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MATHEMATICAL METHODS FOR VIBRATIONS AND THEIR MEASUREMENT

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Abstract

For non-linear dynamic systems transformation of inputs into outputs happens with essential distortions and delay. New methods of mathematical modeling are required in the process of designing of such systems. Expert - statistical method is used in processing of the signals informing about the level of vibration. An approach oriented on identification of the technical condition of the rolling bearing proper and measurement (sometimes one-time) of the parameters of the high-frequency vibration (frequency analysis of the envelope In the paper we present the statistical approach to mechanics.

Key words: *vibration, bearing defects detection, signals filtering*

1. INTRODUCTION

Within the framework of internationalcooperation with St. Peterburg University of Information Technologies, Mechanics and Optics training-scientific laboratory of automatic assembling of optical products is being created. The primary task of this project: to automatize the selective picking of the optic-mechanical units and to place them into the jacket with the optimum combination. To attain the stated goal it is necessary to control the accuracy of the assembly unit placing into the jacket. There is a probability of the component misalignment and its wedging in the jacket. „Expertstatistical" method is applied during processing of the signals informing about the level of vibration. Vibration test bench with PT 500.04 PC Based Evaluation Software is used to collect diagnostic information and to carry out statistical recognition of states. In this paper we examine statistical properties of vibration signals providing bearing defect frequency analysis. To detect bearing failure by vibration monitoring we determine the typical bearing defect frequencies. Every time a defect is over rolled it causes a peak in the vibration signal. The repetition of these peaks depends on the position of the defect on the bearing (inner or outer ring or on a rolling element, etc.) and the bearing geometry and speed. PT 500.04 PC Based Evaluation Software is used to assist in these analyses.

In the paper we verify the exactness of calculated defect frequencies carrying out measurement experiments. Using statistical methods we compare experimental results with the date presented in the PT 500.04 PC Based Evaluation Software. Also we obtain our estimations of the bearing defect frequencies. Also in the paper one special method for detection of bearing defects is considered. The result of the process enables the users to analyse the signal and draw conclusions on the state of the bearing without interference noise from structural phenomenon in the machine.

2. FUNDAMENTAL ANALYSIS OF VIBRATION

2.1 Methods of treating of the physically realized signals

Physically realized signals are investigated. The method of analysis of the physically realized signals generated by difficult dynamic systems with damage elements is offered. The statistical approach to analysis of signals is used. The dynamic essence of the processes generating vibrations signals [1] is known. Modern nonlinear dynamics consider analysis of signals as process of identifying of dynamic systems on experimental data. Process of identifying is carried out always without estimations of accuracy of the measurement results.

Let's esteem dynamic system as differential equations [2]:

$$H \cdot u(t) = f(t) \qquad (1)$$

Where

H – the linear differential operative; $u(t)$ – the output signal of the amplifier PT 500.04 PC.

The decision of this system can be periodic or exponential functions as only these functions have to themselves similar derivatives. In experiments with the damaged bearings there should be steady decisions, but can be found and unstable decisions. In work characteristic dynamic decisions such as periodic functions as the question is structural stability were studied. In the given work antifriction bearings with known faults were studied. The system of vibration control with the software PT 500.04 PC is very beneficial as the exploratory tool to the specialist on diagnostic. In this system the set of algorithms of check of a percussive state of mechanical units is realized. Specialists can complicate or simplify analysis of percussive cues. Such variety of algorithms testifies to absence of uniform mathematical model of statistical recognition of faults. The system of vibration control PT 500 can become the universal collector of the diagnostic information if system to supplement digital measuring device Fluke 45. Thus, it is possible having augmented bulk of the collected data on percussive processes in the mechanism (Figure 1). By results of work it is possible to judge opportunities of the advanced test bench and the software. Magnification of bulk of the collected data is necessary for statistical treating results of measuring. For definition of modes of vibration in system of vibration control PT 500.04 use piezoelectric one component accelerometer AS-020. With the help of two sensors it is possible to measure only two projections of a vector of speed-up of vibration to fulcrums of the sensor. Accommodation of a lot of one component sensors demands further advancing of the test stand. Sensitive elements of sensors are inevitably disposed in two different points of space. There are two measuring points with different coordinates; they cannot be esteemed as projections of a vector of speed-up of vibration. Nevertheless, this circumstance is ignored, that distorts the information on a level of vibration of mechanical units in dots of check.

