controller), i.e. $\bar{x}(\cdot)$ is the solution of (6a) driven by the input $\bar{u}(\cdot) : \left[t_i + T_p\right] \rightarrow \mathbf{U}$ with the initial condition $x(t_i)$. The distinction between the real system variables and the variables in the controller is necessary, since even in the nominal case the predicted values will not be the same as the actual *closed-loop values*.

The cost functional J minimized over the control horizon $T_p \leq \delta > 0$ is typically given by:

$$J\left(x(t_i), \bar{u}(\cdot)\right) := \int_{t_i}^{t_i+T_p} F\left(\bar{x}(\tau), \bar{u}(\tau)\right) d\tau + E\,\bar{x}(t_i + T_p) \quad (7)$$

where the stage cost $F: \mathbf{X} \times \mathbf{U} \rightarrow \mathbf{R}^+$ is often assumed to be continuous, satisfies $F(0, 0)=0$, and lower bounded by a class $\mathbf{K}$ function $\alpha_F : \alpha_F(\|x\|) \leq F(x, u)\ \forall\ (x, u) \in \mathbf{X} \times \mathbf{U}$.

The terminal penalty term E and the so called terminal region constraint $\bar{x}(t_i + T_p) \in \Im$ might or might not be present. These are often used to enforce nominal stability.

The stage cost can for example arise from economical and ecological considerations. Often, a quadratic form for F is used:

$F(x, u) = x^T Q\, x + u^T R\, u$ (8)

with $Q \geq 0$ and $R > 0$.

The state measurement enters the system via the initial condition in (6a) at the sampling instant, i.e. the system model used to predict the future system behavior is initialized by the actual system state. Since all state information is necessary for the prediction, the full state must be either measured or

estimated.

The solution of the optimal control problem (6) is denoted by $\bar{u}(\cdot; x(t_i))$. It defines the open-loop input that is applied to the system until the next sampling instant t_i+1:

$$u(t; x(t_i)) = \bar{u}(t; x(t_i)), \quad t \in [t_i, t_{i+1}) \tag{9}$$

The control $u(t; x(t_i))$ is a feedback, since it is recalculated at each sampling instant using the new state measurement. In comparison to sampled-data *NMPC* for continuous time systems, in instantaneous *NMPC* (also often referred to as receding horizon control) the input is defined by the solution of *Problem 1* at all times: $u(x(t)) = \bar{u}^{*}(t; x(t))$, i.e. open-loop input is applied (De Nicolao 2000, Mayne 2000).

The solution of (1) starting at time t_1 from an initial state $x(t_1)$, applying an input $u:[t_1, t_2] \to \mathbf{R}^m$ is denoted by $x(\tau; u(\cdot), x(t_1))$, $\tau \leq [t_1, t_2]$. If it will refer to an admissible input as:

Definition 2.1 (Admissible input) *An input* $u : [0, T_p] \to \mathbf{R}^m$ *for a state* x_0 *is called admissible, if it is:*

a) piecewise continuous

b) $u(\tau) \in \mathbf{U} \; \forall \; \tau \in [0, T_p]$

c) $x(\tau; u(\cdot)), x_0) \in \mathbf{X} \forall \tau \in [0, T_p]$, *d)* $x(T_p; u(\cdot), x_0) \in \Im$

Definition 2.2 (Value function) The value function V(x) of the open-loop optimal control *Problem 1* is defined as the *minimal value of the cost for the state x*: $V(x) = J\left(x, \bar{u}^{*}(\cdot; x)\right)$

The value function plays a central role in the stability analysis of *NMPC*, since it often serves as a Lyapunov function candidate (Allgower 1999, Mayne 2005).

Typically, no explicit controllability assumption on the system is considered in NMPC. Instead, the stability results are based on the assumption of initial feasibility of the optimal control problem, i.e. the existence of a input trajectory $u(\cdot)$ s. t. all constraints are satisfied.

The unequalness of the predicted and the closed-loop trajectories has two immediate consequences. Firstly, the actual goal to compute a feedback such that the performance objective over the *infinite horizon* of the closed-loop is minimized is not achieved. In general it is by no means true that the repeated minimization over a moving *finite horizon* leads to an optimal solution for the infinite horizon problem. The solutions will often differ significantly if a short finite horizon is chosen. Secondly there is in general no guarantee that the closed-loop system will be stable. It is indeed easy to construct examples for which the closed-loop becomes unstable if a short finite horizon is chosen. Hence, when using finite prediction horizons special attention is required to guarantee stability.

