LOGICAL MODELLING AND CORRELATION ANALYSIS OF THE RETINA NEURAL STRUCTURES WITH “ON” (“OFF”)-CENTER
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Abstract
Neural connections between photoreceptors, horizontal and bipolar cells in the structure of the retina with the “on” (“off”) - center are analyzed. A logical equivalent of these elements and a two-threshold model of excitation of a bipolar cell are proposed. A comparative correlation analysis of threshold models of artificial neurons and a white noise filter is demonstrated. The spectral algorithm for the synthesis of an artificial bipolar neuron is described. The high efficiency of application in a switching neurostructures of the logical analogue of a structure with the “on” (“off”) - center is noted.

Key words: structure with “on” (“off”) - center, bipolar cell, correlation analysis, noise filter

1. INTRODUCTION
Optimality of the commutational neural networks depends, to a great extent, on the element base that is used in controlling of the network channels switches. At the previous conference [1] noise filters were represented as an example of the effective elements.

There were considered two different kinds of filters which were formed by mixing (summing according to mod 2) of the control elements exit signals. The first group of noise filters was composed of the sums according to mod 2 of the local filters exit signals
\[ \mathcal{G}(x) = \varphi_1(x) \oplus \varphi_2(x) \oplus \ldots \oplus \varphi_m(x), \]
where the local filter
\[ \varphi_i(x) = \lambda_1 \cdot \lambda_2 \cdot \ldots \cdot \lambda_r(i), i = 1 \div m, \]
is the composition of simple linear filters
\[ \lambda_j(x) = \lambda_{j1} x_1 \oplus \lambda_{j2} x_2 \oplus \ldots \oplus \lambda_{jn} x_n, \lambda_{jk} \in \{0,1\}, k = 1 \div n. \]
These filters are simple enough as far as their structure is concerned, but at the same time, while solving the tasks at a bigger scale, the number of \( m \) local filters applied for the formation of the effective noise filter proves to be too big.

The second group got the name “neural noise filters”, and it is defined as a sum of exit signals of the neurons,
\[ \mathcal{G}(x) = \nu_1(x) \oplus \nu_2(x) \oplus \ldots \oplus \nu_m(x), \]
where artificial neurons \( \nu_i(x), i = 1 \div m, \) possess threshold activation. An example is given where it is clearly seen that neuron filters are more efficient than the filters of the first group. But due to the fact that there is a great variety of neurons, there is a necessity to find out what type of neurons will be the most efficient in the composition of a filter. In connection with this, it shall be noted that there is a possibility of the existence of certain structures in the biological prototypes of the neuron networks, which will perform the role of the effective noise filter. The retina structures, for example, that got the name “ganglious cells” with "on"("off") - center may present a certain interest.

The aim of this work is to present a logical model of the retina structure with "on"("off")-center and correlation analysis of this model’s efficiency.

2. NEURAL TWO-THRESHOLD MODEL OF THE RETINA STRUCTURE WITH “OFF”-CENTER
In the previous work [2] the conclusion was made on the basis of the analysis of the ganglious cell with “on”-center reaction, that the corresponding neuron structure in the first approximation performs the function of “sum according to module 2”. But this structure has a multitude of inputs from receptors whereas the weights of these inputs are different. Signals from the center of the reception field, for example, have greater weight in regard to the signals of peripheral receptors. Of course, it
shall be taken into account in the structure of the model. Let us consider the composition and the structure of the retina.

According to the modern vision [3], organisation of the ganglionic cells reflects the organisation of bipolar cells passively. Bipolar cells have receptive fields with the center and periphery. The reaction of the center is predetermined by a direct input from a small group of receptors; in its turn, periphery is determined by an indirect way from a wider area of receptors, linked with horizontal cells, which, in their turn, transmit signals to bipolar cells. Exits of horizontal cells always form inhibitory synapses. A bipolar cell sends the only dendrite to the receptors that forms a synapse with one receptor (it is always a conus), or dendrite can split into twigs which contact by synapses with more than one receptor. These receptors constitute the center of the receptive field.

