LOCATION, PRICE, AND WELFARE IN THE OLIGOPOLY WITH ONE ONLINE FIRM

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Abstract

We utilize a barbell model, in which both the physical firms' costs and markets are asymmetric, to examine the effects of increasing the number of firms by introducing one online firm into the markets on physical firms' location choices, outputs, price, and welfare under Cournot competition. We show that introducing one online firm may induce the two physical firms to switch their optimal locations from agglomeration at the urban market to separate at different markets, may decline the total sales and raise the price level in the urban market, and may worsen the welfare level.

Keywords: *Online Firm, Location Choices, Market Asymmetry, Cournot Competition*

1. INTRODUCTION

Online shopping has been more and more popular in past decades. The popularity of online shopping continues to erode sales of physical retailers. For example, Best Buy, the largest physical retailer of electronics in the U.S., reported its tenth consecutive quarterly reduction in sales in August 2014.[1] Therefore, how the physical firms strategically adjust their locations in response to the negative influence of online shopping is an important issue deserved to study.

Next, conventional wisdom indicates that a rise in the number of firms will increase the total output, fall the price level, and improve the welfare in a non-spatial framework through enhancing the competition in the industry. However, increasing the number of firms by introducing one online firm may induce the physical firms to mitigate the competition through moving their locations farther away from each other in a spatial model. This reduced competition may lead the traditional results to be reversed. Thus, it is interesting to examine the impacts of introducing one online firm on the output, price, and welfare in a spatial framework.

It is characterized in a spatial framework that online shopping is location-irrelevance, which can save transportation costs for consumers while suffering a distaste cost.[2] As the true price faced by consumers is the price charged by the online firm plus the distaste cost per unit of output, the presence of the distaste cost lowers the online firm's perceived demand through decreasing the consumers' willingness to pay. By contrast, the consumers buying from the physical firms bear transportation costs but incur no distaste cost.

The extant theoretical literature includes Balasubramanian (1998), Bouckaert (2000), Liu *et al*. (2006), Loginova (2009), and Guo and Lai (2014, 2017). In particular, Guo and Lai (2017) adopt a Hotelling (1929) linear city model, in which the left side of the line denotes the densely populated urban area while the right side represents the loosely populated rural area. They show that the entry (introduction) of an online firm will force the surviving physical firms to move toward the left urban area while the online firm will occupy the right rural area under Bertrand competition in the long-run equilibrium. However, the location patterns are more diversified in the real world, depending upon the market sizes, the distaste costs and the marginal costs of the physical firms. For instance, we can observe that for the products whose distaste costs are high, such as apparel and jewelry, etc., the low-cost (high-cost) physical stores are located in the urban (rural) area, and meanwhile the online firm can serve both areas.

[1] Please refer to the following website: https://en.wikipedia.org/wiki/Online_shopping.

[2] The distaste cost includes the delay in receiving the product, the inability of consumers to inspect (i.e. touch, smell, hear, or directly see) the product beforehand, and uncertainty about the sellers' credit and shipping safety.

In order to highlight the role of spatial barriers in the model where urban and rural markets are contained, we employ the barbell model to describe the co-existence of a high demand urban area and a low demand rural area. There are two distinctly asymmetric markets located at the opposite endpoints of the line segment with unit length, respectively.[3] No consumers are residing inside the line segment. Thus, we can denote the big market as the urban area and the small market as the rural area.

The existing literature can be categorized into two strands, viz industrial organization strand and location strand. The industrial organization strand contains the following papers. Balasubramanian (1998) utilizes a circular city model to study the price competition between the online and physical firms; Bouckaert (2000) analyzes free-entry competition between stores and mail order businesses, and finds that compared to Salop's model, fewer firms are active with free-entry. Liu *et al*. (2006) show that an incumbent physical retailer can deter the online entry of a pure-play e-tailer by strategically refraining from entering online. Loginova (2009) proves that under some parameter configurations, physical firms will raise their prices in response to the presence of online firms, and welfare goes down.

The location strand includes the following studies. Guo and Lai (2014) obtain that entry of an online firm will induce physical firms to move away from the opposite endpoints to locate inside the line segment in a Hotelling model with a quadratic transport cost function. Guo and Lai (2017) show that surviving physical firms eventually move toward densely populated urban area after the introduction of one online firm.

Increasing the number of firms through introducing one online firm, we obtain the following results. First, it may induce the two physical firms to switch their optimal locations either from agglomeration at the urban market to separate at the urban and rural markets, or from separation to agglomerate at the urban market. Second, the total sales may decline and the price level may rise in the urban market. Lastly, the welfare level may worsen.

The remainder of this paper is organized as follows. Section 2 sets up a basic model and examines the physical firms' location choices in the absence of an online firm. Section 3 introduces one online firm to compete with the physical firms, and explores the physical firms' location choices. Section 4 compares the optimal locations between the absence and presence of an online firm. Section 5 examines the effects of introducing one online firm on the welfare, output, and price level. The final section concludes the paper.

