

**THE HYDRODYNAMIC EFFECTS ON THE WALLS
OF CONTAINER DEVELOPED DURING AN EARTHQUAKE**

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Abstract

Ground-supported tanks are used to store a variety of liquids. During earthquake activity the liquid exerts impulsive and convective pressures (sloshing) on the walls and bottom of the rectangular tank. This paper provides theoretical background for calculating of hydrodynamic pressures developed during an earthquake in rectangular container - endlessly long shipping channel. Analytical diagrams of hydrodynamics pressures, eq. (5), (12) and (14) are used for computing of hydrodynamic pressures on the wall of the rectangular tank. The problem was numerically modelled by using FEM formulation in software CosmosM.

Key words: earthquake, rectangular tank, impulsive pressures, convective pressures, finite element method

INTRODUCTION

Liquid storage tanks are considered essential lifeline structures. Large-capacity ground-supported tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes [2 – 12]. Knowledge of pressures and forces acting on the walls and bottom of containers during an earthquake is important for good analysis and design of earthquake resistant structure/facility – tanks, rectangular endlessly long shipping channel, which are made from steel or concrete.

HYDRODYNAMIC PRESSURES ON THE WALL OF TANK

Consider a rectangular container as shown in Figure 1, and at the instant under consideration let the surface of the fluid be horizontal and let the walls of the container have a horizontal acceleration $\ddot{u}_o$ in the x – direction, [1].

Let it be required to find the pressures on the walls of the container due to the acceleration $\ddot{u}_o$. Let the fluid have a depth H , a length $2L$ and a unit thickness, Figure 1a. It is seen that the action of the fluid is similar to that which would be obtained if the horizontal component of fluid velocity $\dot{u}$, were independent of the y coordinate; that is, imagine the fluids to be constrained by thin, massless, vertical membranes free to move in the x – direction, and let the membranes be originally spaced a distance dx apart.

When the walls of the container are given an acceleration the membranes will be accelerated with the fluid and the fluid will also be squeezed vertically with respect to the membranes.

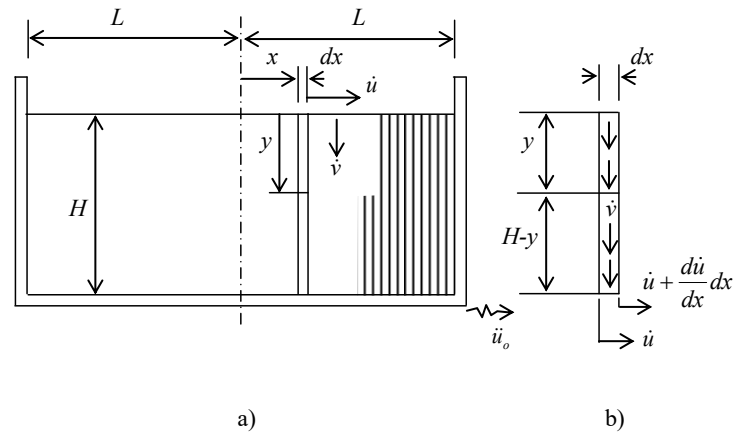


Figure 1. Rectangular tank is filled with water

As shown in Figure 1b, since the fluid is restrained between two adjacent membranes, the vertical velocity $\dot{v}$ is dependent on the horizontal velocity $\dot{u}$ according to

$$\dot{v} = (H - y) \frac{d\dot{u}}{dx}. \quad (1)$$

This is an equation specifying the constraint on the fluid flow. As the fluid is considered incompressible, it follows that the acceleration $\ddot{v}$ is proportional to the velocity $\dot{v}$ and the acceleration $\ddot{u}$ is proportional to the velocity $\dot{u}$, and the pressure in the fluid between two membranes is given by the standard hydrodynamical equation:

$$\frac{\partial p}{\partial y} = -\rho \ddot{v}, \quad (2)$$

where ρ is density of the fluid.