Vertebrate photoreceptors are evidently more depolarised in the dark, as they possess a lower membrane potential in comparison with the usual nerve cells at rest. Depolarization causes a continuous release of the mediator from the endings of their axons, exactly as it takes place in the regular receptors in case of stimulation. The light, while increasing the potential on the photoreceptor membrane, i.e. hyperpolarizing it, decreases the emission of the mediator. Consequently, light stimulation switches off a photoreceptor.

Bipolar cells like receptors and horizontal cells do not generate impulses as ganglion cells carry out this mission. But here again it is necessary to speak about “on” reaction, implying depolarization as a response to the light stimulus, and consequently, enhanced release of the mediator in the output synapses, and also about “off” reaction, meaning hyperpolarization and the decrease in the mediator output/release.

Bipolar cells with "off"-center shall have excitory input synapses from the receptors of the center, as receptors themselves are switched off (hyperpolarized) by the light; whereas the input synapses of the bipolar cells with the "on"-center shall be inhibitory.

In accordance with [4] photoreceptors together with horizontal cells and bipolars form so called triad. Such a triad is composed of a presynaptic membrane of a photoreceptor, which is entered by a bipolar dendrit, and at the sides of it the branches of the horizontal cells enter the dendrit. As a result of such links between photoreceptors, horizontal cells and bipolars, the center of bipolars receptive field is formed directly in the process of signal transmission from photoreceptors, while the periphery is formed indirectly via horizontal cells as a result of excitatory or inhibitory synapses activity.

In accordance with the facts presented above from [3], Fig.1 displays a logical scheme of excitatory function of the bipolar cell with an “off”-center. Variables $x_i \div x_{m1}$ at the inputs of the circuit are marked by the light stimuli. Proceeding from the fact that the reaction of photoreceptors is inverse to the light stimulation, the receptors here are represented by invertors, i.e. logical elements NOT.

Synaptic links with center receptors, congruent to inputs $x_i \div x_k$, are excitary, that is why they are characterized by positive weight values $d_i > 0 \div a_k > 0$.

Horizontal cells, collecting signals from the multitude of receptors from the periphery perform the function of the logical element OR. But due to the fact that horizontal cells possess inhibitory synaptic functions at the output, their resultant logical function is NOR. In essence, horizontal cells, besides the function of highlighting the contrasts in images, also form local filters together with photoreceptors. It is the conjunction of the signals from receptors covered by horizontal cells connections. So, for example, according to the rules of binary logic for light stimuli $x_i, x_{i+1}, \ldots, x_{i+t}$ at the output of the corresponding horizontal cell a signal $\varphi_i = \overline{x_i} \lor \overline{x_{i+1}} \lor \ldots \lor \overline{x_{i+t}} = x_i x_{i+1} \ldots x_{i+t}$ is being formed. Synaptic connection with a binary cell for this “on” signal shall be inhibitory and be characterized by negative weight, - the value $-a_{k+1}$.
Bipolar function, i.e. function of a bipolar cell, shall correspond to the equal reaction of the cell both to the prevailing sum of the excitatory stimuli, and the prevailing sum of the inhibitory stimuli. Consequently, taking into consideration the reciprocal interaction of the central and the peripheral zones of the reception field, displayed in the form of the excitatory and inhibitory impact on the bipolar cell membrane, and taking into account the conclusions made in the research [2], a two-threshold model of the function with retina structure with an "off"-center shall be accepted:

$$bi(x) = \begin{cases} 
1, & \text{if } \sum_i a_i \psi_i \geq p_2; \\
1, & \text{if } \sum_i a_i \psi_i \leq p_1; \\
0, & \text{if } \sum_i a_i \psi_i \in (p_1, p_2) 
\end{cases}$$

where $\psi_i \in \{x_{ij}, \varphi_i\}$, considering the fact that separate receptors $x_{ij}$ can be connected with a bipolar cell directly, moreover, considering that there are different variants $\varphi_i$ of connecting receptors with horizontal cells; $(p_1, p_2)$- numeric interval from $p_1$ to $p_2$.