2. THE BASIC MODEL

Consider a spatial framework, in which there are two distinctly asymmetric markets, denoted by markets *A* and *B*, located at the opposite endpoints of the line segment with unit length, respectively. Assume that market *A* is the urban (big) market while market *B* is the rural (small) market. Initially, there are two physical firms, 1 and 2, who can locate at any point along a line segment as shown in Figure 1. The firms sell a homogeneous product to consumers residing only in either of the two markets and engage in Cournot competition. Firm 1 (2) is a more (less) efficient firm, whose marginal cost is assumed to be nil (a constant *c*). The locations of firms 1 and 2 are assumed to be x_1 and x_2 apart from the left endpoint, respectively, and $x_1 \leq x_2$.

[3] The barbell model was first proposed by Hwang and Mai (1990) and subsequently employed by Gross and Holahan (2003), Liang *et al*. (2006), Wang *et al*. (2016), *etc*.

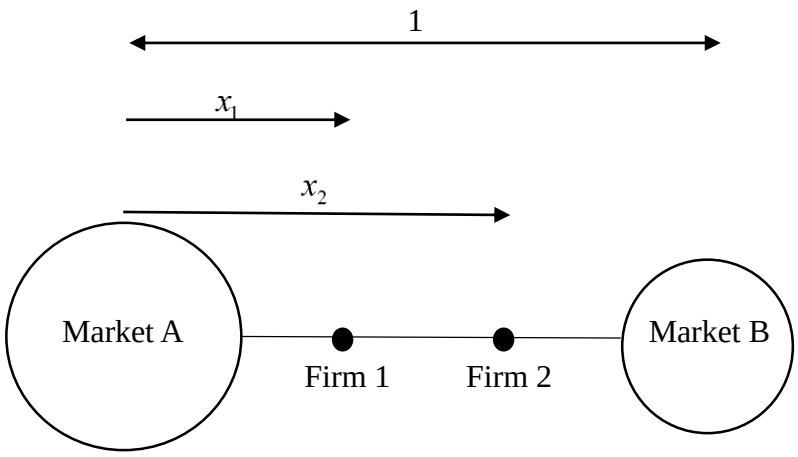

Figure 1. The spatial framework

The demand functions are expressed as follows:

$$q^A = \gamma(1 - p^A),$$
$$q^B = (1 - p^B). \tag{1}$$

where q^i, and p^i are the quantity demanded and delivered price in market i ($i = A, B$); and $\gamma > 1$ is a measure of relative market size where market A (B) represents the urban (rural) market.

The game in question is a two-stage game. In the first stage, the two physical firms simultaneously select their locations. In the second stage, the firms simultaneously choose their quantities. The sub-game perfect equilibrium of the model is solved by backward induction, beginning with the final stage.

In stage 2, each firm's profit can be defined as the sum of its profits from markets A and B as follows:

$$\pi_1^N = \left(p^A - tx_1\right)q_1^{AN} + \left(p^B - t|1 - x_1|\right)q_1^{BN}, \tag{2.1}$$

$$\pi_2^N = \left(p^A - c - tx_2\right)q_2^{AN} + \left(p^B - c - t|1 - x_2|\right)q_2^{BN}, \tag{2.2}$$

where the superscript "N" denotes variables associated with the absence of an online firm; q_i^j ($i = 1, 2$) are firm i's sales in market j ($j = A, B$); and t is the transport rate.

As the basic model is similar to that in Liang *et al*. (2006) except that firms' marginal costs are asymmetric, in what follows we shall jump to the analysis in stage 1 directly.

In stage 1, we can derive the reduced profit functions as follows:

$$\pi_1^N = \frac{\gamma}{9}\left[1 + c - t\left(2x_1 - x_2\right)\right]^2 + \frac{1}{9}\left[1 + c - t\left(1 - 2x_1 + x_2\right)\right]^2, \tag{3.1}$$

$$\pi_2^N = \frac{\gamma}{9}\left[1 - 2c - t\left(2x_2 - x_1\right)\right]^2 + \frac{1}{9}\left[1 - 2c - t\left(1 - 2x_2 + x_1\right)\right]^2. \tag{3.2}$$

By solving the first- and second-order conditions for optimal locations, we find that profit functions for firms 1 and 2 are all strictly convex with respect to location x_j ($j = 1, 2$), respectively, which implies a

corner solution. The Nash equilibrium locations can be derived by comparing the profits at the two corners, i.e., markets *A* and *B*.

Note that given the assumptions of $x_1 \geq x_2$, there exist three possible solutions: $(x_1, x_2) = (0, 0), (0, 1)$ and $(1, 1)$. However, the last can be ruled out as firm 1' profits are necessarily higher to locate at market *A* than at market *B*, should firm 2 choose to locate at market *B*. This is derivable by the following condition:

$$\pi_1^N(0,1) - \pi_1^N(1,1) = \frac{4t}{9}[(\gamma - 1)(1 + c) + t] > 0. \tag{4.1}$$

The solution can therefore be derived by comparing the profits of firm 2 at the two markets given firm 1 to stay at market A as:

$$\pi_2^N(0,0) - \pi_2^N(0,1) = \frac{4t\gamma}{9}\left\{\left(1 - \frac{1}{\gamma}\right) - 2c\left(1 - \frac{1}{\gamma}\right) - t\right\} > (<)0,$$
$$\text{if } \gamma > (<)\gamma^N = \left[1 + \frac{t}{1 - 2c - t}\right]. \tag{4.2}$$

Recall that $\gamma > 1$. Eq. (4.2) shows that the physical firms will agglomerate at market A when market A is sufficiently big and firm 2's marginal cost is sufficiently small, i.e., $\gamma > \gamma^N = \left[1 + \frac{t}{1 - 2c - t}\right]$, while they separate to locate at the opposite endpoints, respectively, otherwise.