2.1. Hydrodynamic impulsive pressures

The acceleration $\ddot{u}_0$ thus produces an increase of pressure on one wall and a decrease of pressure on the other wall of

$$p_w = \rho \ddot{u}_0 H \left(\frac{y}{H} - \frac{1}{2} \left(\frac{y}{H} \right)^2 \right) \sqrt{3} \tanh \sqrt{3} \frac{L}{H}. \quad (3)$$

In dependence on $\xi = z/H$, where distance z is distance (height) from the bottom of tank to the free surface of the liquid $z = H - y$, the hydrodynamic pressures on wall are given

$$p_{HDw} \left(\frac{z}{H} \right) = \rho \ddot{u}_0 H \frac{\sqrt{3}}{2} \left(1 - \left(\frac{z}{H} \right)^2 \right) \tanh \sqrt{3} \frac{L}{H}. \quad (4)$$

Hydrodynamic impulse pressures are as well given by

$$p_{HDw}(\xi) = C_{iw}(\xi) \rho H A_g(t); \quad (5)$$

the free-field motion of the ground is here represented by the peak value of $A_g(t)$ is denoted by a_g . ρ is mass density of liquid. The distribution of hydrodynamic pressures on the wall of tank $p_{HDUw}(\xi)$ along the height of wall is given by eq. (5), where the function $C_{iw}(\xi)$ (eq. 6) is shown in Figure 2 in dependence on slinness of tank γ . The function $C_{iw}(\xi)$ presents function of partitions of the pressures on the wall for $\xi = 0$ (i.e. at the wall of the tank) at the bottom of the tank and for $\xi = 1$ (i.e. at the wall of the tank) at the free surface. Tank's slinness is given by relation $\gamma = H/R$, where H is the height of filling of fluid in the container and L is half of width of container.

$$C_{iw}(\xi) = \frac{\sqrt{3}}{2} (1 - \xi^2) \tanh \sqrt{3} \frac{L}{H}. \quad (6)$$

$ksi = z/H$

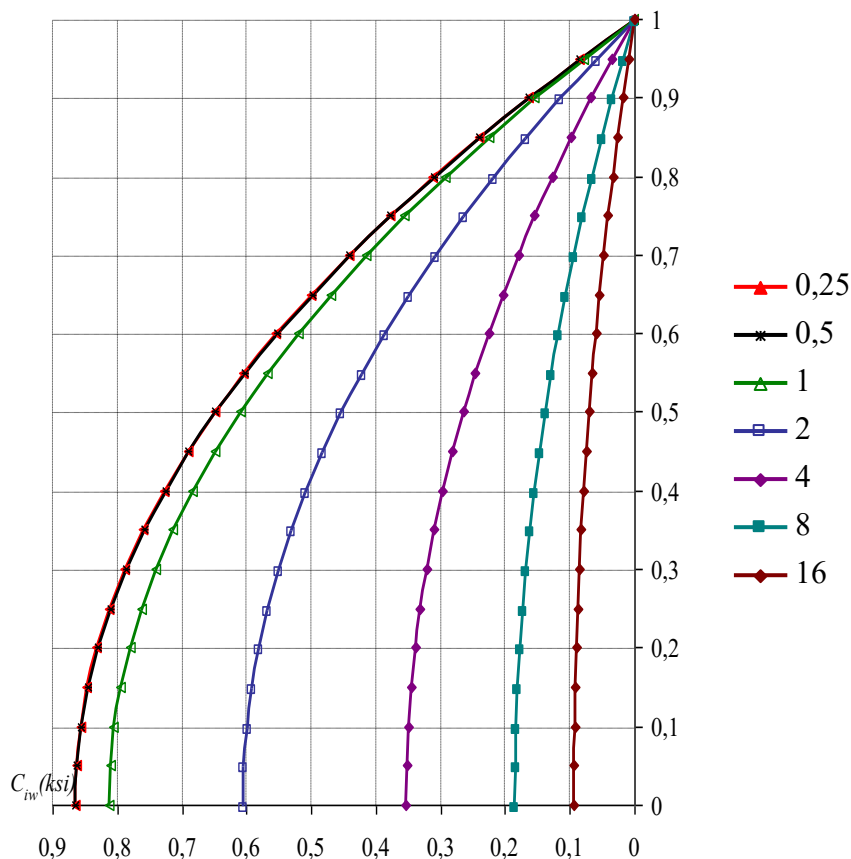


Figure 2. Functions $C_{iw}(\xi)$ of partitions of the impulse pressures on the wall of tank in dependence on slinness of the tank γ .

2.2. Hydrodynamic convective pressures

The effect of the impulsive pressures is to excite the fluid into oscillations. To examine the fundamental mode of vibration, consider the fluid to be constrained between rigid membranes that are free to rotate as shown in Figure 3.

