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MODELLING OF NONHOLONOMIC DYNAMICAL SYSTEM

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Abstract

Creation of training –and-scientific laboratory for automatic assembly of the optical products is envisaged in the project. Training-and-scientific laboratory is an automated flow line of the dynamic system control. The laboratory is intended to conduct studies and scientific-research work with the problems of mechanical engineering and instrument making. Complex of automated assembly is created proceeding from the structure of the lens. The main aim of the project: To render automatic selective sampling of the optic-mechanical units and to arrange them into jacket in an optimum combination.

Key words: *mathematical modeling, nonholonomic constraints, selective sampling of the optic-mechanical units*

1. INTRODUCTION

Nonholonomic dynamic system – is any controlled dynamic system where object of the control travels in space and time. Geometric and kinematic constraints are imposed on such systems. The constraints are considered nonholonomic if they are presented by non-integrable equations. Series arrangement of elastic and frictional element where constraint's reaction is expressed by function of deformation and by the rate of deformation of the elastic element can be an example. For non-linear dynamic systems transformation of inputs into outputs happens with essential distortions and delay. New methods of mathematical modeling are required in the process of designing of such systems. While mathematical model compiling decomposition method of the system assumes breakdown of the system by dividing nonholonomic constraints into subsystems. Mathematical model for each subsystem is developed and it combines selected autonomous subsystems into the common system. Equation of the dynamic system motion can be got by superposition of the system of nonholonomic constraints on the mechanical system.

Creation of training –and-scientific laboratory for automatic assembly of the optical products is envisaged in the project. Training and scientific laboratory is an automated flow line of the dynamic system control. The laboratory is intended to conduct studies and scientific-research work with the problems of mechanical engineering and instrument making. Complex of automated assembly is created proceeding from the construction of the lens. The main aim of the project: To render automatic selective sampling of the optic-mechanical units and to arrange them into jacket in an optimum combination. Demonstration educational center is supposed to be simple in realization. It will give possibility to conduct laboratory works at stationary and remote places. Economic feasibility of such project is the main criterion for selection of the elements and assembly components of the system.

2. THE RESEARCH

2.1 Aspects of modeling of nonholonomic dynamic system

The dynamical system is a set of nodes that are in collaboration [1]. The simulated system consists of a set of interrelated components of a technological system of the assembling line of micro-objectives. In composing of the mathematical model based on the method of decomposition, nonholonomic relations were divided into subsystems. The technological system of the assembling line consists of two subsystems: the subsystem of assembly of the lens in a frame and control subsystem of the micro-objectives. The subsystem of assembly of the lens includes the following components:

- the executive device- manipulator, through which occurs the transfer and positioning of the optical elements;
- MP-measuring value pick-up, receiving the data to assess the accuracy of the physical properties of the optical elements;
- the device for selection of lenses and frames for mounting of lens with the help of the ring.

Selection of components is based on data obtained after the evaluation of the physical properties of the optical elements.

The control subsystem of the micro-objectives can be described as a model implementing the following methods of measurement. The differential method (see Fig. 1,2), directly compares the unknown output quantity (measuring value pick-up MP) with the reference quantity (input quantity, reference value pick-up RP), the difference between these two quantities being measured with a differential measuring instrument DM. Its indication is another input quantity. For the balance method, the differential measuring instrument DM is replased by a nullindicator NI. To balance of an adjustment pick-up AP is necessary, which generates another input quantity. In this case, the model function is the sum of the of the input quantities and the square of the standard uncertainty associated with the sum of the squares of the standard uncertainties of the input quantities. The substitution method is similar to the differential method except that the difference between the output and reference quantities is not directly measured but that the two quantities are measured one after each other using the same measuring instrument and forming the ratio of the two quantities. As with the substitution method the output and reference quantities are of almost the same value and the two are measured using the same instrument, both quantities are strongly correlated.