The intuition behind this result can be stated as follows. The optimal locations of the firms are jointly determined by the following three effects: the market-size effect, the relatively efficient effect, and the competition effect, represented by the terms in the brace on the right-hand side of (4.2) in that order.

The market-size effect indicates that both firms tend to locate toward the larger market to scramble for the higher demand, which is positively related to the value of γ. The relatively efficient effect will force the less efficient firm to move away from its rival to reduce the rival's competitiveness at its home market due to the existence of spatial barriers. Thus, firm 2 tends to move toward market *B*. The larger *c* is, the stronger will be the relatively efficient effect. The competition effect will induce both firms to take apart to reduce the competition between firms due to the existence of spatial barriers. This effect becomes higher as the transport rate goes up.

By attributing to the above three effects, when market A is sufficiently big and firm 2's marginal cost is sufficiently small, i.e., $\gamma > \gamma^N = \left[1 + \frac{t}{1 - 2c - t}\right]$, the market-size effect outweighs the competition and the relatively efficient effects so that the firms will agglomerate at the larger urban market. On the contrary, if market A is not excessive large and firm 2's marginal cost is sufficiently large, i.e., $\gamma < \gamma^N = \left[1 + \frac{t}{1 - 2c - t}\right]$, such that the competition and the relatively efficient effects outweigh the market-size effect, then the firms will take apart and locate at the opposite endpoints of the line segment. We can thus establish:

Proposition 1. *Assuming that the physical firms engage in Cournot competition, they will agglomerate at the urban market when the urban market is sufficiently big and the less efficient firm's marginal cost is sufficiently small,* i.e., $\gamma > (1 - 2c)/(1 - t - 2c)$, *while taking apart to locate at different markets, respectively, otherwise*.

3. INTRODUCING ONE ONLINE FIRM

In this section, we examine the physical firms' optimal locations in response to the introduction of one online firm, denoted as firm *0*, into the basic model.

As we have explained in the Introduction section, the consumer's distaste cost from online purchase lowers the online firm's perceived demand through decreasing the consumers' willingness to pay for the product. Accordingly, the online firm's profit function can be expressed as:

$$\pi_0^O = \left(p^A - k\right)q_0^{AO} + \left(p^B - k\right)q_0^{BO}, \tag{5}$$

where the superscript "*O*" denotes variables associated with variables in the presence of an online firm; q_0^j denotes the online firm's sales at market *j* (*j* = *A*, *B*); and (p^j – *k*) represents the online firm's perceived demand in market *j*.

In stage 2, by solving the profit-maximizing conditions for each firm's outputs at the two markets, we obtain:

$$q_0^{AO} = \{\gamma[1-3k+c+t(x_1+x_2)]\}/4, \tag{6.1}$$

$$q_1^{AO} = \{\gamma[1+k+c-t(3x_1-x_2)]\}/4, \tag{6.2}$$

$$q_2^{AO} = \{\gamma[1+k-3c-t(3x_2-x_1)]\}/4, \tag{6.3}$$

$$q_0^{BO} = [1-3k+c+t(2-x_1-x_2)]/4, \tag{6.4}$$

$$q_1^{BO} = [1+k+c-t(2-3x_1+x_2)]/4, \tag{6.5}$$

$$q_2^{BO} = [1+k-3c-t(2-3x_2+x_1)]/4. \tag{6.6}$$

In stage 1, by substituting (6) into firms' profit functions, we derive the reduced profit functions as:

$$\pi_0^O = \frac{\gamma}{16}[1-3k+c+t(x_1+x_2)]^2 + \frac{1}{16}[1-3k+c+t(2-x_1-x_2)]^2, \tag{7.1}$$

$$\pi_1^O = \frac{\gamma}{16}[1+k+c-t(3x_1-x_2)]^2 + \frac{1}{16}[1+k+c-t(2-3x_1+x_2)]^2, \tag{7.2}$$

$$\pi_2^O = \frac{\gamma}{16}[1+k-3c-t(3x_2-x_1)]^2 + \frac{1}{16}[1+k-3c-t(2-3x_2+x_1)]^2. \tag{7.3}$$