The summarize the key characteristics and properties of *NMPC* are:

- *NMPC* allows the direct use of nonlinear models for prediction;
- *NMPC* allows the explicit consideration of state and input constraints;
- In *NMPC* a specified time domain performance criteria is minimized *on-line*;
- In *NMPC* the predicted behaviour is in general different from the closed loop behaviour;
- For the application of *NMPC* typically a real-time solution of an open-loop optimal control

problem is necessary.

To perform the prediction the system states must be measured or estimated.

Basing the applied input on the solution of an optimal control problem that must be solved on-line is advantageous and disadvantageous at the same time. First, and most important, this allows to directly consider constraints on states and inputs which are often difficult to handle otherwise. Furthermore, the desired cost objective, the constraints and even the system model can in principle be adjusted on-line without making a complete redesign of the controller necessary. However, solving the open-loop optimal control problem, if attacked blindly, can be difficult or even impossible for large systems.

2.2 State feedback NMPC and nominal stability

One of the most important questions in *NMPC* is whether a finite horizon *NMPC* strategy does guarantee stability of the closed-loop or not. The key problem with a finite prediction and control horizon is due to the difference between the predicted open-loop and the resulting closed-loop behaviour. Ideally, one would seek for an *NMPC* strategy which achieves closed-loop stability independently of the choice of the parameters and, if possible, approximates the infinite horizon *NMPC* scheme as well as possible. An *NMPC* strategy that achieves closed-loop stability independently of the choice of the performance parameters is often referred to as an *NMPC* approach *with guaranteed stability*. Different approaches to achieve closed-loop stability using finite horizon lengths exist. Here only some central ideas are reviewed and no detailed proofs are given. Moreover, no attempt is made to cover all existing methods. Most of the technical details are left out for reasons of a simple presentation.

Finite Horizon NMPC Schemes with Guaranteed Stability: Different possibilities to achieve closed-loop stability using a finite horizon length exist. Most of these approaches modify the standard *NMPC* setup such that stability of the closed-loop can be guaranteed independently of the plant and performance specification. This is usually achieved by adding suitable equality or inequality constraints and suitable additional penalty terms to the standard setup. The additional terms are generally not motivated by physical restrictions or performance requirements but have the sole purpose to enforce stability. Therefore, they are usually called *stability constraints*. One possibility to enforce stability with a finite prediction horizon is to add the so called *zero terminal equality constraint* at the end of the prediction horizon, i.e.

$$\bar{x}\left(t+T_p\right)=0 \tag{10}$$

is added to *Problem 1*(Wright 1996, Biegler 2000, Chen 1982). This leads to stability of the closed-loop, if the optimal control problem has a solution at t=0. Similar to the infinite horizon case the feasibility at one sampling instant does imply feasibility at the following sampling instants and a decrease in the value function.

The main advantages of a zero terminal constraint are the straightforward application and the conceptual simplicity (Keerthi 1988).

Many schemes exist that try to overcome the use of a zero terminal constraint of the form (10). Most of them use the *terminal region constraint* $\bar{x}\left(t+T_p\right)\in\Im$ and/or a *terminal penalty term* $E\left(\bar{x}\left(t+T_p\right)\right)$ to enforce stability and feasibility. Typically the terminal penalty E and the terminal region E are determined off-line such that the cost function:

$$J\left(x(t),\bar{u}(\cdot)\right)=\int_{t_i}^{t_i+T_p}F(\bar{x}(\tau),\bar{u}(\tau))d\tau+E(\bar{x}(t_i+T_p)) \tag{11}$$

gives an upper bound on the infinite horizon cost and guarantees a *decrease in the value function* as the horizon recedes in time.

We do not go into details about the different approaches. Instead we state the following theorem, which gives conditions for the convergence of the closed-loop states to the origin. It is a slight modification of the results given in (Fontes 2000, Chen 1997).

