INFLUENCE OF ANHARMONICITY CURRENT-PHASE DEPENDENCE ON PROPERTIES OF WEAK LINKS OF JOSEPHSON TYPE

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Abstract

In work is considered the influence of anharmonicity current-phase dependence on properties of weak links of Josephson type (WLJT). The introduction of coefficient anharmonicity k *develops the model anharmonicity dependence current-phase of weak links* $j(\varphi) = j_c\left(\sin\varphi - \sin(k\varphi)/k\right)$. *Is established, that in WLJT anharmonicity current-phase dependence increases size of plasma frequency. Is shown the feature of dependence of a supercurrent from the normalized magnetic flux on fluxoid quantum in weak links with anharmonic current-phase dependence response of top at meaning of the normalized flux* $\hat{\Phi}/\hat{\Phi}_0 = 0$, *and also occurrence of minima in fractional meanings normalized flux* $\hat{\Phi}/\hat{\Phi}_0$. *On the basis of modelling researches of dynamics of plasma frequency is interpreted the occurrence fractional steps Shapiro on current-voltage characteristic Josephson of structures with anharmonicity dependence current-phase at influence of a microwave of a signal.*

Key words: *weak links, anharmonicity current-phase dependence, Josephson junctions*

1. INTRODUCTION

The study of properties superconducting weak links (SWL) belong to one of the central directions of physics of the condensed condition. Now to study SWL is devoted the large number both theoretical and experimental works [1, 2]. Stimulus of this activity is the following reasons:

a) a number of properties SWL was found by applications in creation of unique physical devices, and the idea develops about creation of new materials for superconducting electronics, basically for devices perspective millimeter and submillimeter of ranges of waves [3];

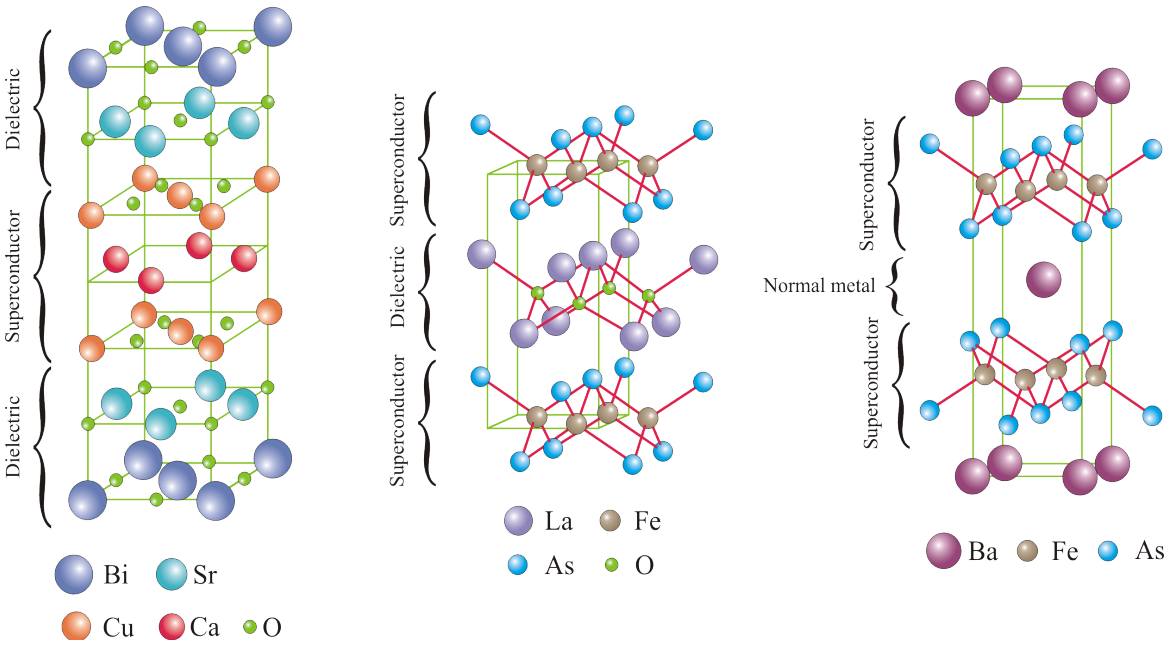
b) Increase of critical temperature with the help Josephson of carry of a charge between neighbors superconducting clusters, resulting to macroscopic high-temperature superconductivity (HTSC) [4];