By differentiating (7.2) and (7.3) with respect x_1 and x_2, respectively, we obtain:

$$\partial\pi_1^O/\partial x_1 = [3t(-q_1^{AO} + q_1^{BO})]/2, \tag{8.1}$$

$$\partial^2\pi_1^O/\partial x_1^2 = [9t^2(\gamma+1)]/8 > 0, \tag{8.2}$$

$$\partial\pi_2^O/\partial x_2 = [3t(-q_2^{AO} + q_2^{BO})]/2, \tag{8.3}$$

$$\partial^2\pi_2^O/\partial x_2^2 = [9t^2(\gamma+1)]/8 > 0. \tag{8.4}$$

Eq. (8) shows that profit functions for firms 1 and 2 are all strictly convex with respect to location x_j (*j* = 1, 2), respectively, which implies a corner solution. The possible solutions are: (x_1, x_2) = (0, 0), (0, 1) and (1, 1). Likewise, the last solution can be ruled out by the following condition:

$$\pi_1^O(0,1) - \pi_1^O(1,1) = \frac{3t}{16}\{[2(1+c+k)-t](\gamma-1)+2t\} > 0. \tag{9.1}$$

Similarly, the solution can be derived by comparing the profits of firm 2 at the two markets given firm 1 to stay at market *A* as:

$$\pi_2^O(0,0)-\pi_2^O(0,1)=\frac{3t}{16}\{2(\gamma-1)+2(\gamma-1)(k-3c)-3t(\gamma-1)-2t\}>(<)0,$$
$$if\ \gamma>(<)\gamma^O=\left[1+\frac{2t}{2(1+k-3c)-3t}\right]. \quad (9.2)$$

Recall that $\gamma > 1$ and eq. (6.3) that $q_2^{AO}(0,1)=\{\gamma[1+k-3c-3t]\}/4>0.$ It follows that the term $[2(1+k-3c)-3t]$ is positive. Eq. (9.2) shows that the physical firms will agglomerate at market *A*, when market *A* is sufficiently large, firm 2's marginal cost is sufficiently small, and the online firm's distaste cost is sufficiently high, i.e., $\gamma>\gamma^O=\left[1+\frac{2t}{2(1+k-3c)-3t}\right]$, while they separate to locate at the opposite endpoints, respectively, otherwise.

The intuition is as follows. The optimal locations of the firms are jointly determined by the following four effects: the market-size effect, the relatively efficient effect, the competition intensification effect, and the competition effect, denoted as the terms in the brace on the right-hand side of (9.2) in that order. The market-size, and the competition effects are the same as those defined in Section 2. Introducing one online firm creates an extra competition intensification effect and meanwhile modifies the relatively efficient effect compared with no online firm.

The competition intensification effect indicates that introducing one online firm will intensify the competition in the market through increasing the number of firms, which will force the less efficient firm to move farther away from its rival toward the small market.[4] In addition, introducing one online firm modifies the relatively efficient effect. The modification arises because a rise in the distaste cost *k* will attract firm 2 to move toward the big market due to firm 2 becoming relatively more efficient while a higher *c* will still induce firm 2 to move toward the small market.

Based on the above analysis, when market *A* is sufficiently big, the distaste cost is sufficiently high, and firm 2's marginal cost is sufficiently small, i.e., $\gamma>\gamma^O=\left[1+\frac{2t}{2(1+k-3c)-3t}\right]$, the sign of the relatively efficient effect becomes positive and meanwhile the market-size and the relatively efficient effects outweigh the competition and the competition intensification effects such that the firms will agglomerate at the urban market. On the contrary, when market *A* is not excessive big, the distaste cost is sufficiently small, and firm 2's marginal cost is sufficiently high, i.e., $\gamma<\gamma^O=\left[1+\frac{2t}{2(1+k-3c)-3t}\right]$, the sign of the relatively efficient effect becomes negative and meanwhile the competition, the relatively efficient, and the competition intensification effects dominate the market-size effect such that the two firms will take apart. Thus, we can establish the following proposition:

Proposition 2. *Introducing one online firm will cause the physical firms to agglomerate at the urban market when the urban market is sufficiently big, firm 2's marginal cost is sufficiently small, and the*

[4] Refer to Bakaouka and Milliou (2018, p. 75) for similar definition of the competition intensification effect.

online firm's distaste cost is sufficiently high, i.e., $\gamma > \gamma^O = \left[1 + \frac{2t}{2(1+k-3c)-3t}\right]$, *while they will take apart otherwise*.

Proposition 2 is different from the result in Guo and Lai (2017), in which entry of one online firm will cause the physical firms to move toward the urban area. This difference arises because firms undertake Bertrand competition in the Hotelling's linear city model in Guo and Lai (2017), while they engage in Cournot competition in a barbell model in this paper.

4. LOCATION CHOICES BEFORE AND AFTER INTRODUCING ONE ONLINE FIRM

In this section, we study the effect of introducing one online firm on physical firms' location choices by comparing their optimal locations between after and before introducing one online firm.