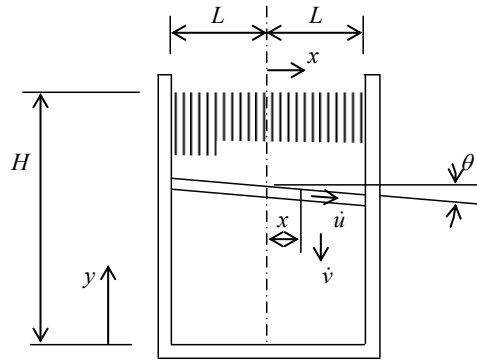


Figure 3. Rectangular tank are filled with water

The constraint is described by the following equations:

$$\dot{u} = \frac{L^2 - x^2}{2} \frac{d\dot{\theta}}{dy}, \quad (7)$$

$$\dot{v} = \dot{\theta} z. \quad (8)$$

The pressure in the fluid is given by

$$\frac{\partial p}{\partial x} = -\rho \ddot{u}. \quad (9)$$

The hydrodynamic pressures are given by

$$p_w = \left(\rho \frac{L^3}{3} \sqrt{\frac{5}{2}} \frac{\cosh \sqrt{\frac{5}{2}} \frac{x}{L}}{\sinh \sqrt{\frac{5}{2}} \frac{H}{L}} \right) \omega^2 \theta_0 \sin \omega t \quad (10)$$

In dependence on $\xi = z/H$, where distance z is distance (height) from the bottom of tank to the free surface of the liquid $z = y$, the convective hydrodynamic pressure is given by a summation of modal terms (sloshing modes), each one having a different variation with time.

$$p_{HDCwn}(\xi) = \sum_{n=1}^{\infty} Q_{cwn}(\xi) \rho L A_n(f_{cn}). \quad (11)$$

As for rectangular tanks, the dominant contribution is that of the fundamental mode, that is:

$$p_{HDCw1}(\xi) = Q_{cw1}(\xi) \rho L A_1(f_{c1}); \quad (12)$$

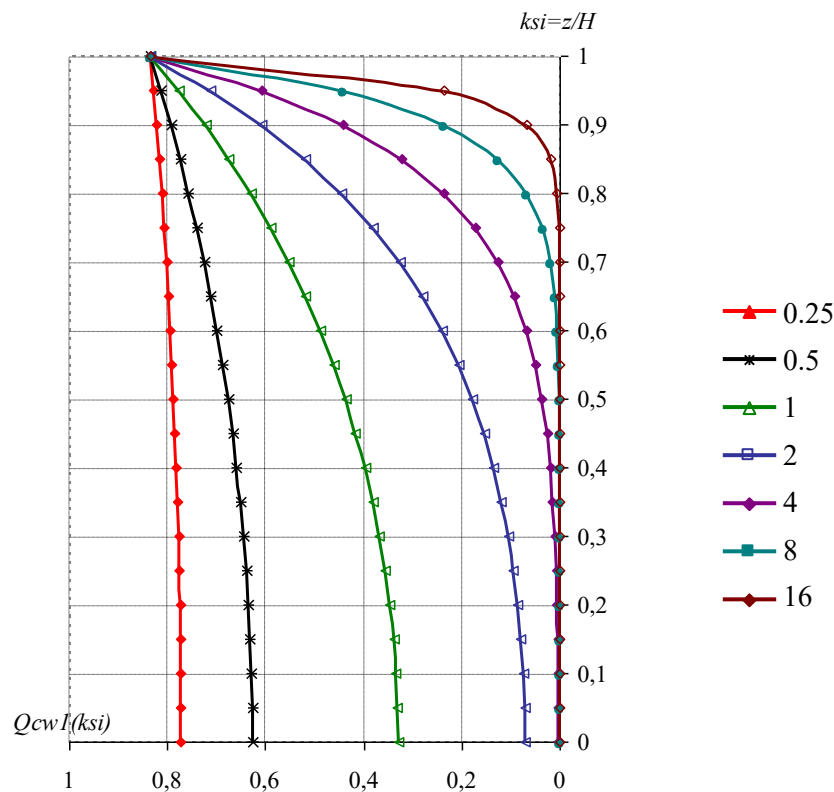


Figure 4. Functions $Q_{cwl}(\xi)$ of partitions of the impulse pressures on the wall of tank, in dependence on slimmess of tank γ

the free-field motion of the ground is here represented by the peak value of $A_I(t)$ is the acceleration response function of a simple oscillator having the frequency of the first mode, the appropriate value of the damping, and subjected to an input acceleration $A_g(t)$. ρ is mass density of liquid. The distribution hydrodynamic pressures on the wall of tank $p_{HDCwl}(\xi)$ along the height of wall is given by eq. (12), where the function $Q_{cwl}(\xi)$ (eq. 13) is represented in Figure 4 in dependence on slimmess of tank γ . The function $Q_{cwl}(\xi)$ present function of partitions of the pressures on the wall for $\xi = 0$ (i.e. at the wall of the tank) at the bottom of the tank and for $\xi = 1$ (i.e. at the wall of the tank) at the free surface. Tank's slimmess is given by relation $\gamma = H/R$, where H is the height of filling of fluid in the tank and L is half of width of container.