Comparing frequencies, proper maximal conditional units of the state of bearings [BCU (Bearing condition unit)], it is possible to distinguish damage of the bearing box with some of uncertainty of measurement. The greatest concern introduces the response of oscillating from a complex of defective mechanical units. Feature of damage of the bearings is imaged by a complex of the frequencies applicable to several kinds of damages. Effective applying of diagnostic is confined to the major problem connected to ambiguous mutual conformity between probable damages and their display in vibrations signals. The situation, when some damages are generated identical variations of spectra of vibration is probable. The decision of this problem can be achieved by increase of detail of the receivable information and perfection of algorithms of treating of vibrations signals. To augment bulk of the data on vibrations processes in the mechanism hooking up of exact digital multimeters will

allow. The problem connected to complication of forming of yardsticks of the importance of variations of diagnostic attributes, can be realized with the help such upgraded the system of vibration control PT 500.04.

The candidate solution of system (1) depends on resistance forces in the broken–down mechanical units. In our researched mechanical system process of damage is developed slowly. High damping as damages in mechanical units are known is realized. If damages were slowly developed, would be observed aperiodic decisions of system (1). It is known, that for rrotary mechanism harmonic signals of vibration with domination in a spectrum of the basic harmonic on rotating speed are characteristic.

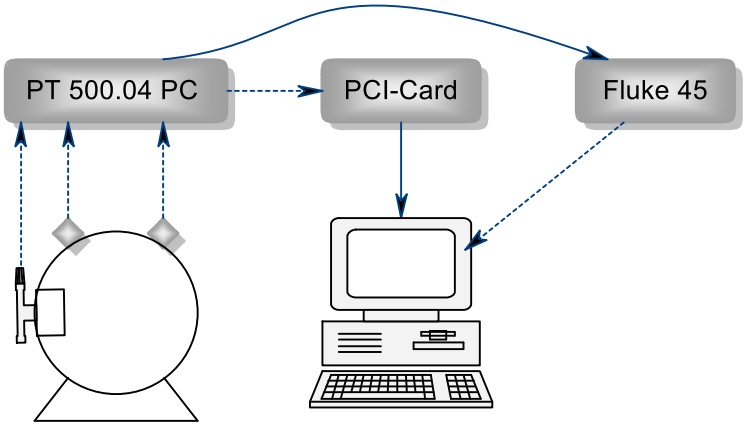


Figure 1. The collector of percussive performances with use of multimeter Fluke 45

2.2 Mathematical methods of nonlinear dynamics

It is possible to apply a known method of nonlinear dynamics to recovery with sufficient fidelity of a phase trajectories of dynamical systems, using physically realized signals $u(t)$. In the given experiment it is possible to receive the two-dimensional phase image connected to a true trajectory of dynamic system in a phase plain. As we deal with real process and real observations a discrete time series is analyzed $u_i(t)$. Digital meter Fluke 45 allows to choose the minimal pitch on time Δt and to realize measuring for equal interims. This fact allows simplifying analysis of time series. The best disclosing of a trajectory on a phase plain is observed at the values of delay close to one quarter of the period of rotating. The received phase image sometimes is expressed in maximum indiscrete line. Quality of an electric network can be estimated on an extent of responsiveness of a phase trajectory to little changes of frequency structure of a researched signal. The coefficient of nonlinear distorting characterizes quality of electric energy. The relevant feature of dynamic systems is occurrence of non-stationary signals. These signals are spawned by a complex of damages in mechanical units. The peer frequency analysis held in programmed Lab VIEW environment, mirrors this fact (Figure 2). Therefore statistical treating of signals with an estimation of equivocation of results of measuring is required.