Fig. 1. Logical scheme of the excitement function of bipolar cell with “off”- center
3. COMPARATIVE CORRELATIONAL ANALYSIS OF THRESHOLD, TWO-THRESHOLD AND BIPOLAR FUNCTIONS

As it has been noted in the work of [5], the quality of correlation characteristics

\[ B_N(\tau) = \sum_{x=0}^{N-1} q(x) \cdot q(x \oplus \tau) \pmod N \]

that were used for signal transmission \( q(x) \) in radio communication (where \( N \) is the number of function \( q(x) \) setting points, for the given length \( N \) and the given number of units in it \( \sum_{x=0}^{N-1} q(x) \), is determined by the value

\[ \Delta_N = B_N(0) - \max_{\tau \neq 0} B_N(\tau) \]

It is connected with the fact that the value of \( \Delta_N \) determines the options for the correction of mistakes in the course of signal transmission with the help of \( q(x) \). In case of application in commutation neural networks, \( \Delta_N \) of the control function \( q(x) \) will determine the efficiency of the source function \( f(x) \) fragment splitting into sub-fragments with high absolute values of differs.

It is pointed out in the same work [5], that the task to find the optimal functions \( q(x) \), for which with

\[ \sum_{x=0}^{N-1} q(x) \]

\( N \) and \( N \) given, maximum of \( \Delta_N \) is reached, is very important but difficult to solve. Following this point of view, the boolean function would possess the best characteristics, for which

\[ B_N(\tau) = \text{const} \]

Another function is in the priority, as well, the function where the autocorrelation is the nearest to the constant value with \( \tau \neq 0 \), i.e., the function is the closest to the pseudorandom sequence of the given length \( N \).

In order to be sure that the presented model of the retina neural structure corresponds to the the demands of application efficiency in the commutation structures, let us consider autocorrelation characteristics of several corresponding functions. To compare their characteristics, four logical functions of 8 variable functions, having close unit values have been chosen:

- threshold \( g(x) \), structural vector \( A_g = (p; a_1, a_2, \ldots, a_8) = (7; 3, 3, 2, 2, 1, 1, 1, 1) \);
- two- threshold \( d(x) \), with vector \( A_d = (p_1, p_2; a_1, a_2, \ldots, a_8) = (3, 9; 3, 3, 2, 2, 1, 1, 1, 1) \);
- bipolar \( bi(x) = bi(x_1, x_2, x_3, x_4, \varphi_3, \varphi_6), \varphi_5 = x_5 \cdot x_6, \varphi_6 = x_7 \cdot x_8 \),
structural vector \( A_{bi} = (p_1, p_2; a_1, a_2, a_3, a_4, a_5, a_6) = (1, 4; 1, 1, 1, 1, 1, 1) \);
- white noise structure, quadratic form \( \beta(x) = x_1 x_2 \oplus x_3 x_4 \oplus x_5 x_6 \oplus x_7 x_8 \).

One can see the corresponding massives of values for the coefficients of autocorrelation characteristics (ACC) \( B(\tau) \), without mention of argument \( \tau \) value, given below.
Massive of coefficients $B_g(\tau)$ ACC $B_g$ of the threshold function $g(x)$.

<table>
<thead>
<tr>
<th>$B_g(\tau)$</th>
<th>110</th>
<th>92</th>
<th>82</th>
<th>74</th>
<th>70</th>
<th>62</th>
<th>54</th>
<th>52</th>
<th>50</th>
<th>46</th>
<th>42</th>
<th>40</th>
<th>38</th>
<th>30</th>
<th>22</th>
<th>18</th>
<th>10</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>23</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>51</td>
<td>2</td>
<td>40</td>
<td>10</td>
<td>26</td>
<td>10</td>
<td>5</td>
<td>23</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1. Distribution of ACC $B_g$ coefficients according to the value of $B_g(\tau)$

Massive of coefficients $B_d(\tau)$ ACC $B_d$ of the two-threshold function $d(x)$.