Theorem 2.1 (Stability of sampled-data NMPC) *Suppose that*:

a) the terminal region $\Im \subseteq \mathbf{X}$ *is closed with* $0 \in \Im$ *and that the terminal penalty* $E(x) \leq C$ *is positive semi-define*

(b) the terminal region and terminal penalty term are chosen such that $\forall x \in \Im$ *there exists an (admissible) input* $u_\Im : [0, \delta] \to \mathbf{U}$ *such that* $x(\tau) \in \Im \forall \tau \in : [0, \delta]$ *and*:

$$\frac{\partial E}{\partial x} f\big(x(\tau), u_\Im(\tau)\big) + F\big(x(\tau), u_\Im(\tau)\big) \leq 0, \quad \forall \tau \in [0, \delta] \tag{12}$$

(c) the NMPC open-loop optimal control problem is feasible for $t=0$

Then in the closed-loop system (1) *with* (9) $x(t)$ *converges to the origin for* $t \to \infty$*, and the region of attraction* R *consists of the states for which an admissible input exists*.

Proof. The proof is given here for sake of completeness. It bases on using the value function as a decreasing Lyapunov-like function. As usual in predictive control the proof consists of two parts: in the first part it is established that initial feasibility implies feasibility afterwards. Based on this result it is then shown that the state converges to the origin.

Feasibility: Consider any sampling instant t_i for which a solution exists (e.g. t_0). In between t_i and t_{i+1} the optimal input $\bar{\bar{u}}(\cdot; x(t_i))$ is implemented. Since no model plant mismatch nor disturbances are present, $x(t_{i+1}) = \bar{x}\left(t_{i+1}; \bar{\bar{u}}(\cdot; x(t_i)), x(t_i),\right)$. The remaining piece of the optimal input $\bar{\bar{u}}(\tau; x(t_i))$, $\tau \in [t_{i+1}, t_i + T_p]$ satisfied the state and input constraints. Furthermore, $\bar{x}\left(t_i + T_p ; x(t_i), \bar{\bar{u}}(\cdot; x(t_i))\right) \in \Im$ and that is known from Assumption (b) of the theorem that for all $x \in \Im$ there exists at least one input $u_\Im(\cdot)$ that renders $\Im$ invariant over δ. Picking any such input we obtain as admissible input for any time $t_i + \sigma$, $\sigma \leq (0, t_{i+1} - t_i]$:

$$\tilde{u}\big(\tau; x(t_i + \sigma)\big) = \begin{cases} \bar{\bar{u}}(\tau; x(t_i)), & \tau \in [t_{i+1} + \sigma,\ t_i + T_p] \\ u_\Im(\tau - t_i + T_p), & \tau \in (t_i + T_p,\ t_i + T_p + \sigma] \end{cases} \tag{13}$$

Specifically, it is given for the next sampling time ($\sigma = t_{i+1} - t_i$) that $\tilde{u}(\cdot; x(t_{i+1}))$ is a feasible input, hence feasibility at time t_i implies feasibility at t_{i+1}. Thus, if (6) is feasible for $t=0$, it is feasible for all $t \geq 0$.

Furthermore, if the states for which an admissible input exists converge to the origin, it is clear that the region **R** that consists of those points belongs to the region or attraction.

Convergence: it is firs show that the value function is decreasing starting from a sampling instant. Also, the value of V at $x(t_i)$ is given by:

$$V(x(t_i)) = \int_{t_i}^{t_i+T_p} F\left(\bar{x}(\tau, \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \bar{\bar{u}}(\tau; x(t_i))d\tau + E(\bar{x}(t_i + T_p, \bar{\bar{u}}(\cdot; x(t_i)), x(t_i))\right) \quad (14)$$

and the cost resulting from (13) starting from any $x(t_i + \sigma, \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \sigma \in (0, t_{i+1} - t_i]$ using the input $\tilde{u}(\tau; x(t_i + \sigma))$, given by:

$$J(x(t_i + \sigma), \tilde{u}(\cdot; x(t_i + \sigma))) = \int_{t_i+\sigma}^{t_i+\sigma+T_p} F(\bar{x}(\tau; \tilde{u}(\cdot; x(t_i + \sigma)), x(t_i + \sigma)), \tilde{u}(\tau; x(t_i + \sigma)))d\tau)$$
$$+ E(\bar{x}(t_i + \tau + T_p; \tilde{u}(\cdot; x(t_i + \sigma)), x(t_i + \sigma))) \quad (15)$$