2.2.1 An estimation of uncertainty of measurement

All results of measuring it are necessary to correct in view of a voltage drop on contact resistances u_c, in view of a proportion beneficial cue/noise. It is necessary to take into account the voltage u_t, generated by a gradient of temperature in an electric network. Last voltage it is measured at absence of a current in measuring circuits. The correction Δu to indications of a digital meter u_1 and u_2 in two processes of measuring is not variable, as conditions of measuring are not variable.

$$u_0 + u_c + u_t = k(u_1 - \Delta u) \quad (2)$$

$$u_t = k(u_2 - \Delta u) \quad (3)$$

After simple transforming we shall receive

$$u_0 = k(u_1 - u_2) - u_c \quad (4)$$

Voltage drop on contact resistances do not distort a beneficial signal as at measuring constant voltage are taken readings from a scale of the digital instrument and at shift of polarity of hooking up of the instrument to a measured circuit.

2.2.2 *Check of power of high-frequency vibration*

In work a quality monitoring of a state of antifriction bearings on high-frequency vibration were compared. Check of power of high-frequency vibration of bearings was carried out; analysis of the form of the high-frequency vibration, raised by short percussive pulses; spectral analysis of fluctuations of power of high-frequency vibration.

Various members of mechanisms vibrate on one or several discrete frequencies. Interplay of these oscillating leads to a complex wave pattern of vibration in the monitored unit. Therefore the measured signal is necessary for analyzing by emanating from him compounding discrete frequencies. The bearing box is esteemed as resonant system, at which one her base frequencies [3] are counted:

$$f_{ord} = \frac{z}{2} \times \frac{n}{60}(1 - \frac{D_w}{d_m} \times \cos\alpha)$$

$$f_{ird} = \frac{z}{2} \times \frac{n}{60}(1 + \frac{D_w}{d_m} \times \cos\alpha) \quad (5)$$

$$f_{bd} = 2\frac{D_w}{d_m} \times \frac{n}{60}(1 - \frac{D_w}{d_m})^2 \times \cos^2\alpha$$

Where

f_{ord} – outer ring defect frequency in Hz,

f_{ird} – inner ring defect frequency in Hz,

f_{bd} – rolling element defect frequency in Hz,

z – the number of rolling elements per row,

n – the speed of the inner ring in rpm,

D_w – rolling element diametr in mm,

d_m – pitch diameter in mm,

α – contact angle in degrees.

On these frequencies characteristic frequencies of arising damages will be modulated. For definition of resonant frequencies the bearing box is esteemed as the unit composed of two thin-wall rings and bodies rolling. The center of the arbor makes complex move under operating of forces of interplay of contacting elements. To each fault there corresponds oscillating with frequency, to aliquot rotating speed of a rotary table. Coefficients of frequency rate are counted on the equations of move of defective units of the bearing box. Amplitudes of speed-up of vibration on resonant frequencies characterize an extent of progressing of damage. The basic signal represents a succession of rectangular pulses. Frequency of these oscillating is used as information argument [4]. Spectral analysis bending around does not allow determining coefficients of frequency rate with the big fidelity. Sample of cues is made non-uniformly.

2.2.3 An estimation of a spectrum of power

If to determine frequency, on which one power of high-frequency vibration reaches a maximum coefficients of frequency rate will be appreciated with sufficient fidelity. The frequency spectrum of power depends on frequency of a spectrum of vibration under the following formula

$$L = \frac{1}{n^2}\left|\sum_{k=0}^{n-1} u_k e^{-i2\pi f t_k}\right|^2 \qquad (5)$$

Where

n- Amount results of discrete measuring of estimations of signals of vibration. After some simplifications the estimation of a spectrum of power is under the formula

$$L = \frac{1}{n^2}\left|\sum_{k=0}^{n-1} u_k \cos 2\pi f t_k - i\sum_{k=0}^{n-1} u_k \sin 2\pi f t_k\right|^2 \qquad (6)$$

Now easily there are frequencies, which correspond to the max value of a spectrum of power. The following equations will allow determining the frequencies applicable to the max value of a spectrum of power

$$-\sum_{k=0}^{n-1} u_k \cos 2\pi f t_k \sum_{k=0}^{n-1} u_k t_k \sin 2\pi f t_k + \\ +\sum_{k=0}^{n-1} u_k \sin 2\pi f t_k \sum_{k=0}^{n-1} u_k t_k \cos 2\pi f t_k = 0 \qquad (7)$$