$$Q_{cwl}(\xi) = 0,833 \frac{\cosh\left(\xi \frac{1}{2} \sqrt{\frac{5}{2}} \frac{H}{L}\right)}{\cosh\left(\frac{1}{2} \sqrt{\frac{5}{2}} \frac{H}{L}\right)}. \quad (13)$$

2.3. Resultant hydrodynamic pressures

For tanks, whose walls can be assumed as rigid, a solution of the Laplace equation for horizontal excitation can be obtained in a form, so that the total pressure is again given by the sum of impulsive and convective pressures by use of absolute summation rule [14]:

$$p_{HDw}(\xi) = p_{HDIw}(\xi) + p_{HDCwl}(\xi). \quad (14)$$

SOLUTION, RESULTS AND DISCUSSION

Let us have a ground supported rectangular endlessly long shipping channel, having dimension of $L = 5$ m and height $H_w = 3$ m. Walls have uniform thickness of 0.25 m. The base slab is $h = 0.4$ m thick. The shipping channel is filled with water to the height 2.6m, Figure 5. There is no roof slab on the tank. The tank is situated on hard soil. Commercial software CosmosM is used for numerical modeling of the problem by using FEM formulation.

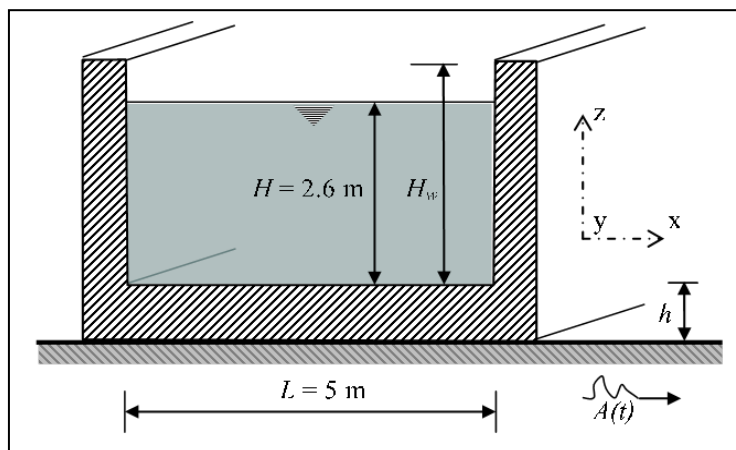


Figure 5. Details of tank geometry

We consider only horizontal earthquake loading by using of recording of real accelerogram Loma Prieta, California (18.10.1989), which is shown in Figure 6 in dependence on time; seismic excitation is only along x - direction. Horizontal earthquake loading starts to act at the time 10 s. Now, only hydrostatic pressure acts at the walls of the shipping channel during time 0 – 10 s, [11 – 14].

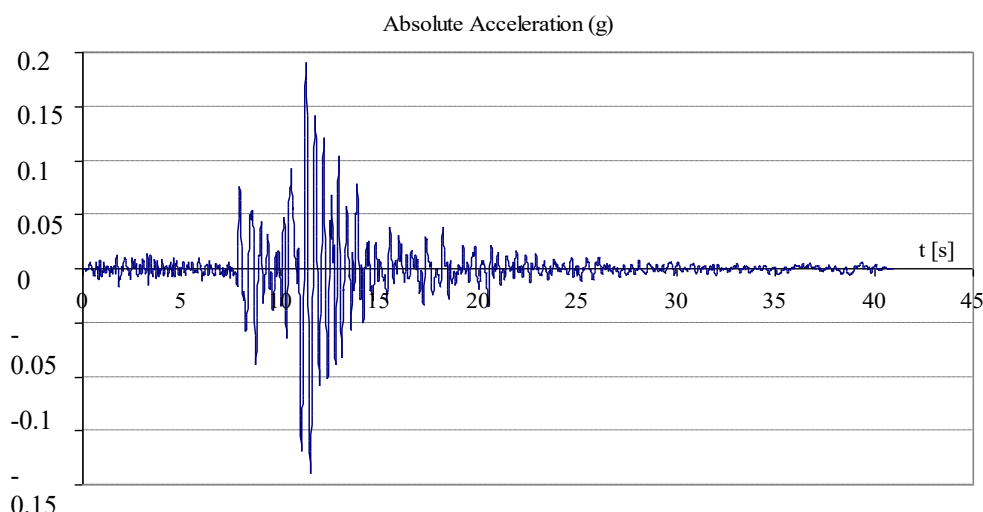


Figure 6. Accelerogram Loma Prieta, California (18.10.1989)