c) finding of some properties of HTSC in layered connections on a basis copper oxide such as $YBa_2Cu_3O_{7+\delta}$, $Bi_2Sr_2Ca_2Cu_3O_{8+\delta}$, $Tl_2Ba_2Ca_2Cu_3O_{10+\delta}$ and new class on the basis of iron such as $LaO_{1-x}F_xFeAs$, $CeO_{1-x}F_xFeAs$, $PrO_{1-x}F_xFeAs$, $SmO_{1-x}F_xFeAs$, representing Josephson the network of weak links, as till now remains up to the end obscure the mechanism Cooper pairing, nature of a normal condition etc. [5-8];

d) use of weak links for theoretical study and description of nonlinear non-stationary processes with the purposes of a clear statement of physical senses and experimental phenomena of these processes etc. [9].

At study of properties SWL the particular interest is represented by influences of anharmonicity current-phase dependence. The origin of anharmonicity current-phase parity various Josephson of structures has a different nature and character [10, 11]. Deviation of meanings superconducting current

$I_S(\varphi)$ from the sine wave form are found experimentally in Josephson of structures with antiferromagnetic by a layer at reduction of thickness [12]; in SNS-, SFS-, SINS-structures at temperature $T << T_c$ [13]; in superlattices consisting of alternating layers of isolator and a superconductor S-I-S- … -S-I-S [14], and also can be shown in SWL on microcracks and HTSC, representing itself layered S-I-S- … -S-I-S -, S-N-S- … -S-N-S- structures (fig. 1, a, b, c).



a) b) c)

Fig. 1. Layered high-temperature superconductors: a) $Bi_2Sr_2Ca_2Cu_3O_{8+\delta}$; b) LaOFeAs; c) $BaFe_2As_2$.

2. MODEL OF ANHARMONICITY CURRENT-PHASE DEPENDENCE IN WEAK LINKS

At construction of model anharmonicity current-phase dependence is necessary to take into account, those processes and mechanisms, which result to anharmonicity of current-phase dependence of weak links. Anharmonicity current-phase dependence is shown at presence of maximum harmonics, i.e. when the sizes of the first harmonic are comparable to second.

Basically, it – own inductance, change of capacity formed streak, low temperature rather from critical, deformation of thickness of a layer, the heterogeneity of a layer dividing superconducting electrodes, fluctuating processes on boundary undressed.

The research of physical properties Josephson junctions (JJ) on a basis HTSC shows, that the amplitude of the second harmonic in the ratio current-phase depends on orientation d-wave superconducting parameter about under the attitude of a plane JJ [15]:

$$I_s(t) = I_{c1}\sin(\omega t + \varphi) + I_{c2}\sin 2(\omega t + \varphi), \quad (1)$$

where $\omega = 2eV/\hbar$.

In work [10] anharmonic character superconducting current is explained by introduction of self-inductance JJ:

$$j(\varphi) = \sin(\varphi - lj(\varphi)), \tag{2}$$

where $l = 2\pi\hat{O}/\hat{O}_0 = 2\pi LI_c/\hat{O}_0$ ($\hat{O}_0$ – quantum of a magnetic flux). The small meaning l enables approximate the equation (2):

$$j(\varphi) = \sin\varphi - l\sin(2\varphi)/2\,. \tag{3}$$

In works [16, 17] was offered the generalized model anharmonicity of current-phase dependence of weak links described by expression (fig. 2):

$$j(\varphi) = j_c\left(\sin\varphi - \frac{1}{k}\sin(k\varphi)\right) \tag{4}$$

where

$$k = f\left(L, T, k_{\text{hl}}, d, k_{\hat{o}\,\ddot{e}}\right) \tag{5}$$

$k > 1$ – coefficient (degree) anharmonicity of dependence current-phase JJ; L – inductance of junction; I_c – critical current; T_c – critical temperature; T – temperature of specimen; d – deformed thickness; k_{hl} – coefficient of heterogeneity of a layer; k_{fl} – fluctuating coefficient.