Under formula (7) the ergodic casual process preselected by a final set of material values $u_k = u(t_k)$ is investigated. Have taken advantage of an estimation of power (Schuster Periodogram) for definition of a harmonic with frequency f, which one gives the greatest contribution to disintegrating to Fourier series. For the lines composed from five steams $(n-1=5)$ of measured physical magnitudes, formula (7) looks like

$$2\cdot(u_0u_1+u_1u_2+u_2u_3+u_3u_4+u_4u_5)\cdot\sin 2\pi f+$$
$$+4\cdot(u_0u_2+u_1u_3+u_2u_4+u_3u_5)\cdot\sin 4\pi f+6\cdot(u_0u_3+u_1u_4+u_2u_5)\cdot\sin 6\pi f+ \quad (8)$$
$$+8\cdot(u_0u_4+u_1u_5)\cdot\sin 8\pi f+10\cdot u_0u_5\cdot\sin 10\pi f=0$$

Augmenting realization of investigated process up to a member $n-1$, formula (7) looks like

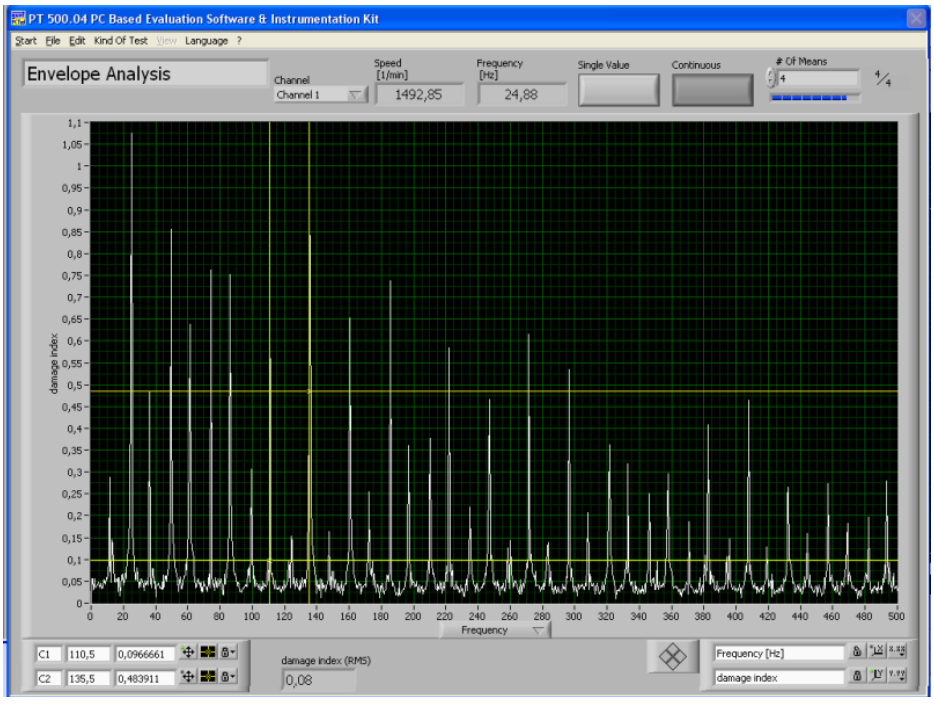
$$\sum_{k=1}^{n-1}2k\cdot(u_0u_1+u_1u_2+\cdots+u_{n-1-k}u_{n-1})\cdot\sin 2k\pi f=0 \quad (9)$$

The formula (9) is fair for even n and for $t_{k-1}-t_k=1s$.

Having constructed Schuster Periodogram, it is possible to determine extreme frequency with the big fidelity, on which one the true spike of the response of fault is observed.

3. THE MAIN TASKS AND EXPERIMENT

On the ground Based Evaluation Software PT 500 the universal collector of the diagnostic information has been collected. The measuring complex is supplemented by digital measuring device Fluke 45. Purity of experiment is demonstrated to calculations of equivocation of measuring.