Reformulation yields:

$$J(x(t_i + \sigma), \tilde{u}(\cdot; x(t_i + \sigma))) = V(x(t_i)) - \int_{t_i}^{t_i+\sigma} F\left(\bar{x}(\tau; \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \bar{\bar{u}}(\tau; x(t_i)))\right)d\tau -$$
$$- E(\bar{x}(t_i + T_p; \tilde{u}(\cdot; x(t_i), x(t_i)))) + \int_{t_i+\sigma}^{t_i+\sigma+T_p} F(\bar{x}(\tau; \tilde{u}(\cdot; x(t_i + \sigma)), x(t_i + \sigma)), \tilde{u}(\tau; x(t_i + \sigma)))d\tau + \quad (16)$$
$$+ E(\bar{x}(t_i + \tau + T_p; \tilde{u}(\cdot; x(t_i + \sigma)), x(t_i + \sigma)))$$

Integrating inequality (12) from $t_i+\sigma$ to $t_i+\sigma+T_p$ starting from $x(t_i+\sigma)$ is obtained zero as an upper bound for the last three terms on the right side. Thus:

$$J(x(t_i + \sigma), \tilde{u}(\cdot; x(t_i + \sigma))) - V(x(t_i)) \le -\int_{t_i}^{t_i+\sigma} F\left(\bar{x}(\tau; \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \bar{\bar{u}}(\tau; x(t_i)))\right)d\tau \quad (17)$$

Since $\tilde{u}$ is only a feasible but not necessarily the optimal input for $x(t_i+\sigma)$, it follows that:

$$V(x(t_i + \sigma)) - V(x(t_i)) \le -\int_{t_i}^{t_i+\sigma} F\left(\bar{x}(\tau; \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \bar{\bar{u}}(\tau; x(t_i)))\right)d\tau \quad (18)$$

i.e. the value function is decreasing along solution trajectories starting at a sampling instant t_i. Especially we have that:

$$V(x(t_i + 1)) - V(x(t_i)) \le -\int_{t_i}^{t_{i+1}} F\left(\bar{x}(\tau; \bar{\bar{u}}(\cdot; x(t_i)), x(t_i)), \bar{\bar{u}}(\tau; x(t_i)))\right)d\tau \quad (19)$$

By assumption, this decrease in the value function is strictly positive for $x(ti)\neq 0$. Since this holds for all sampling instants, convergence can be established as in (Chen 1998) by an induction argument and the application of Barbalat's lemma.

Loosely speaking, E is an F-conform local control Lyapunov function in $\Im$. The terminal region constraint enforces feasibility at the next sampling instant and allows, similarly to the infinite horizon case, to show that the value function is strictly decreasing. Thus stability can be established. Note that this result is nonlocal in nature, i.e. there exists a region of attraction **R** which is of at least the size of $\Im$. Typically the region of attraction resulting from this scheme is much larger than $\Im$ and contains the set of all states for which the open-loop optimal control problem has a feasible solution.

Various ways to determine a suitable terminal penalty term and terminal region exist. Examples are

the use of a control Lyapunov function as terminal penalty E_i (Jadbabaie 2001) or the use of a local nonlinear or linear control law to determine a suitable terminal penalty E and a terminal region $\Im$ (Michalska 1993).

Quasi-infinite horizon *NMPC*: We exemplify the choice of a suitable terminal region constraint and a terminal penalty term considering the so called quasi-infinite horizon NMPC (QIH-NMPC) approach (Findeisen 2001). In *QIHNMPC* E and $\Im$ are obtained on the basis of a locally stabilizing linear control law $u_\Im=Kx$ considering a quadratic stage cost of the form (8). The terminal penalty term E in this case is quadratic and of the form $E(x)=x^TPx$. The following procedure gives a systematic approach to compute the terminal region and a terminal penalty matrix *off-line* (Findeisen 2001), assuming that the Jacobian linearization (A, B) of (1) is stabilizable, where $A := \frac{\partial f}{\partial x}(0, 0)$ and $B := \frac{\partial f}{\partial u}(0, 0)$.