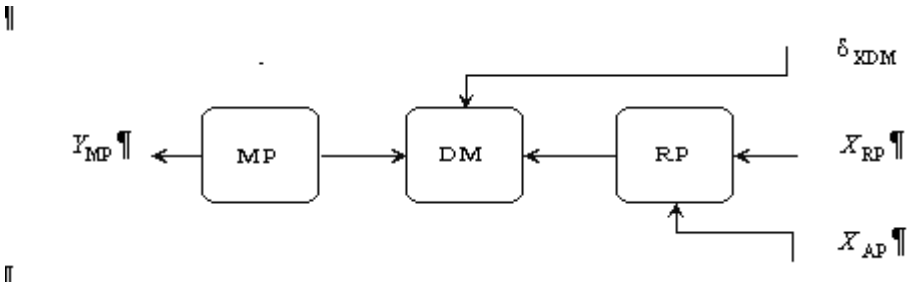


Figure 1 Differential method

The subsystem of assembly of micro-objectives is considered as a multipole described by the system of equation (1) through the *h* parameters. The influence of the environment is taken into account as input random variables.The guided system is able to change its position under the action of controllable and disturbing signals. One part of the controllable signals may be forced to change in the needed direction and the other part provides the information about the state of the system in the previous time. The controllable signals are obeyed to a random distribution.

$$
\begin{aligned}
dy_1 &= \sum_{i=1}^{k} h_{1i} du_i + \sum_{j=k+1}^{n} h_{1j} dx_j \\
dy_2 &= \sum_{i=1}^{k} h_{2i} du_i + \sum_{j=k+1}^{n} h_{2j} dx_j \\
&\vdots \\
dy_n &= \sum_{i=1}^{k} h_{ni} du_i + \sum_{j=k+1}^{n} h_{nj} dx_j
\end{aligned}
\qquad (1)
$$

$U(u_1, u_2, \ldots, u_k)$ – input controllable signals;

$\mathrm{X}(\mathrm{x}_{k+1}, x_{k+2}, \ldots, x_n)$ – input signals of the state;

$\mathrm{Y}(\mathrm{y}_1, y_2, \ldots, y_n)$ – output signals

The input controllable signals and state signals are evaluated taking into account the uncertainties of measurements and are mutually correlated. As the mathematical model describes the nonlinear mechanical system, then the system of equations (1) is a system of linear differential equations with nonlinear coefficients. The transformation of input signals into the output signals comes with significant distortions. In the evaluation of uncertainties of the input and output signals is taken into account an assumption that the reaction of the system to effect of the input signals is not instantaneous, but with some delay. The creation of the technological system of the assembly line does not exclude from the process of simulation an experiment and measurement too. The aim of mathematical modeling is that from the precisely formulated initial data to obtain the fact that is unavailable to the direct observation.We have to consider that this kind of technical facility has not been established yet. The most significant factor on composing of the mathematical model is the realization of the selective selection of optical and mechanical components and placing them into the jacket with optimum combination.Input signals X of the system are determined at any time interval by phase coordinates $x_{k+1}, x_{k+2}, \ldots, x_n$. The process of operation of the system depends on output signals $\mathrm{Y}(\mathrm{y}_1, y_2, \ldots, y_n)$, that determine the successive states of the system over time. Phase trajectories depend on parameters of the system h_{ji} that are varied with the change of the input controllable signals $U(u_1, u_2, \ldots, u_k)$. The proposed invariant model is designed to simulate the physical and information processes that occur in the system. The system belongs to a class of statistical models, as between the functions and arguments exist a statistical correlation. The level of abstraction of the mathematical model corresponds to the model of a “black box”. Recorded differential equations of motion (1) in general form have to be brought to a form, which are convenient for integration.