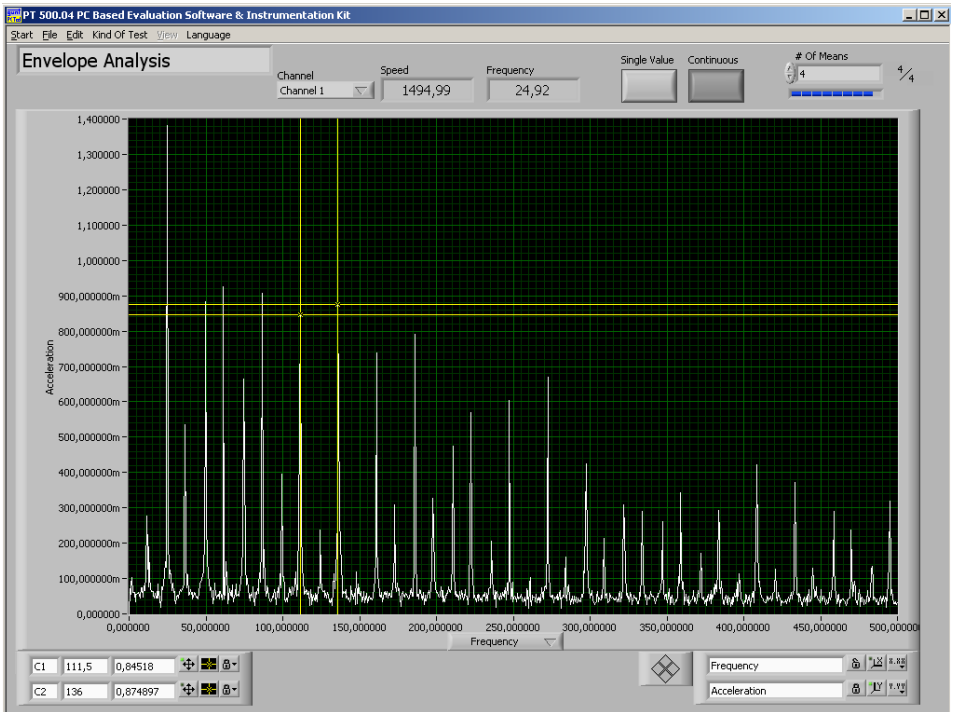


Figure 2. Peer analysis bending around

The form of measuring is represented on Figure 3. By results of measuring it is drawn up periodogram Schuster, shown on Figure 4.