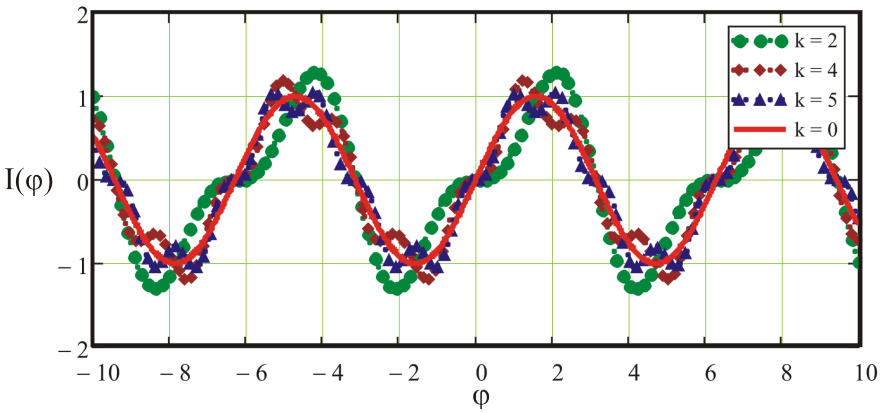


Fig. 2. Possible variants anharmonicity of current-phase dependence in weak links at different meanings of coefficient (degree) anharmonicity.

3. INFLUENCE ANHARMONICITY OF CURRENT-PHASE DEPENDENCE ON PROPERTIES OF WEAK LINKS OF JOSEPHSON TYPE

3.1 Influence of anharmonicity dependence current-phase on plasma frequency WLJT

The By one of parameters WLJT reacting on change of a parity the current-phase is plasma frequency ω_{∂} [9, 18].

For definition of influence anharmonicity a current - phase on plasma frequency we use the equation of phase dynamics of WLJT:

$$\frac{d^2\varphi}{dt^2} + a \cdot \frac{d\varphi}{dt} + \sin\varphi = i \tag{6}$$

where φ – phase, $i = I/I_c$ – the normalized current or constant component of the current, enclosed to junction.

It is known, that at influence of pulse currents observe a deviation of current-phase dependence from the law of a sine. As a pulse current is used cosine a pulse (fig. 3) [19].

We transform the equation (6) for our case

$$\frac{d^2\varphi}{dt^2} + a \cdot \frac{d\varphi}{dt} + \sin\varphi - \frac{\sin(k\varphi)}{k} = i + i_{\text{imp}} \cos\omega t - i_{\text{imp}} \cos\theta\,. \tag{7}$$

where φ – phase, i – constant component, i_{imp} – pulse component current, θ – cutoff angle.

At $\theta = 90^\circ$ right part (7) will be transformed on (7): $i + i_{\text{imp}}\left(\cos\omega t - 1\right)$. For calculation ω_p we shall take advantage of a method I.N.Askerzade [18]: a phase part (7) we break into 2 parts: quickly varying φ_{dyn} and slowly varying φ_{sl}:

$$\varphi = \varphi_{\text{dyn}} + \varphi_{\text{sl}} \tag{8}$$

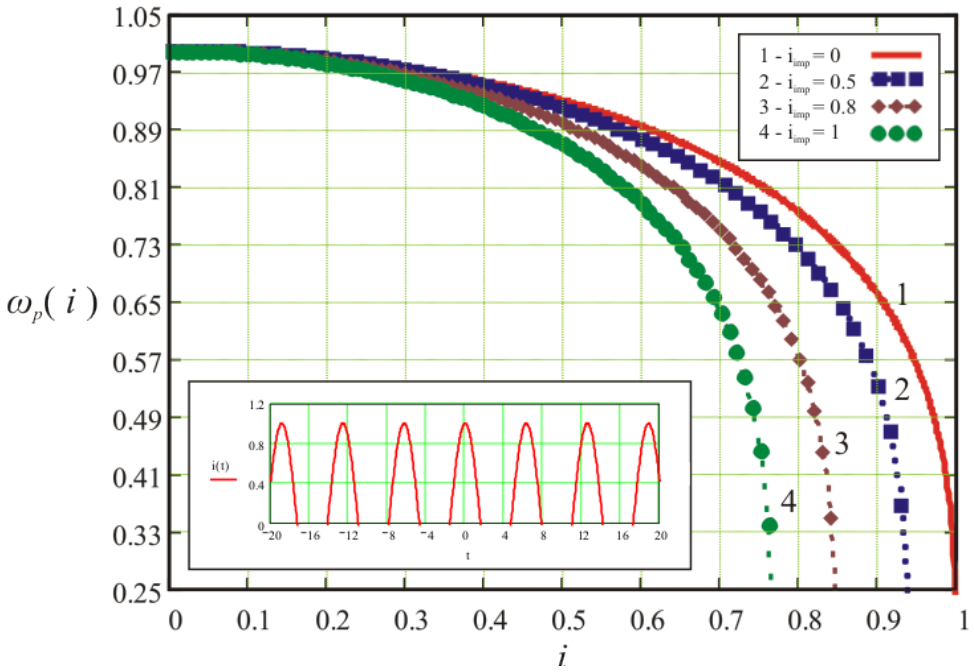


Fig. 3. Dependence of plasma frequency ω_p on size of a pulse current i_{imp} for weak superconductivity with harmonic by a ratio current-phase

Having substituted meanings of a phase (8) in expression (7) и taking into account insignificant size of change of a slowly varying part of a phase $d\varphi_{sl}/dt \Rightarrow 0$:

$$\frac{d^2\varphi_{dyn}}{dt^2} + a \cdot \frac{d\varphi_{dyn}}{dt} + \sin\left(\varphi_{dyn} + \varphi_{sl}\right) - \frac{1}{k}\sin(k(\varphi_{dyn} + \varphi_{sl})) = i + i_{imp}(\cos\omega t - 1). \quad (9)$$

The results of the numerical decision of the equation (9) are given in a fig. 3. For a numerical method of the decision of the given differential equation the traditional methods, as a method Runge-Kutty are not suitable, in connection with simultaneous presence quickly varying φ_{dyn} and slowly varying φ_{sl} parts of a phase. Therefore to the decision (9) numerical method have applied the operator radau for rigid system of the differential equations, which returns a matrix of the decisions of the differential equation specified derivative and the entry conditions on a certain interval on a method radau.

In figure 4 the change of plasma frequency with increase of coefficient (degree) anharmonicity is shown. For account the meanings of coefficient anharmonicity k = 0, 0.5, 0.8, 1.5, 2 are chosen.

With increase of coefficient anharmonicity the meaning of plasma frequency is increased too. The curve (triangle) corresponds to meaning k = 0.5, rectangulars – k = 0.8, rhombus – k = 1.5. Here case k = 0 is described harmonic character of parity by a current-phase (continuous curve). Hence, anharmonicity parities the current-phase increases sizes of plasma frequency.