Step 1 : Solve the linear control problem based on the Jacobian linearization (A, B) of (1) to obtain a locally stabilizing linear state feedback $u_\Im=Kx$.

Step 2 : Define $A_K:=A+BK$, and choose a constant $\kappa\leq[0, \infty)$ satisfying $\kappa<-\lambda_{max}(A_K)$ and solve the Lyapunov equation:

$$(AK+\kappa I)^T P + P(AK+\kappa I) = -(Q+K^T RK) \tag{20}$$

to get a positive definite and symmetric P.

Step 3 : Find the largest possible α_1 defining a region

$$\Im_1 := \{x \in \mathbf{R}^n \mid x^T Px \leq \alpha_1\} \tag{21}$$

such that $Kx\leq\mathbf{U}$, for all $x\leq\Im_1$.

Step 4 : Find the largest possible $\alpha\leq(0, \alpha_1]$ specifying a terminal region

$$\Im := \{x \in \mathbf{R}^n \mid x^T Px \leq \alpha\} \tag{22}$$

such that the optimal value of the following optimization problem is non-positive:

$$\max_x \{x^T P\varphi(x) - \kappa \cdot x^T Px \leq \alpha\} \tag{23}$$

where $\varphi(x) := \mathbf{f}(x, Kx) - A_K x\, \phi(x) := \mathbf{f}(x, Kx) - AKx$.

This procedure allows to calculate E and $\Im$ if the linearization of the system at the origin is stabilizable. If the terminal penalty term and the terminal region are determined accordingly, the open-loop optimal trajectories found at each time instant approximate the optimal solution for the infinite horizon problem.

Remark 2.1 *Theorem 2.1 allows to consider the stabilization of systems that can only be stabilized by feedback that is discontinuous in the state* (Fontes 2000)*, e.g. nonholonomic mechanical systems. This is possible since the optimal input signal is applied over the sampling time open-loop to the system. For such systems, however, it is in general rather difficult to determine a suitable terminal region and a terminal penalty term - for instance, the QIH-NMPC method cannot be used.*

Remark 2.2 *The use of a terminal inequality constraint leads to computational and feasibility advantages compared to the infinite horizon and zero terminal constraint approach. No zero terminal constraint must be met in finite time. The solution time necessary for solving the open-loop optimal control problem is decreased, since no "boundaryvalue" problem stemming from the zero terminal*

constraint must be solved. Furthermore, note that in NMPC it is in general not necessary to find always an optimal solutions of Problem 1 in order to guarantee stability (Magni 1997, Qin 2003). *Only a feasible solution leading to a decrease in the value function is necessary. This can be utilized to decrease the necessary on-line solution time and makes the practical application more robust*.

Summarizing, the nominal stability question of *NMPC* in the state feedback case is well understood. Various *NMPC* approaches that guarantee stability exist.

3. MATERIAL AND METHODS

The ethanol is the most important organic compound which has a wide application in different industry fields: food, perfumery-cosmetic, chemical, millwright, etc. In recent 10 - 15 years enormous attention is paid to the ethanol production as a fuel.

The kinetic model of the process is:

$$\begin{aligned}
\frac{dX}{dt} &= \mu_m \frac{S}{k_s + S} X - \frac{F}{V} X \\
\frac{dS}{dt} &= \frac{F}{V}(S_{in} - S) - \frac{1}{Y_S} \frac{\mu_m S}{k_s + S} X \\
\frac{dE}{dt} &= \frac{1}{Y_E} \frac{\mu_m S}{k_s + S} X - \frac{F}{V} E \\
\frac{dV}{dt} &= F
\end{aligned} \tag{24}$$

where: X – cell concentration, g/l; S – substrate concentration, g/l; E – ethanol concentration, g/l; S_{in} – feed concentration, g/l; t – time, h; $Y_{S,E}$ – yield coefficients of the substrate and the ethanol; μ_m – maximum specific growth rate of biomass, h^{-1}; k_s – the Michaelis-Menten constant; F – feed flow rate, l/h.

The initial conditions and the kinetics constants is:

$X(0) = X_0 = 0.27$ g/l; $S(0) = S_0 = 31.30$ g/l; $S_{in} = 100$ g/l; $E(0) = E_0 = 0.47$ g/l; $V(0) = V_0 = 1.2$ l;

$\mu_m = 0.342\ h^{-1}$; $k_s = 5.436$ g/g; $Y_S = 5.762$ g/g; $Y_E = 1.985$.