Figure 7 shows elastic response spectrums, which are calculated for base acceleration from the figure 6, for 0.5 %, 2 %, 4 %, 5 % and 7 % damping.

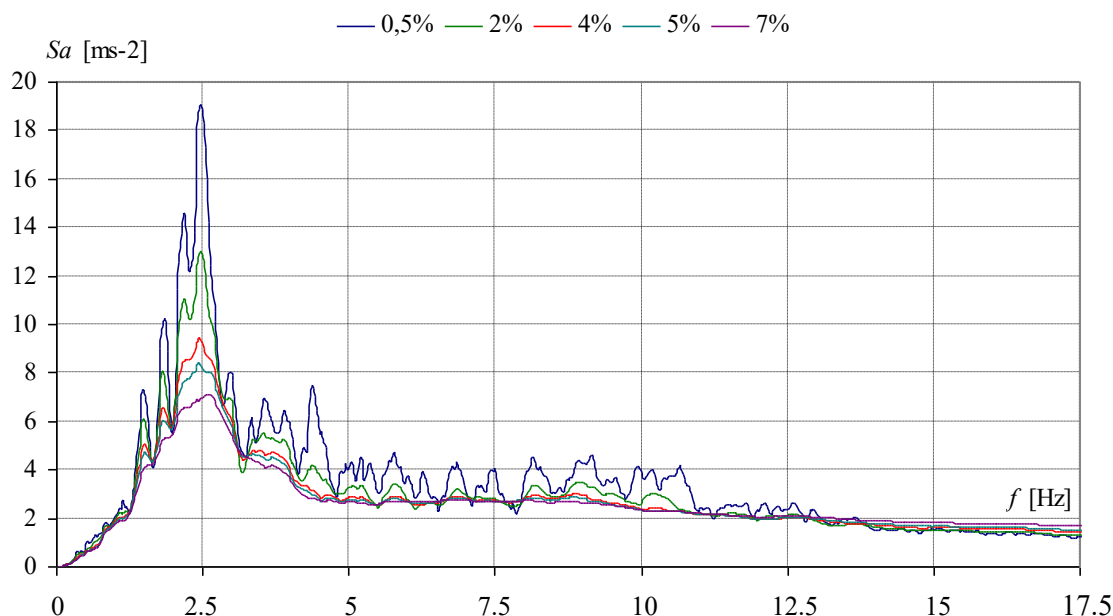


Figure 7. Elastic response spectrums for accelerogram Loma Prieta from Figure 6

Total base shear and bending moments of ground supported tank at the bottom of the wall can be also obtained by base shear from impulsive pressures and convective pressures – flow, vibration of walls. Impulsive pressures are obtained from equations (4); hydrodynamic convective pressures are obtained from equations (11). Figure 8 shows the parts of base shear V caused only by influence of seismic loading at the bottom of the wall of reservoirs and figure 9 shows the part of bending moments M caused only by influence of seismic loading at the bottom of the wall of reservoirs.

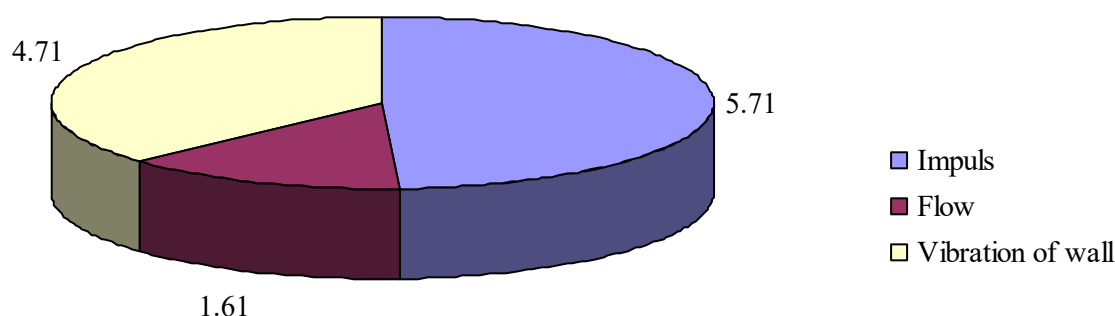


Figure 8. Comparison parts of base shear V caused only by influence of seismic loading at the bottom of the wall of reservoirs