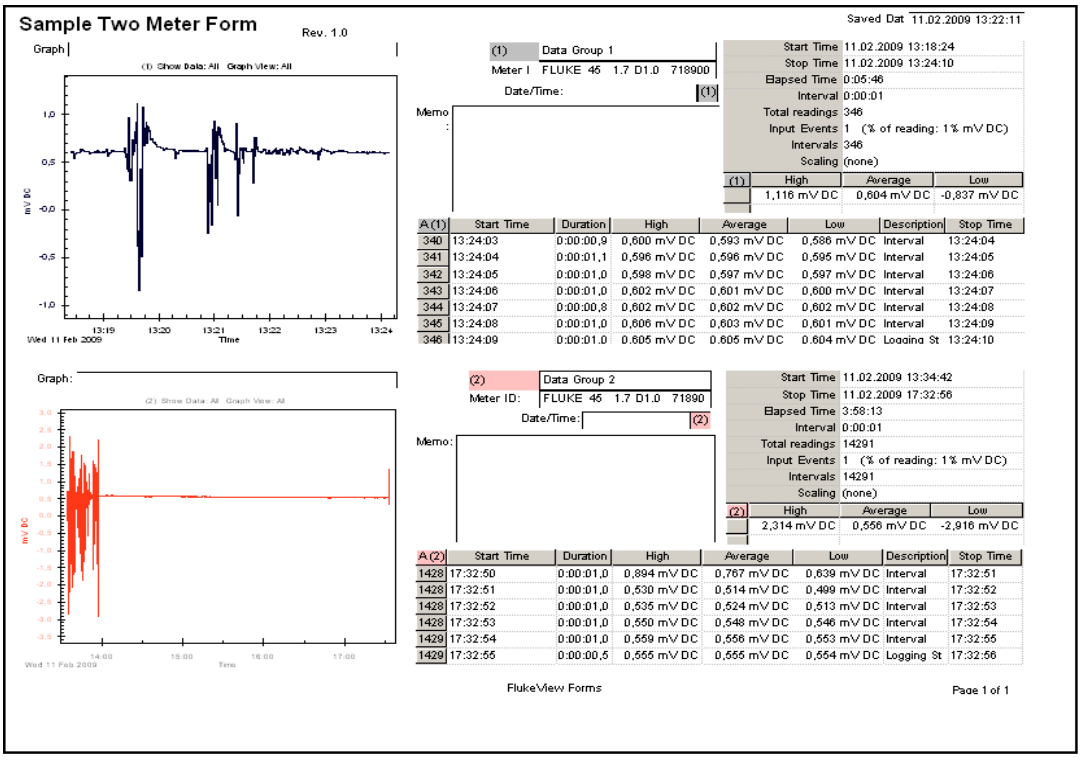


Figure 3. A time series

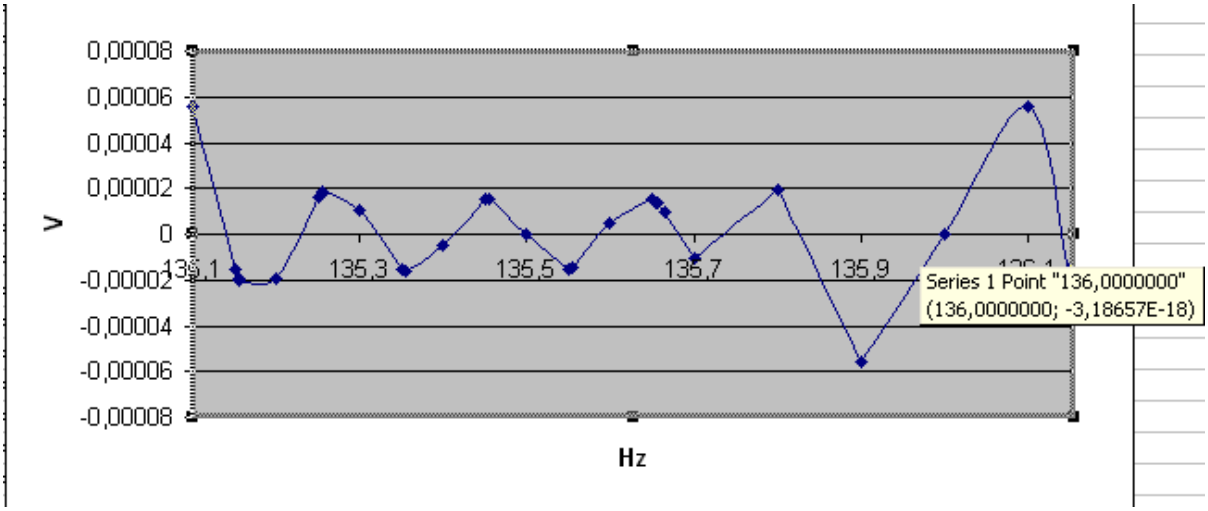


Figure 4. Periodogram Schuster

That fact that the essential spike and essential droop on a spacing of frequencies in neighborhood of characteristic frequency $136\,\mathrm{Hz}$ is observed, guesses that on the given frequency there can be a fault at the bearing box. On the other hand, the chart of mean values of responses of vibration rather is uniform in the other spacing of frequencies. The opportunity of fault on these frequencies should be excluded

4. CONCLUSIONS

The held experiments and treating of results of measuring have allowed formulating following deductions:

- In the work the method of statistical processing of the results of measurements proposed with the uncertainty $3{,}5\cdot 10^{-15}\,mV$ allowed to isolate the frequency, at which was modulated the defect on the race of the internal ring of the bearing
- The Measuring equipment proposed in work is the universal collector of the information. This installation will allow to effect in regular intervals sample of cues with any pitch
- The method of application Proposed in work will allow is one valued to determine possible{probable}□troubles on their □displays in percussive cues

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