4. NONLINEAR MODEL PREDICTIVE CONTROL TO A BIOTECHNOLOGICAL PROCESS FOR ETHANOL PRODUCTION

An added advantage of using a predictive controller is the fact that the decisions made for fed-batch fermentors during the course of the process operation have a significant impact on the final product quality. In such a scenario the model predictive control algorithm proves beneficial since it can predict the effect of the past input moves on the future system behavior. Since process feedback is incorporated into the algorithm at every time step the predicted future constraint violations can be accounted for and subsequent input moves can be chosen sothat they can prevent these violations from taking place. The model predictive control algorithm is also well known for its explicit on-line handling of manipulated variable constraints. On the other hand, constraint handling is not easily incorporated with classical feedback techniques; furthermore classical feedback controllers respond

only after the disturbance manifests itself in the process output thereby limiting their applicability for constrained fed-batch control problems (Fontes 2000).

This problem of implementing the optimal policy and ensuring that the system follows the optimal trajectory in the presence of disturbances has received a limited previous consideration. Attention is thus focused on designing a controller to track the optimal policy and on approaches for the disturbance compensation for the closed-loop control problem (Chen 1998).

The control scheme for the fed-batch bioreactor is implemented in two stages and the substrate feed rate serves as the manipulated variable. Initially a constrained optimal control problem is solved off-line in order to determine the manipulated variable profile that maximizes the end of the batch product concentration for the product of interest while maintaining a pre-specified fixed final volume. The next step involves tracking of the optimal control trajectory in a *closed-loop operation* (Fontes 2000).

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The first part of the *NMPC* algorithm is the specification of the reference trajectory which may be as simple as a step change to a new setpoint or as it is common for batch processes - a trajectory that the system must follow. At the present time k, the reference trajectory has a value $r(k)$.

Also at k, consider the predicted process output over a future prediction horizon p. A suitable controller model of the process is used to obtain the projected behavior of the output over the prediction horizon by simulating the effects of the past inputs applied to the actual process (value $\hat{y}(k)$ at the current time).

The same controller model is used to calculate a sequence of m current and future manipulated variable moves in order to satisfy a specified objective function. Here m is the move horizon. A common objective function is to minimize the sum of the squared deviations of the predicted controlled variable values from a time-varying reference trajectory over the prediction horizon based on the system information available at the current time k. The minimization also takes into account the constraints that may be present at the state, output, or the manipulated variable. Conceptually the

problem is similar to the constructing of the inverse of the controller model to determine the sequence of m moves that most closely achieve the specified output behavior over the predicted horizon.

However, due to unmodeled disturbances and modeling errors there may be some deviations between the actual observed output $y_m(k)$ and the predicted output behavior. Due to these deviations the computed future manipulated variable moves are no longer appropriate and hence only the first of the computed manipulated variable moves, $\Delta\mathbf{u}(k)$, is implemented on the actual process. The error $d(k)=y_m(k) - \hat{y}(k)$ is calculated and is used to update the future measurements. The optimization is carried out again based on this new horizon and using the updated system information and the process continues. Since the horizon recedes at the next time step, this is also known as a receding horizon control problem. However, in the case of batch systems where the final time of the process operation is specified the available prediction horizon and the window of opportunity for control shrink as the batch is close to completion. Consequently the value of the prediction horizon in the control algorithm successively decreases as the end of batch is close.