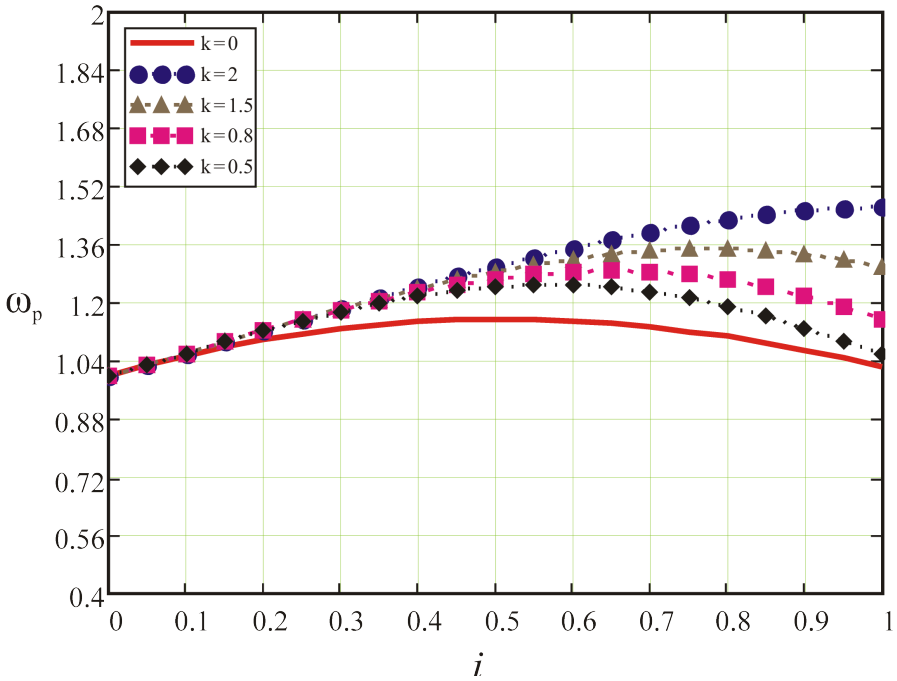


Fig. 4. Dependence of plasma frequency from anharmonicity of current-phase at various meanings k

3.2 Influence of anharmonicity dependence current-phase on current-voltage characteristic WLJT and interpretation of formation of a fractional steps Shapiro

For definition of influence of anharmonicity dependence current-phase on current-voltage characteristic (CVC) WLJT, we shall take advantage of dependence between plasma frequency ω_{∂} and parameter Makkamber-Stjuart $\beta = \left(\omega_{\tilde{n}}/\omega_{\partial}\right)^2$ (dimensionless capacity, where $\omega_{\tilde{n}}$ – characteristic frequency).

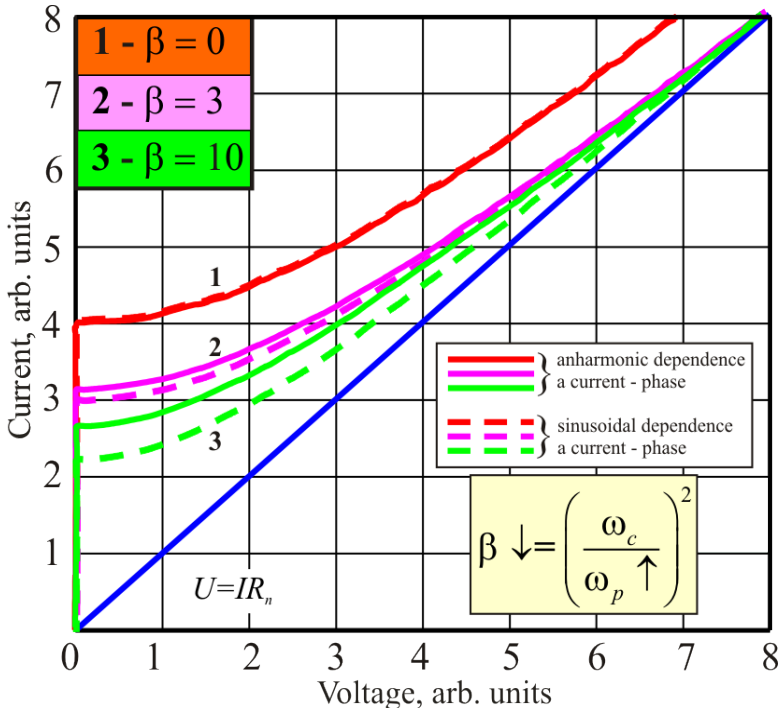


Fig. 5. Current-voltage characteristic WLJT with harmonic (dotted) and anharmonic (continuous) current-phase dependences.

Numerical modeling CVC WLJT with harmonic current-phase dependences has shown, that with increase of coefficient (parameter) Makkamber-Stjuart $\beta = \left(\omega_{\tilde{n}}/\omega_{\partial}\right)^2 = 2\pi I_c R_n^2 C/\hat{O}_0$ its hysteresis (fig. 5, continuous curves) is increased, and at anharmonicity dependence of current-phase the hysteresis CVC WLJT (fig. 5, dotted curves) decreases. The degree of reduction of a hysteresis CVC WLJT depends on meaning of coefficient β, with increase of capacity (or β) WLJT the reduction of a hysteresis CVC is shown intensive, and at $\beta < 1$ do not observe change in CVC. It is explained to that in case of a deviation meaning the current-phase from sine wave dependence is increased plasma frequency ω_p, which back is proportional to coefficient β. Results in reduction and by that to reduction of a hysteresis CVC WLJT (fig. 5) [17].