Recall that the critical values of the relative market size for the less efficient firm to locate at the urban market before and after introducing one online firm are γ^N and γ^O, respectively. In what follows we illustrate the loci of the relationships between the critical value γ^N and firm 2's marginal cost c as well as the critical value γ^O and c by use of Figures 2 and 3, in which the horizontal axis denotes firm 2's marginal cost and the vertical axis represents the critical values γ^i (i = N, O). We can figure out from (4.2) that curve γ^N is positive sloping and strictly convex,[5] and from (9.2) that curve γ^O is also positive sloping and strictly convex.[6] Moreover, we can calculate the difference in the intercepts between curves γ^N and γ^O by substituting c = 0 into γ^N and γ^O as:

$$\gamma^N(c=0) - \gamma^O(c=0) = \frac{t(2k-t)}{(1-t)[2(1+k)-3t]} > (<) 0, \text{ if } k > (<) \frac{t}{2}. \tag{10}$$

Eq. (10) shows that when the distaste cost is high, i.e., $k > t/2$, the intercept of curve γ^N is higher than that of curve γ^O. Meanwhile, we can derive that $\gamma^N = \gamma^O$, when $c = k - (t/2)$. Thus, given that the distaste cost is high, i.e., $k > t/2$, we can illustrate the loci of curves γ^N and γ^O by use of Figure 2 where γ^N is larger than, equal to or smaller than γ^O when c is smaller than, equal to or larger than $k - (t/2)$.

[5] By differentiating γ^N with respect to c, we obtain:
$\partial\gamma^N/\partial c = (2t)/\left[(1-t-2c)^2\right] > 0$, and $\partial^2\gamma^N/\partial c^2 = (8t)/\left[(1-t-2c)^3\right] > 0$.

[6] By differentiating γ^O with respect to c, we obtain:
$\partial\gamma^O/\partial c = (12t)/\left\{\left[2(1-3c+k)-3t\right]^2\right\} > 0$, and $\partial^2\gamma^O/\partial c^2 = (144t)/\left\{\left[2(1-3c+k)-3t\right]^3\right\} > 0$.

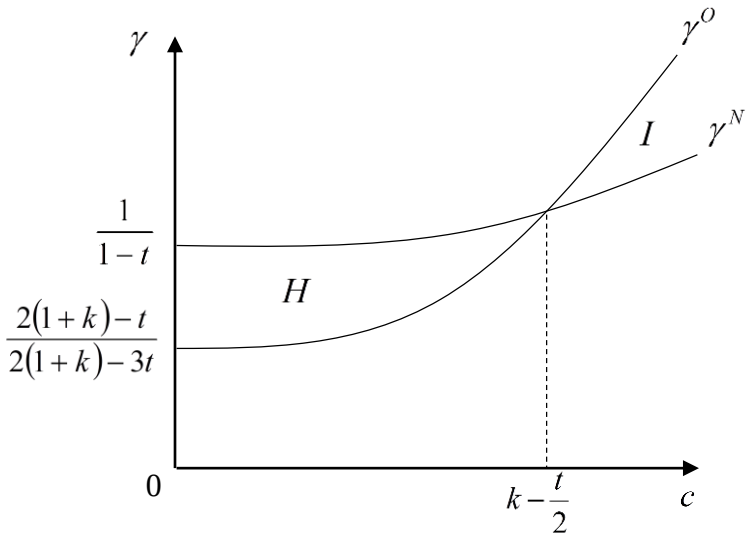

Figure 2. The loci of curves γ^N and γ^O in situation where $k > t/2$

In Figure 2, we find from Proposition 1 that the physical firms will agglomerate at the urban market in the area above curve γ^N, while taking apart in the area beneath curve γ^N. We also find from Proposition 2 that by introducing one online firm, the physical firms will agglomerate at the urban market in the area above curve γ^O, while taking apart in the area beneath curve γ^O. Next, provided that the distaste cost is high, i.e., $k > t/2$, when the relative market size is medium, i.e., $\gamma^O < \gamma < \gamma^N$, and firm 2's marginal cost is low, i.e., $c < k - \frac{t}{2}$, corresponding to area *H*, physical firms will choose to separate before introducing one online firm, while they will agglomerate at the urban market after introducing one online firm. Thus, introducing one online firm will induce the less efficient firm to move from the rural market to the urban market.

Intuitively, given a high distaste cost, when the relative market size is medium, the market-size effect is dominated by the relatively efficient and the competition effect such that firm 2 chooses to separate and locates at the rural market before introducing one online firm. Introducing one online firm creates an extra competition intensification effect and modifies the relatively efficient effect. The competition intensification effect will force the less efficient firm to move away from the urban market to reduce the competition. Next, the sign of the relatively efficient effect will become positive, which will induce the less efficient firm to move toward the urban market. As the distaste cost is high, the relatively efficient effect will outweigh the competition intensification effect so that introducing one online firm will induce the physical firms to move from separation to agglomerate at the urban market.

Based on the above analysis, we can obtain:

Proposition 3. *Provided that the distaste cost is high, when the relative market size is medium, and firm 2's marginal cost is low, corresponding to area H in Figure 2, introducing one online firm will induce the physical firms to move from separation to agglomerate at the urban market.*

Proposition 3 is significantly different from the result in Guo and Lai (2017), in which the online firm can only serve the rural region. By contrast, the online firm can serve both the urban and the rural regions in this paper.

Next, when market *A* is big, i.e., $\gamma^N < \gamma < \gamma^O$, and firm 2's marginal cost is sufficiently high, i.e., $c > k - \frac{t}{2}$, corresponding to area *I* in Figure 2, we find that the physical firms will agglomerate (separate) at the urban market before (after) introducing one online firm.