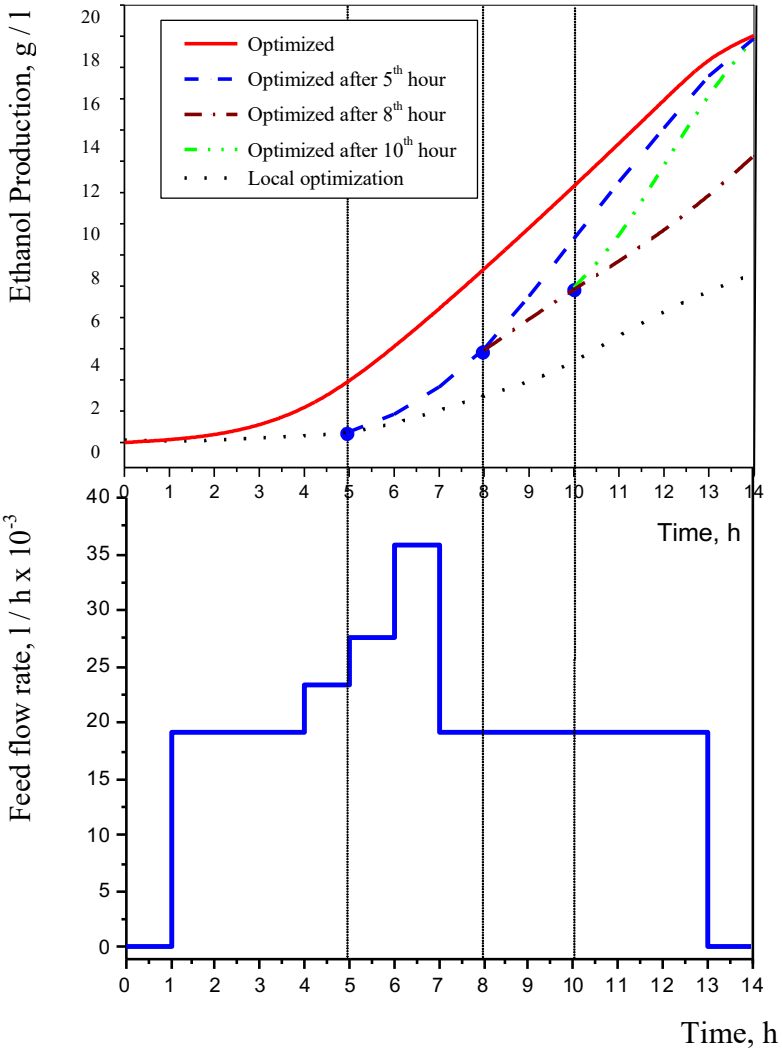


Fig. 2. *NMPC* to fermentation process for Ethanol production

At the next time instant $k + 1$, the process measurement is again taken and the horizon is shifted forward by one step. The optimization is carried out again based on this new horizon and using the updated systems information and the process continues. Since the horizon recedes at the next time step it is also known as a receding horizon control problem.

NMPC involves the solution of an optimization problem on-line in order to determine the optimal inputs over a future time horizon.

Updating the disturbance estimate and solving the optimization at each time step compensates the unmeasured disturbances and model inaccuracy (which cause actual system outputs to be different from the model outputs). Usually the problem is formulated so that the objective is minimized subject to certain system constraints, for example bounds on the magnitude of current and future inputs or outputs. This ability to handle constraints in an optimal way is the primary advantage of the model predictive control over other design schemes. The primary disadvantage of *MPC* relative to other techniques is its inability to deal with model uncertainty.

An algorithm for application of *MPC* for the investigated process is developed. The results are shown on **Fig. 2.** The 5th hour is chosen for a first control point . As it may be noted there is a diversion from the reference profile marked on the figure by "—", accordingly the optimal profile is changed – *NDP* profile. The second point is at 8th hour. The third point is at 10th hour. The obtained control guarantees the robustness and stability of the biomass production.

The optimization criterion is – maximal biomass concentration at the end of the process (criterion *J*).

5. CONCLUSION

Model predictive control for linear constrained systems has been proven as a useful control solution for many practical applications. It is expected that the use of nonlinear models in the predictive control framework, leading to nonlinear model predictive control, results in improved control performance and allows the direct use of first principle based models. However, the consideration of nonlinear models also poses challenging theoretical, computational and implementational problems.

The results from the optimal do not not have a feedback and it does not guarantee robustness to process disturbances.

NMPC is developed for guarantee robustness to process disturbances for a fed-batch fermentation process for the Ethanol production.. The method is carried out with an aim control of disturbance of the optimal control variable (feeding rate). For local optimization of choice optimization hour, an algorithm is applied in order to find an optimal profile of the control variable.

The developed control algorithm ensures maximal Ethanol production at the end of the process and guarantees a feedback on disturbance as well as robustness to process disturbances.

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