One of features of properties anharmonic WLJT is the occurrence on CVC fractional (or semi-integer) steps Shapiro under influence high-frequency or microwave frequency (superhigh-frequency) of radiation. Within the framework of the given work of formation fractional of steps Shapiro the under influence of anharmonicity dependence a current-phase and reduction of a local superconducting

branch CVC under influence high-frequency or microwave frequency signal is explained by increase of size of plasma frequency [20-22].

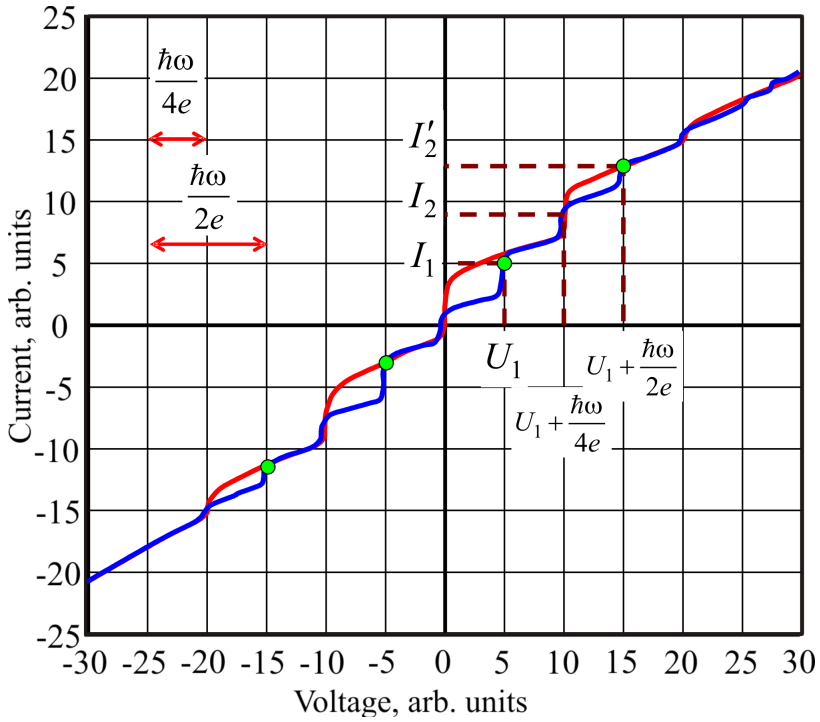


Fig. 6. To an explanation of process of fractional (or semi-integer) steps Shapiro on CVC WLJT with anharmonicity of current-phase dependence.

When the influence of a microwave of a signal reduces ω_p, anharmonicity dependence a current-phase increases ω_p, it results in change of coefficient $\beta = \left(\omega_c/\omega_p\right)^2$ determining superconducting a branches CVC WLJT. The increase under action anharmonicity dependence a current-phase occurs in "points" $n\hbar\omega/4e$ ($n = \pm 1,\ \pm 2,\ ...$) and reduces superconducting of a branch CVC WLJT (fig. 6, the circles designate points of formation fractional (or semi-integer) of steps Shapiro).

3.3 Features of quantum interference under influence anharmonicity dependence of current-phase

In work [23] it was experimentally revealed, that in normalized magnetic flux the maximal supercurrent $I_{c\,\max}$ JJ depends on size of a magnetic flux $\hat{\Phi}$ penetrating into contact. The received dependence $I_{c\,\max}\left(\hat{\Phi}/\hat{\Phi}_0\right)$ is similar to dependence of intensity of light at diffraction on a single crack from distance up to the central point and demonstrates wave properties superconducting currents.

For revealing features of quantum interference WL with anharmonicity dependence by current-phase we shall consider a case of influence of a strong magnetic field on them. For this case of an external magnetic field satisfies a condition $H_0 >> \hat{\Phi}_0/\left(2\pi\lambda_J d\right)$.

Integrate expression of phase dynamics JJ at influence of a strong external magnetic field:

$$\varphi(x) = \frac{2\pi d}{\Phi_0} I_0, \tag{10}$$

in result:

$$\varphi(x) = \frac{2\pi d}{\Phi_0} I_0 x + C \tag{11}$$

where C – constant integration.