Intuitively, given these conditions, introducing one online firm will keep the sign of the relatively efficient effect negative such that the relatively efficient, the competition intensification, and the competition effects outweigh the market-size effect. Consequently, the physical firms will switch their optimal locations from agglomeration to take apart at different markets. Thus, we have:

Proposition 4. *Provided that the distaste cost is high, when the urban market is big, and firm 2's marginal cost is sufficiently high, corresponding to area I in Figure 2, introducing one online firm will induce the physical firms to switch their optimal locations from agglomeration to take apart at different markets*.

Proposition 4 is sharply different from the result in Guo and Lai (2017), in which all of the surviving physical firms will move toward the urban region. We can observe in the real world that for high distaste cost products, such as apparel, and jewelry, low-cost physical stores are located in the urban area while high-cost physical stores are in the rural area. Moreover, Proposition 4 is also different from the result in Liang *et al*. (2006) where both firms will agglomerate at the urban market when the urban market is sufficiently big.

We proceed to analyze the case where the distaste cost is small, i.e., $k < t/2$. We find from (10) that the intercept of curve γ^N is lower than that of curve γ^O. Meanwhile, we can derive that $\gamma^N = \gamma^O$ when $c = k - (t/2) < 0$. This implies that curves γ^N and γ^O can never cross each other and curve γ^O always lies above curve γ^N, when firm 2's marginal cost is greater than zero. Thus, the loci of curves γ^N and γ^O are shown Figure 3 where γ^N is always larger than γ^O.

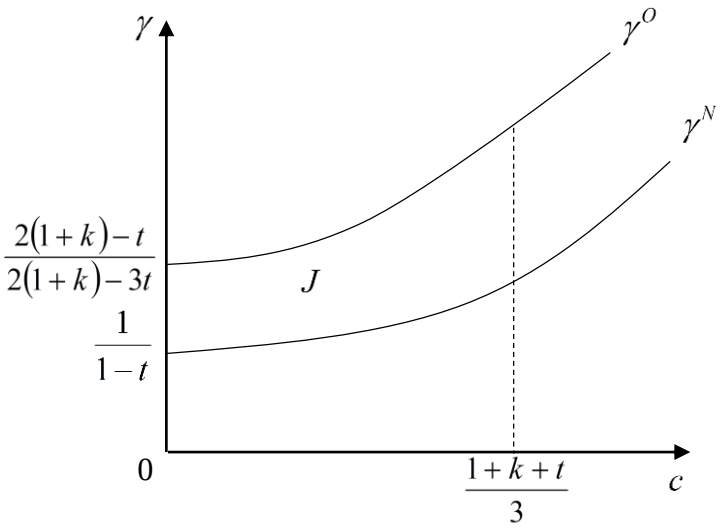

Figure 3. The loci of curves γ^N and γ^O in situation where $k < t/2$

Figure 3 shows that when market *A* is big, i.e., $\gamma^N < \gamma < \gamma^O$, and firm 2's marginal cost is low, i.e., $c < (1+k+t)/3$, corresponding to area *J* in Figure 3, the physical firms will agglomerate at the urban market in the absence of an online firm, while they will separate in the presence of an online firm.

The intuition behind this result is as follows. Introducing one online firm will cause the influence of the relatively efficient effect to become insignificant because both the distaste cost and firm 2's marginal cost are low. This will lead the competition intensification, and the competition effects to outweigh the

market-size effect so that introducing one online firm will induce the two physical firms to switch their optimal locations from agglomeration to take apart at different markets. Next, By substituting $(x_1, x_2) = (0,1)$ into (6.6), we obtain that $q_2^{BO} = (1-3c+k+t)/4 < 0, if\ c > (1+k+t)/3.$

Similarly, we find from (6.3) that $q_2^{AO} = \gamma(1-3c+k-3t)/4 < 0, if\ c > (1+k-3t)/3.$ Introducing one online firm will drive firm 2 out of the markets when firm 2's marginal is higher than $(1+k+t)/3$. Thus, we have the following proposition:

Proposition 5. *Provided that the distaste cost is low, when the urban market is medium, and firm 2's marginal cost is low, corresponding to area J in Figure 3, introducing one online firm will switch physical firms' locations from agglomeration to separation, while driving firm 2 out of the markets when firm2's marginal cost is higher than* $(1+k+t)/3$.

Proposition 5 is also significantly different from the results in Guo and Lai (2017) and in Liang *et al*. (2006).

5. THE EFFECTS OF INTRODUCING ONE ONLINE FIRM ON THE WELFARE, OUTPUT, AND PRICE

In this section, we explore the welfare, output, and price between after and before introducing one online firm. In order to discover the interesting outcomes that introducing one online firm may worsen the welfare level, decrease the total sales and raise the price level, in what follows we concentrate our analysis on the following two areas in Figure 2.

5.1. The area above regions I and H in Figure 2

This area denotes the case where the physical firms agglomerate at the urban market regardless of the presence of an online firm. We find from Figure 2 that the following conditions are contained, i.e., the distaste cost is high, i.e., $k > t/2$, and the relative market size is sufficiently big.