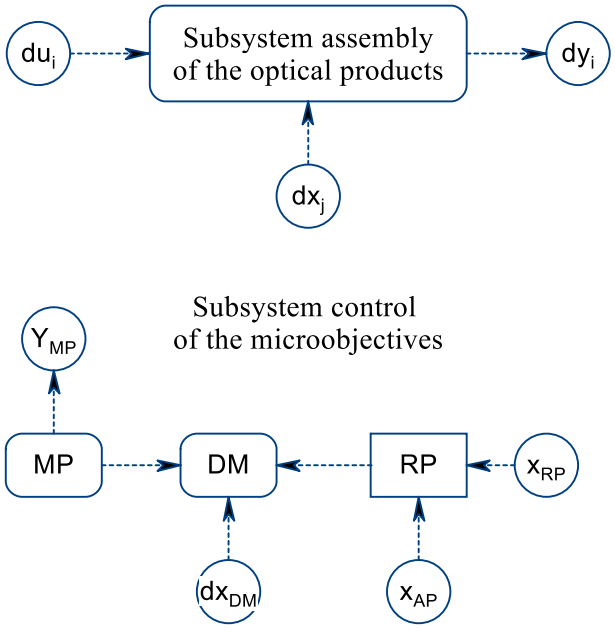


Figure 2 Block diagram of the system

In general, the number of input variables may be numerous quantity. Many of the input values are correlated with each other. For measuring of reduction of the dimension, the correlation coefficient of the input values is introduced. Assuming that the input values $U(u_1,u_2,\ldots,u_k)$ and $X(x_{k+1},x_{k+2},\ldots,x_n)$ depend on a numerous uncorrelated variables $Q(q_1,q_2,\ldots,q_l)$, and then the evaluation of dispersion associated with X is expressed by equation

$$(dx_j)^2 = \sum_{l=1}^{m} \left(\frac{\partial F}{\partial q_l}\right)^2 \cdot (dq_l)^2 \quad (2)$$

For evaluation of another input values, the similar equation can be expressed

$$(du_i)^2 = \sum_{l=1}^{m} \left(\frac{\partial G}{\partial q_l}\right)^2 \cdot (dq_l)^2 \quad (3)$$

The evaluated covariation associated with U, and X is expressed as

$$dx_j \cdot du_i = \sum_{l=1}^{m} \frac{\partial F}{\partial q_l} \cdot \frac{\partial G}{\partial q_l} \cdot (dq_l)^2 \quad (4)$$

The covariation is zero, if none of the uncorrelated variables $Q(q_1,q_2,\ldots,q_l)$ is not common to the functions *F* and *G*. Only those members, for whom $\frac{\partial F}{\partial q_l} \neq 0$ и $\frac{\partial G}{\partial q_l} \neq 0$ at a given *m,* are contributed to the sum expressed by the equation (4).

2.2 Mathematical method of nonlinear dynamics

The planned mathematical method investigates the Port-Controlled Hamiltonian model of nonholonomic dynamical systems [4]. Nonholonomic constraints on motion can be expressed in terms of nonintegrable linear velocity relationships $F(Q) \times Q' = 0$. Recently [2], this klass of nonholonomic constraints has been broadened as to encompass affine velocity relationships $G(Q) + F(Q) \times Q' = 0$. PCH model incorporates directly nonholonomic constraints and broadening the class of nonholonomic constraints allows us to propose in a PCH formulation a control algorithm for dynamical systems where nonholonomic constraints on velocities are imposed by control objectives [1] and not by the structure of the system itself [3,5,6]. As an application, the energy based robust control is studied of finite dimensional underactuated mechanical systems.

2.3 Main goals and the planned research

The main direction of the research is connected with creation of the laboratory for automatic assembly of optical products on the base of Virumaa College. Students and engineers of enterprises-partners will be engaged in this work. Automated selective sampling of the optic-mechanical units and spacing rings as well as arranging their optimum combination into jacket will be carried out on the flow line. It is necessary to solve the following problems for this:

- to select optimum sequence of the optic-mechanical elements and spacing rings for arranging them into jacket . The assembled unit has to meet quality requirements;
- to get necessary technical facilities;
- to develop succession of optic-mechanical units assembling;
- to investigate and to carry out optimum supply of the parts and their connection;
- to assure quality control of the assembling operation;
- to solve problems of the sorted units utilization.

3. CONCLUSIONS

To study the real physical system, the mathematical model has been developed with the same ratio and properties. The developed mathematical model has several advantages:

- it allows carrying out a primary study of the system;
- with the help of the developed mathematical model and using the laws of mathematical statistics, we can perform all calculations with great accuracy;
- on the basis of the mathematical model computational programs are implemented, which allow to replace expensive and time-consuming real experiments. With these tools we can numerous times and in a wide range to change the input signals and conditions of the operation of the designed system.

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