Substituting (11) in expression for density of a current with anharmonicity of parity a current-phase (4) and integrate on lengthways weak links, we find size of a complete current through weak links

$$I_s = j_c \int_{-L/2}^{L/2} \left[\sin\left(\frac{2\pi x I_0 d}{\Phi_0} + C_1 \right) - \frac{1}{k} \sin\left(\frac{2\pi x k I_0 d}{\Phi_0} + C_2 \right) \right] dx. \tag{12}$$

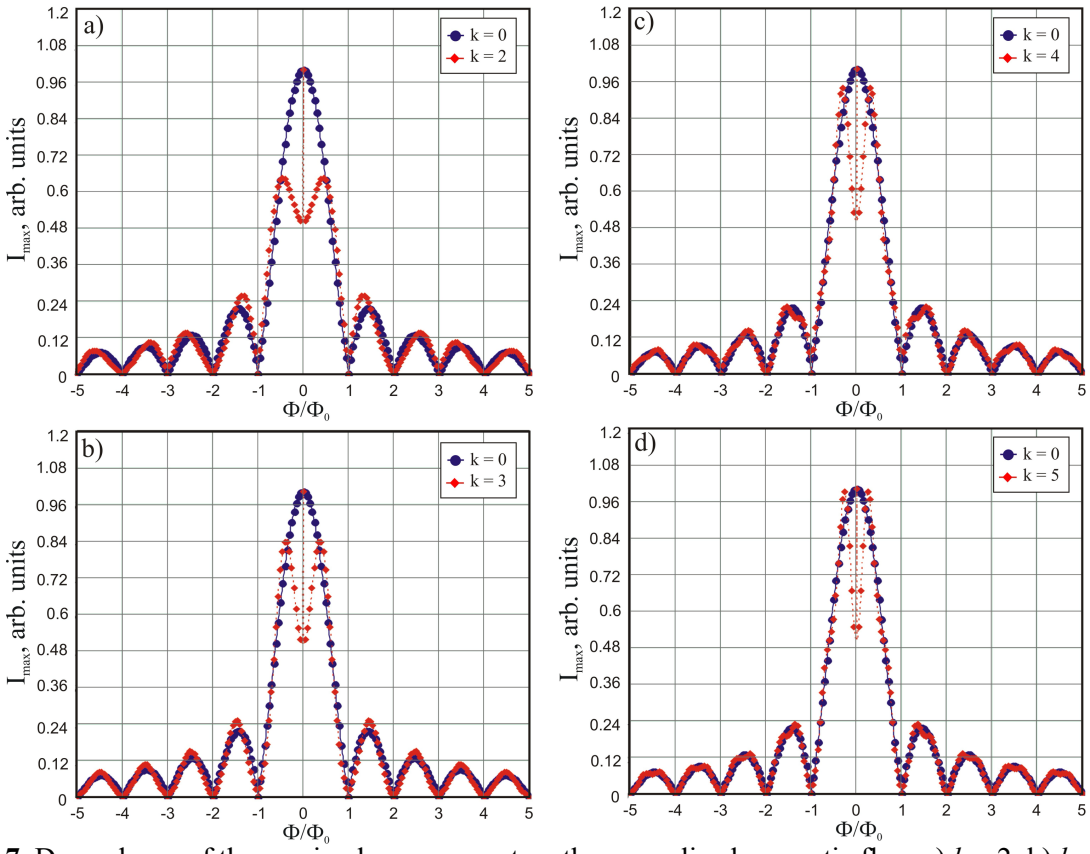


Fig. 7. Dependence of the maximal supercurrent on the normalized magnetic flux: a) $k = 2$; b) $k = 3$; c) $k = 4$; d) $k = 5$.