By subtracting (6) from the corresponding outputs in situation where an online firm is absent, we can then obtain the differences in outputs between after and before introducing one online firm as follows:

$$q^{AO^*} - q^{AN^*} = \frac{\gamma(1-3k+c)}{12} > 0 \text{ and }, q^{BO^*} - q^{BN^*} = \frac{1-3k+c+2t}{12} > 0 \qquad (11)$$

$$q_1^{BO^*} - q_1^{BN^*} = q_2^{BO^*} - q_2^{BN^*} = \frac{-(1-3k+c)-2t}{12} < 0, \text{ and}$$

$$q_1^{AO^*} - q_1^{AN*} = q_2^{AO*} - q_2^{AN*} = \frac{-\gamma(1-3k+c)}{12} < 0, \qquad (12)$$

where q^{jO^*} and q^{jN^*} (j = *A*, *B*) denote the total sales of market j in the presence and absence of an online firm, respectively.

Eq. (11) shows that the total sales in markets *A* and *B* after introducing one online firm are greater than those before introducing one online firm. This result occurs because the competition is intensified by increasing the number of firms. Next, by attributing to the business-stealing effect, a rise in the number of firms will decrease the incumbent firms' output.

We are now in a position to examine the difference in welfare levels between after and before introducing one online firm as follows:[7]

[7] We find from (13) that $31c^2 + 207k^2 + 7 - 90k - 162kc + 38c < 0$ requires $(7+31c)/69 < k$, and $31t + 31c + 19 - 81k < 0$ requires $[31(t+c)+19]/81 < k$. As $\{[31(t+c)+19]/81\} - \{(7+31c)/69\} = (248 - 124c + 713t)/1863 > 0$, it follows that only the condition, $[31(t+c)+19]/81 < k$, prevails. Next, we also find from (6.1) that $k < (1+c)/3$ is required to ensure that the sales of the online firm in market A are positive. By manipulating these two conditions, we obtain that the

$$SW^{O^*}(0,0) - SW^{N^*}(0,0) =$$

$$\left(\frac{1}{2}\right)\left\{\left(q^{AO^*} - q^{AN^*}\right)\left[2 - \frac{1}{\gamma}\left(q^{AO^*} + q^{AN^*}\right)\right] + \left(q^{BO^*} - q^{BN^*}\right)\left[2 - \left(q^{BO^*} + q^{BN^*}\right)\right]\right\}$$

$$-k\left(q_0^{AO^*} + q_0^{BO^*}\right) - \left[t\left(q_1^{BO^*} - q_1^{BN^*}\right) + c\left(q_2^{AO^*} - q_2^{AN^*}\right) + (c+t)\left(q_2^{BO^*} - q_2^{BN^*}\right)\right]$$

$$= \frac{1}{288}(1+\gamma)(31c^2 + 207k^2 + 7 - 90k - 162kc + 38c)$$

$$+\frac{1}{72}t(31t + 31c + 19 - 81k) < 0, \text{if } \frac{31(t+c)+19}{81} < k < \frac{1+c}{3} \text{ and } t < \frac{4(2-c)}{31}. \quad (13)$$

The social welfare consists of the consumer's surplus and the producer's surplus in two markets. The welfare difference can be rewritten as the terms on the right-hand side of the first equality in (13), in which the first term can be denoted as the competition intensification effect, the second term as the online firm's cost effect, and the third term as the business-stealing effect.

The competition intensification effect indicates that introducing one online firm increases the number of firms such that the competition is intensified. By (11), the competition intensification effect can make the welfare difference be positive by increasing the total sales in markets *A* and *B*. Next, the online firm's cost effect may lead the welfare difference to negative through adding the online firm's distaste cost. Finally, we find from (12) that each firm's output will be lower in the presence than that in the absence of one online firm. It follows that the business-stealing effect can result in a positive welfare difference via lowering the transportation and production costs. Accordingly, the welfare difference is determined by the balance of the above three effects.

We find from (13) that when the distaste cost is high and the transport rate is low, i.e., $k > \frac{31(t+c)+19}{81}$ and $t < \frac{4(2-c)}{31}$, the online firm's cost effect prevails while the business-stealing effect is weak.[8] Introducing one online firm will worsen the welfare level. This result occurs because the cost structure in the markets is deteriorated due to introducing one less efficient online firm. The same result can carry over to other areas in Figures 2 and 3. Thus, we can establish the following proposition:

Proposition 6. *Suppose that the physical firms agglomerate at the urban market, regardless of the presence of an online firm. Introducing one online firm will worsen the welfare level if the distaste cost is high and the transport rate is low, i.e.,* $k > \frac{31(t+c)+19}{81}$ *and* $t < \frac{4(2-c)}{31}$.

Proposition 6 is sharply different from the traditional wisdom, in which increasing the number of firms will improve the welfare level by intensifying the competition.

5.2. The area in region I of Figure 2

This area represents the case where the physical firms agglomerate in the absence while separate in the presence of an online firm. We find from Figure 2 that the following restrictions have to be fulfilled: $k > t/2$, $\gamma^N < \gamma < \gamma^O$, and $c > k - \frac{t}{2}$.