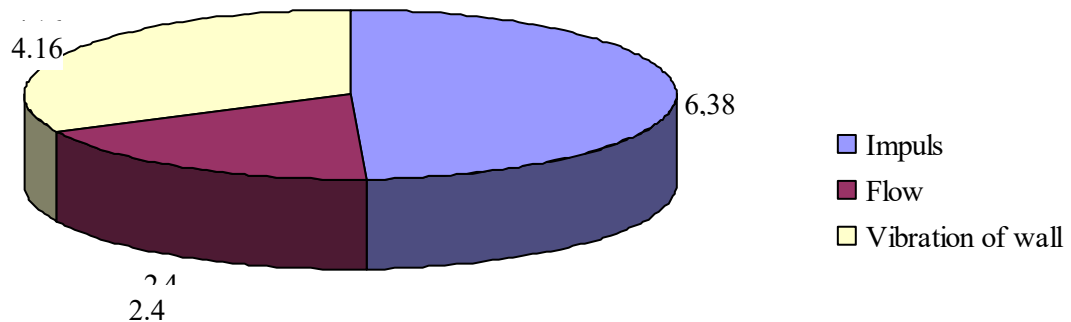


Figure 9. Comparison parts of bending moment M caused only by influence of seismic loading at the bottom of the wall of reservoirs

Figure 10 shows comparison of base shear V and bending moment M caused only by influence of seismic loading at the bottom of the wall of reservoirs and of total base shear V and total bending moment M caused by influence of all effects, which can occur as a result of influence of water on the wall of reservoirs during seismic loading at the bottom of the wall of reservoirs.

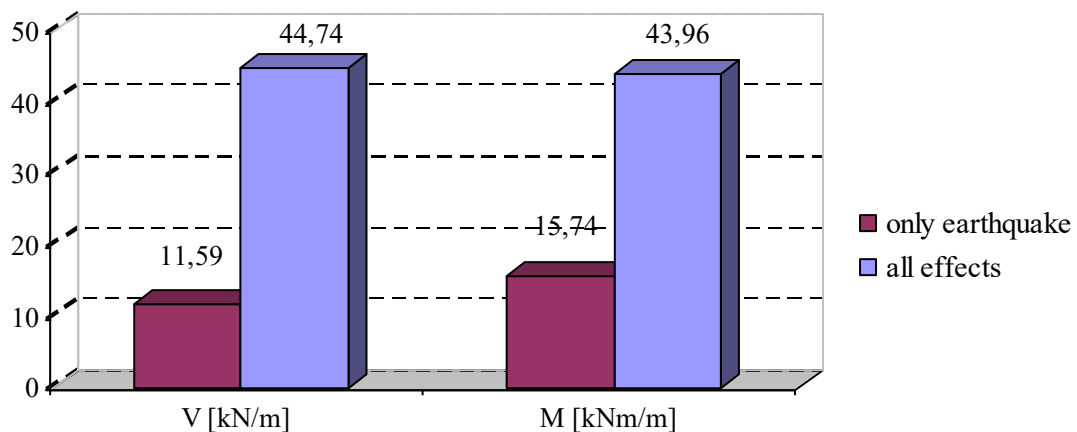


Figure 10. Comparison of base shear V and bending moment M only caused by influence of seismic loading at the bottom of the wall of reservoirs and of total base shear V and total bending moment M caused by influence of all effects, which can occur as a result of influence of water on the wall of reservoirs during seismic loading at the bottom of the wall of reservoirs

CONCLUSION

Figure 8 shows parts of hydrodynamic effects of fluid at the wall of the shipping channel developed during an earthquake. Individual parts are: hydrodynamic impulsive pressures, which are obtained from equations (4); hydrodynamic convective pressures, which are obtained from equations (11); and vibrations of walls. Figure 9 shows parts of hydrodynamic effects of fluid at the wall of the shipping channel developed during an earthquake. Influence of water on the wall of reservoirs during seismic

loading (akcelerogram Loma Prieta, California (18.10.1989)) at the bottom of the wall of reservoirs is seen from figure 10, which shows comparison of influence of only hydrodynamic effects and of all effects of water on the wall.

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