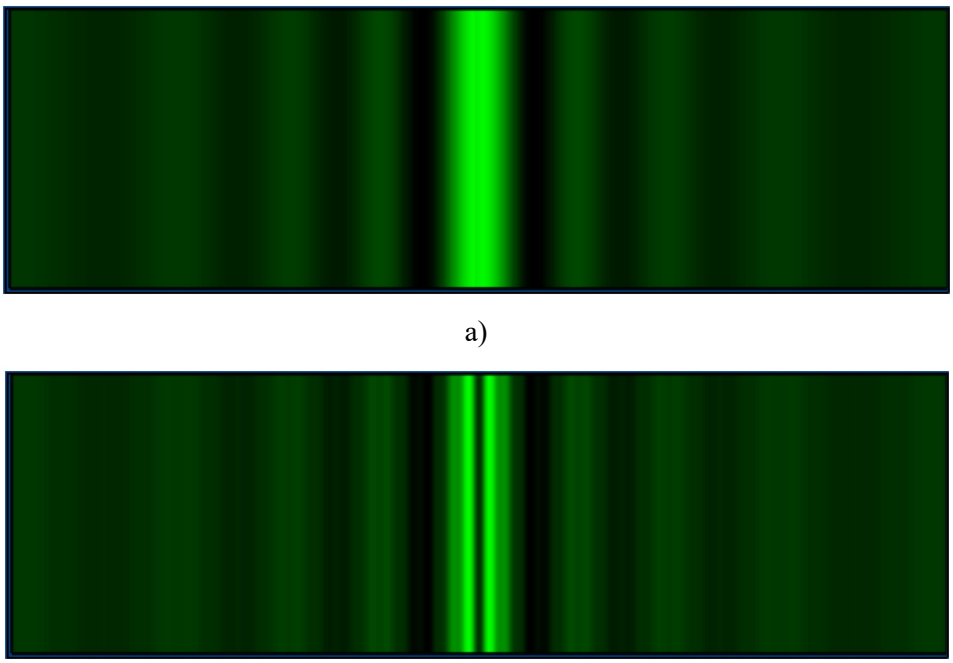
As a result of integration (12) and using expression $\hat{O} = L\acute{I}_0 d$ we shall receive:

$$I_{\max} = I_c \left| \frac{\sin(\pi \hat{O}/\hat{O}_0)}{\pi \hat{O}/\hat{O}_0} - \frac{1}{2k} \frac{\sin(k\pi \hat{O}/\hat{O}_0)}{\pi \hat{O}/\hat{O}_0} \right|. \quad (13)$$

For meanings $k = 2,3,4,5$ the dependence of the maximal supercurrent on the normalized magnetic flux is given in figure 7.

As it is visible, from a fig. 7 feature of dependence of a supercurrent from the normalized magnetic flux (quantum interference) in "anharmonic" JJ as against "harmonic" is formations of two maxima of a current, the response of top at meaning of the normalized flux at $\hat{O}/\hat{O}_0 = 0$, and also occurrence of minima in fractional meanings of the normalized magnetic flux [24, 25].

In a fig. 7 the diagrams of dependence the maximal supercurrent – normalized magnetic flux are given at k = 2, 3, 4, 5, and in a fig. 8 – interference figure in single weak links for harmonic and anharmonic current-phase dependence.



a)

b)

Fig. 8. Visualization of interference figures in single weak links with harmonic (a) and anharmonic (b) current-phase dependence.

4. CONCLUSION

By introduction of coefficient anharmonicity *k* developed model anharmonicity of current-phase dependence SWL.

On the basis of the differential equation describing dynamics of SWL with anharmonicity of current-phase dependence, is determined the influence of anharmonicity current-phase dependence on plasma

frequency SWL. Is revealed, that anharmonicity parities the current-phase increases size of plasma frequency.

Is established, that anharmonicity of current-phase dependence results in reduction of a hysteresis CVC SWL. The degree of reduction of a hysteresis CVC SWL depends on meaning of parameter Makkamber-Stjuart β, with increase of capacity of SWL the reduction of a hysteresis CVC is shown intensive, and is not observed at $\beta < 1$ change in CVC.

On the basis of researches modelling dynamics of plasma frequency is interpreted the occurrence fractional (or semi-integer) steps Shapiro on CVC Josephson of structures with anharmonicity current-phase dependence at influence of a microwave of a signal.

Within the framework of the theory Ferrel-Prange for SWL with anharmonicity the current-phase dependence receives expressions describing influence of a strong magnetic flux on WLJT as with anharmonicity of parity current-phase.

Is shown, that feature of dependence of supercurrent on the normalized magnetic flux (quantum interference) in "anharmonic" JJ as against "harmonic" is formations of two maximum of a current, the response of top at meaning of the normalized flux at $\hat{O}/\hat{O}_0 = 0$, and also occurrence of minima in fractional meanings of minima in fractional meanings of the normalized magnetic flux.

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