By subtracting (6) from the corresponding outputs in situation where an online firm is absent, we obtain the differences in outputs between the presence and absence of one online firm as follows: $q^{AO^*} - q^{AN^*} = \frac{\gamma(1-3k+c-3t)}{12} > (<)0$, and $q^{BO^*} - q^{BN^*} = \frac{1-3k+c+5t}{12} > 0$, (14)

$$q_1^{AO^*} - q_1^{AN*} = \frac{-\gamma(1-3k+c-3t)}{12} > (<)0,\ q_2^{AO^*} - q_2^{AN*} = \frac{-\gamma(1-3k+c+9t)}{12} < 0,$$

condition $t < [4(2-c)]/31$ is needed to ensure that $[31(t+c)+19]/81 < (1+c)/3$. Thus, we can conclude that the sign of (13) is negative, if $[31(t+c)+19]/81 < k < (1+c)/3$ and $t < [4(2-c)]/31$.

[8] Recall footnote 7. The condition, $k < (1+c)/3$, is required to ensure that the sales of the online firm in market A are positive.

$$q_1^{BO^*} - q_1^{BN^*} = \frac{-(1-3k+c)-5t}{12} < 0, \text{and } q_2^{BO^*} - q_2^{BN^*} = \frac{-(1-3k+c)+7t}{12} > (<)0. \quad (15)$$

Eq. (14) shows that the difference in the total sales of market *A* between after and before introducing one online firm is indeterminate. It may become negative if the transport rate is sufficiently high, say [$t > (1-3k+c)/3$]. This difference hinges upon the following two effects. First, a rise in the number of firms intensifies the competition and thus increases the total sales. Second, introducing one online firm will move the physical firm from agglomeration to separate, which will mitigate the competition in market *A* due to the existence of spatial barriers. Moreover, the higher the transport rate is, the greater the competition will be reduced. Thus, the difference in total sales of market *A* will become negative when the transport rate is sufficiently high such that the latter effect outweighs the former.

By use of (1), the difference in the prices of market *A* is opposite to the difference in the outputs, which can be expressed as $p^{AO*}-p^{AN*} = [-(q^{AO*} - q^{AN*})/\gamma]$. Thus, we obtain a striking result that increasing the number of firms by introducing one online firm may raise the price in the urban market, when the transport rate is sufficiently high.

Eq. (14) also shows that the difference in the total sales of market B between the presence and the absence of an online firm is definitely positive. The reason can be stated as follows. Recall that the physical firms agglomerate at market *A* in the absence while separate in the presence of an online firm. Introducing one online firm will move firm 2 to market *B* and then increasing its sales at market *B* through a lower transportation cost.

We find from (15) that the difference in firm 1's outputs of market *A* between the presence and the absence of an online firm is negative (positive) if the transport rate is low (high), while that in market *B* is definitely negative. Next, the difference in firm 2's outputs of market *B* between the presence and the absence of an online firm is negative (positive) if the transport rate is low (high), while that in market *A* is definitely negative. The same intuition as those in (14) applies to these results.

Based on the above analysis, we have:

Proposition 7. *Given* $k > t/2$, $\gamma^N < \gamma < \gamma^O$, *and* $c > k - \frac{t}{2}$ *such that introducing one online firm leads the physical firms from agglomeration to separation, which may raise the price and lower the sales in the urban market if the transport rate is sufficiently high*.

Proposition 7 is sharply different from the traditional wisdom, in which increasing the number of firms will decrease the price level but increase the total sales.

6. CONCLUDING REMARKS

We have introduced one online and two physical firms into the barbell model, in which the markets and the cost of the physical firms are asymmetric. The focus of this paper has been on the effects of introducing one online firm on the physical firms' location choices, outputs, price, and welfare levels under Cournot competition. We have derived several interesting results as follows.

First, provided that the distaste cost is large, i.e., $k > t/2$, introducing one online firm will induce the two physical firms to switch their optimal locations from separation to agglomerate at the urban market when the urban market is medium, i.e., $\gamma^O < \gamma < \gamma^N$, and the less efficient firm's marginal cost is low, i.e., $c < k - \frac{t}{2}$, while they will move from agglomeration to take apart at different markets when the less efficient firm's marginal cost is sufficiently high, i.e., $c > k - \frac{t}{2}$.

Next, given a low distaste cost, i.e., $k < t/2$, introducing an online firm will induce the physical firms to move from agglomeration to take apart at different markets when the urban market is large, i.e., $\gamma^N < \gamma < \gamma^O$, and the less efficient firm's marginal cost is low, i.e., $c < (1+k+t)/3$, while it will drive the less efficient firm out of the markets when the less efficient firm's marginal cost is higher than $(1+k+t)/3$.

Lastly, suppose that the physical firms agglomerate in the absence while separate in the presence of an online firm. Increasing the number of firms by adding an online firm may raise the price and lower the sales in market A if the transport rate is sufficiently high, while it may increase firm 1's output in market A and firm 2's output in market B if the transport rate is high. Moreover, introducing a less efficient online firm may worsen the welfare level by worsening the cost structure, if the distaste cost is high.

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