<table>
<thead>
<tr>
<th>$B_d(\tau)$</th>
<th>104</th>
<th>80</th>
<th>70</th>
<th>54</th>
<th>52</th>
<th>50</th>
<th>46</th>
<th>42</th>
<th>40</th>
<th>38</th>
<th>36</th>
<th>34</th>
<th>32</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>36</td>
<td>1</td>
<td>3</td>
<td>36</td>
<td>32</td>
<td>4</td>
<td>24</td>
<td>30</td>
<td>8</td>
<td>36</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 2. Distribution of ACC $B_d$ coefficients according to the value of quantity $B_d(\tau)$
Massive of coefficients $B_{bi}(\tau)$ ACC $B_{bi}$ of the bipolar function $bi(x)$

| $B_{bi}(\tau)$ | 101 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 |
|----------------|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $m$            | 1   | 15 | 15 | 70 | 155 |

Table 3. Distribution of ACC $B_{bi}$ coefficients according to the value of quantity $B_{bi}(\tau)$

Massive of coefficients $B_{\beta}(\tau)$ of autocorrelational characteristics $B_{\beta}$ of quadratic form functions (white noise) $\beta(x) = x_1x_2 \oplus x_3x_4 \oplus x_5x_6 \oplus x_7x_8$

<table>
<thead>
<tr>
<th>$B_{\beta}(\tau)$</th>
<th>120</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1</td>
<td>255</td>
</tr>
</tbody>
</table>

Table 4. Distribution of ACC $B_{\beta}$ coefficients according to the value of quantity $B_{\beta}(\tau)$
It is evident that for $\tau \neq 0$ value $B_{\beta}(\tau) = 56 = \text{const}$.

**Fig. 2.** Graphic representation of coefficients $A K X B(\tau)$ distributions:
- g) threshold function $g(x)$;
- bi) function $b_i(x)$ of bipolar cell excitement;
- d) two-threshold function $d(x)$;  
- $\beta$) function $\beta(x)$ of the white noise filter.

Tables and charts Fig.2 of distribution of autocorrelation $B(\tau)$ coefficient values demonstrate the most significant, for the bipolar function $b_i(x)$, correlation with the function of $B(x)$ white noise filter. Characteristic $B_{bi}$ is the closest to the constant value.

In connection with all the information presented above, it shall be noted that the retina bipolar cell represents one of neuron noise filters variants, which was reported in the article [1]. Indeed, it can be written this way $b_i(x) = v_1(x) \oplus v_2(x)$, where neurons $v_1, v_2$ have a conventional one threshold representation, and the corresponding structural vectors for them differ only by the amounts of the
threshold: \( A_1 = (p_1; a_1, a_2, \ldots, a_n), \ A_2 = (p_2; a_1, a_2, \ldots, a_n), \ p_2 > p_1. \) In case of such view, the synthesis of function \( bi(x) \) is performed in the way described in [1].

4. THE SPECTRAL METHOD OF BIPOLAR ELEMENT SYNTHESIS

In case of programmed realisation of artificial neuron network the use of bipolar functions diminishes the costs on network reaction determination.

The bipolar function synthesis is similar to the spectral method of threshold function calculation in operation that was made in the research [6]. For instance, at first, the first dominant \( S_f(\omega) \) of spectre \( S_f \) of the initial function \( f(x), x \in X \) in the segment of X determination field. In accordance with the value of the binary argument \( \omega \) as the first variable bipolar function \( bi(\lambda) \), \( \lambda = (\lambda_1(x), \lambda_2(x), \ldots, \lambda_n(x)) \) the filter is formed, where \( \lambda_i(x) = \omega_1 x_1 + \omega_2 x_2 + \ldots + \omega_n x_n \), \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \), \( x = (x_1, x_2, \ldots, x_n) \). So, weight \( a_i = 1 \) is allocated to the first input of the bipolar element. Values \( w(x) \) of excitation level for every artificial bipolar neuron have been calculated \( X \setminus f w(x) = a_1 \cdot \lambda_i(x) \). Calculations are analogous in case of defining the weight and linear filter for the second input of the bipolar element. Splitting of fragment \( X \) into sub-fragments is done in advance:

\[
X = X_0 \cup X_1, \quad X_0 \cap X_1 = \emptyset, \quad X_0 = \{ x \mid w(x) = 0 \}, \quad X_1 = \{ x \mid w(x) = 1 \}.
\]

Spectra \( S_f^0, S_f^1 \) for sub-fragments \( X_0, X_1 \) are calculated accordingly. After that, dominants \( S_f^0(\omega) \) and \( S_f^1(\omega) \) of the spectrum \( S_f \) is determined. Its argument will determine the value of the linear filter \( \lambda_2(x) \) of the bipolar element second input, that corresponds to weight \( a_2 = 1 \).

Having calculated \( w(x) = a_1 \lambda_1(x) + a_2 \lambda_2(x) \) and the corresponding splitting \( X = X_0 \cup X_1 \cup X_2 \), it could be found that the value of argument \( X \), for which \( f(x) = 1 \), has mostly been spread in the “polar” sub-fragments \( X_0, X_2 \). Next steps shall ensure the gain of this polar distribution of \( X \) argument value.

In the simplified variant, it may be executed in the following way. Let, for example, linear filters and weights for \( k \) inputs of the bipolar element have been determined. Excitement levels

\[
w(x) = \sum_{i=1}^{k} a_i \lambda_i,
\]

are calculated. The common spectrum \( S_f \) will provide the solution of the problem of the polar distribution of the argument \( X \) values, provided

\[
S_f = \sum_{i=q+1}^{k} S_i^q - \sum_{j=1}^{q} S_i^j, \quad q \approx k / 2.
\]

The efficiency of linear filters application \( \lambda(x) \) will decrease at a certain period because of the decrease of fragment [7] \( f(x) \) value for sub-fragments in \( X_t, \ t \in [q - r, q + r] \), \( r = 1, 2, \ldots \). In this case, it is necessary to determine the local filter using method [6]:

\[
\phi(x) = \lambda_1 \cdot \lambda_2 \cdot \ldots \cdot \lambda_k.
\]
The given expression is a limited mathematical description of the horizontal cell in the combination with retina receptors. It is natural that functions of the horizontal cell are considerably wider and (1) refers to the task of signal formation designed for image recognition.

After local and linear filters definitions for $m$ inputs have been made, it will turn out that further absolute values increment of function $f(x)$ differs in polar sub-fragments $X_0, X_1, \ldots, X_{p1}$ и $X_{p2}, X_{p2+1}, \ldots, X_m$ after calculations of the subsequent inputs prove to be minor. In this case, the process of synthesis is completed, and indices $p_1, p_2$ define the threshold values of the bipolar element.

5. CONCLUSION

It is evident that a logical model of the neurostructure with ‘on’ centre is also represented by a bipolar element. It is similar to the model with “off“- center with the precision of the inversion symbols, representing inhibitory effects, and they possess opposite scale signs, which form synapse effects on the bipolar cell membrane. It is quite understandable that the logical model of the neurostructure with "on"- center can also be represented by a bipolar element.

A bipolar element possesses wider functional opportunities in comparison with the traditional artificial neuron. The efficiency of the bipolar function is stipulated by its having partial anti-self-duality, and the ability to vary diversely the number and multiplicity of the local filters at the bipolar element inputs. Being noise filters in essence, bipolar functions approximate complicated logical functions of bigger size at different points of their application more precisely, as complicated functions also possess the indications of “noisiness“.

Use of bipolars as controlling elements in switching neural networks will provide a substantial decrease/cut in the networks volume and will lead to the simplification of their structure.

REFERENCES


